

# Sensitivity of System Reliability to Usage Profile Changes

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## ABSTRACT

Usage profiles and component reliability are two important factors in software system reliability estimation. To assess the sensitivity of a system's reliability to the usage profile and to the reliability of its components, a Markov based system model is used. With the help of this model, the maximum sensitivity to one change or the statistical sensitivity to many independent changes can be estimated. Advantages and limitations to this approach are discussed and finally the theory is applied to an example to show its validity.

## Keywords

software reliability, sensitivity analysis, usage profile, Markov model

## 1. INTRODUCTION

Software plays an increasingly important role in today's society. Reliable software is a prerequisite for most functions in our daily life. Power supply, banking, transportation, medical care etc. depend on reliable software. Extreme events, like bad weather conditions, terror threats or diseases, may stress the society, and hence the software systems. The current status of software reliability assessments is far from good [16]; the control over the software reliability under extreme conditions is even worse. To our knowledge, the issue of assessing the software reliability for extreme conditions, is not explored to any larger extent.

The software reliability depends not only on the defects, residing in the software systems, but also on how the software is used, i.e. the usage profile [10]. E.g. a contributing factor to the power blackout in North America, August 2003, was a software failure in an alarm system [15]. The "bug was triggered by a unique combination of events and alarm conditions on the equipment it was monitoring". As software reliability assessments primarily are based on the normal usage profile, extreme events are not taken into account.

In this paper, we present an initial quantitative study on the sensitivity of the reliability estimate to changes in the usage profile. The long term goal is to assess software reliability risks with respect to extreme usage conditions.

The paper is outlined as follows: after the discussion of some related work in Section 2, Section 3 discusses the theory and in Section 4 the theory is applied and checked to an example from [14].

## 2. RELATED WORK

### 2.1 Usage Profiles and Reliability

The usage profile, or the operational profile, is the characterization of the users' operative utilization of a software system. The usage is characterized in terms of the user-initiated events and the probability for these events [9]. The usage profile is mirrored in the utilization of the software and its components. Thus, a state-based model of the software may be developed, with the probabilities from the usage profile, determining the transition probabilities of the system model. The states in the model may represent either important states in the operation of the system or system components between which controlled is passed. This approach is proposed by Cheung [2], later refined by Siegrist [17] and used by Poore et al [14].

In order to use the system model for reliability estimation, an explicit failure state has to be added. The direct unreliability is then expressed as the probability for a failure event in each system model state or system component. The reliability of the entire system, for a given usage profile, can then be calculated from the system model [2].

This model can be used to model all kind of systems where a failure and a success state are present, and where the reliability can be defined as the probability to terminate in the success state. An example of such a system could be a server where users log in for a session to execute a number of transactions and to finally conclude with a successful termination of the session.

Systems without a terminal success state where the reliability is measured as the mean time to failure (such as most monitoring systems), are discussed in [18], and a sensitivity analysis of these systems will be the topic of future work.

## 2.2 Sensitivity Analysis

As the software reliability depends on the usage profile, the question arises how sensitive the reliability estimate is to changes or uncertainties in the usage profile. The question has been addressed from two different perspectives; 1) based on software reliability growth models (SRGM), and 2) based on Markov models.

In the first approach, the estimated parameters of the SRGM are adjusted, based on random changes in the operational profile. Musa analyses the relative error in failure intensity and the relative error in the operational profile for a single operation [11]. Chen et al. investigate the sensitivity of the reliability estimates to errors in the operational profile with simulation [1]. A single error is injected in the operational profile and the effects in the reliability estimates are investigated. Crespo, Matrella and Pasquini analyse the predicted reliability growth for different operational profiles [3, 13]. The real reliability growth is calculated with the Nelson reliability model [12, 4]. Wesslén, Runeson and Regnell model the usage with a Markov model and simulates the impact on the Nelson reliability estimate, based on multiple random changes in the usage profile [19].

The second approach uses a Markov chain to model the system and its usage. Thus, Poore et al analyze the sensitivity of a system's reliability to the reliability of its components [14]. This approach is extended by Yacoub et al. [20]. Lo et al present an analytic approach, also based on a component model, which identifies the most sensitive parameter, component reliability and transition flow [8].

In [5] and [6] Goševa-Popstojanova et al discuss two different methods for sensitivity analysis based on Markov models: the method of moments and Monte Carlo simulation. Though both methods give very good results for small systems, they require a huge amount of calculation and don't scale up well for large systems. They also don't provide a good way to get a quick estimate without lengthy calculations or simulations.

## 3. SENSITIVITY ANALYSIS

In this section we will first shortly repeat some theoretical basics from [17] in Subsection 3.1, then the maximal theoretical sensitivity of the reliability estimate to one change in the operational profile is discussed in Subsection 3.2. Next the statistical sensitivity to many random changes is discussed in Subsection 3.3 and finally some limitations of the theory are further examined in Subsection 3.4.

### 3.1 Definitions

In this paper we will use the Markov model as proposed in [17] to model a system's usage, behaviour and reliability. The states of the model can either represent system states or system components between which control is passed, the following analysis can be used in both cases.

In the model used here, there are two main assumptions. First, the Markov property means that the future behaviour of the system is determined only by the current state of the system and not by the history of the system. Secondly, this model assumes that the system contains exactly two terminal states: a success state  $t$  and a failure state  $f$ . This

means that a run of the system will always terminate in one of these two states. Next to the two terminal states, the system also contains  $n$  transient states  $1, 2, \dots, n$ . State 1 represents the initial state.

The dynamics of the faultless system, without the failure state, are described by a Markov chain with state space  $1, 2, \dots, n, t$ , and with transition matrix  $P$ , where  $p_{ij}$  is the probability to go from state  $i$  directly to state  $j$ .

In the imperfect system, every state has a designated reliability  $r_i$ , which means it has a probability  $1 - r_i$  of failing and entering the failure state  $f$ . The dynamics of the faulty system are described by a Markov chain with state space  $1, 2, \dots, n, t, f$  and with transition matrix  $\hat{P}$ , given as follows:

$$\begin{aligned} \hat{p}_{ij} &= p_{ij} \times r_i && \text{for } i = 1, \dots, n \text{ and} \\ &&& \text{for } j = 1, \dots, n, t \\ \hat{p}_{if} &= 1 - r_i && \text{for } i = 1, \dots, n \\ \hat{p}_{tt} &= \hat{p}_{ff} = 1 \\ \hat{p}_{ti} &= 0 \text{ for } j = 1, \dots, n, f \\ \hat{p}_{fi} &= 0 \text{ for } j = 1, \dots, n, t \end{aligned}$$

The method for computing system reliability from these transition probabilities is based on standard Markov chain theory [7].

Let  $\hat{Q}$  denote the restriction of the transition matrix  $\hat{P}$  to the transient states  $1, 2, \dots, n$ , so the transition matrix  $\hat{P}$  without the last two rows and without the last two columns. Then the matrix

$$V = \sum_{k=0}^{\infty} \hat{Q}^k = (I - \hat{Q})^{-1} \quad (1)$$

is called the potential matrix of the system. Each value  $v_{ij}$  gives the number of expected visits to state  $j$  before terminating when the system is currently in state  $i$ . Since state 1 is the starting state of the system, the system's expected number of transition periods before terminating is the sum of the elements of the first row of the matrix  $V$ .

$$\text{expected \# of periods} = \text{sum}(V_1) = \sum_{j=0}^n V_{1j} \quad (2)$$

This expected number of periods can also be seen as the expected number of events causing a state change in the system during one run of the system from start to either successful termination or failure. The events can be either user actions or internal system events and are defined together with the states in the system model.

Let

$$T = \begin{pmatrix} t_1 \\ \vdots \\ t_n \end{pmatrix} = \begin{pmatrix} \hat{p}_{1t} \\ \vdots \\ \hat{p}_{nt} \end{pmatrix} \quad \text{and} \\ F = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} = \begin{pmatrix} \hat{p}_{1f} \\ \vdots \\ \hat{p}_{nf} \end{pmatrix}$$

be the column vectors containing the probabilities to go directly from a given state to state  $t$  or  $f$  respectively. Then the probability  $s_i$  to finally end up in terminal state  $t$  given that the systems is currently in state  $i$ , or in other words the overall chance on success starting from state  $i$ , can be calculated as follows:

$$S = \begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} = V \times T = (I - \hat{Q})^{-1} \times T \quad (3)$$

In a similar way the chance  $x_i$  to finally end up in terminal state  $f$  given that the systems is currently in state  $i$ , can be calculated as follows:

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = V \times F = (I - \hat{Q})^{-1} \times F \quad (4)$$

Since the system must always terminate in one of the two terminal states

$$s_i = 1 - x_i \quad \text{for} \quad i = 1, \dots, n \quad (5)$$

Because we assume that state 1 is the initial state of the system, the reliability of the whole system is simply

$$s_1 = \sum_{i=1}^n v_{1i} \times t_i = V_1 \times T \quad (6)$$

with  $V_1$  the first row of the matrix  $V$ . In [17] it is also shown that the sensitivity of the system's reliability to the reliability  $r_i$  of a state  $i$  is bounded by the expected number of visits to state  $i$  in the faultless system starting from state 1, which can also be written as

$$(I - Q)_{1i}^{-1}$$

where  $Q$  denotes the restriction of the transition matrix  $P$  of the faultless system to the transient states  $1, 2, \dots, n$ .

In the following sections we will build further upon the theory from [17] to investigate much further how changes in the transition matrix  $\hat{P}$  influence the total reliability of the system.

### 3.2 Maximum Sensitivity

In the previous section we saw how we can calculate the reliability of a system from its state transition matrix  $\hat{P}$ . For some systems, small changes in the matrix  $\hat{P}$  can cause relatively big changes in the reliability of the whole system. In this section we will look at how to calculate the maximum effect of one or more small changes.

To make a distinction between the old system without the small changes and the new system with the small changes, we will use an accent to indicate the variables of the new system. So we investigate the difference between the reliability  $s_1$  of the system with transition matrix  $\hat{P}$  and the reliability  $s'_1$  of the system with transition matrix  $\hat{P}'$

Let  $\delta_{ij} = \hat{p}'_{ij} - \hat{p}_{ij}$  denote the change to  $\hat{p}_{ij}$  and let  $\delta_{Rel} = s'_1 - s_1$  denote the resulting change in the total reliability of the system.

First of all, it's important to notice that it is impossible to change only one probability  $\hat{p}_{ij}$  in the matrix since the sum of the probabilities of one row has to equal 1.

Therefore, the most simple change we can make to the system is to only change one  $\hat{p}_{kt}$  and the corresponding  $\hat{p}_{kf}$ , or in other words, to only change the probabilities of immediate success and failure. This does not change the dynamic properties of the system, only the terminating probabilities.

$$\delta_{kt} = t'_k - t_k = f_k - f'_k = -\delta_{kf} \quad (7)$$

With this change, we do not change the total probability to terminate from any state, just the probability to terminate to the two terminal states. Since this change has no effect on the matrix  $\hat{Q}$  and therefore also not on  $V$  as defined in equation (1). From the equations (6) and (7) it is immediately clear then that

$$\delta_{Rel} = v_{k1} \times \delta_{kt} \quad (8)$$

This means that the reliability changes linearly with the size of the initial change, where the slope is determined by  $v_{i1}$ , the expected number of visits to state  $i$ . This result is very logical, since because of the change we made, for every visit to the state  $i$  we have an extra  $\delta_{it}$  more chance to terminate to state  $t$ .

The situation is more complex when we make changes to the matrix  $\hat{Q}$ . Let's first assume that we only make a change to the transition probabilities  $\hat{p}_{kl}$  and  $\hat{p}_{kf}$ . This means we only change the matrix  $\hat{Q}$  in one location.

$$\delta_{kl} = \hat{p}'_{kl} - \hat{p}_{kl} = f_k - f'_k = -\delta_{kf} \quad (9)$$

It can be easily checked that the matrix  $V$  will then change in the following way:

$$v'_{ij} - v_{ij} = \frac{\delta_{kl} \times v_{ik} \times v_{lj}}{1 - \delta_{kl} \times v_{lk}} \quad (10)$$

Therefore the change in the reliability of the whole system is exactly

$$\delta_{Rel} = \sum_{j=1}^n \frac{\delta_{kl} \times v_{1k} \times v_{lj} \times t_j}{1 - \delta_{kl} \times v_{lk}} = \delta_{kl} \times v'_{1k} \times s_l \quad (11)$$

In a similar way we can deduce that when we change the transition probabilities  $\hat{p}_{kl}$  and  $\hat{p}_{kt}$ , the reliability of the entire system is affected by an amount.

$$\delta_{Rel} = -\delta_{kl} \times v'_{1k} \times (1 - s_l) \quad (12)$$

For small changes these results can be approximated and simplified by replacing  $v'_{1k}$  by the old value  $v_{1k}$ .

This last change discussed does no longer represent a simple change in the reliability of one state or of one transition but a real change in the usage profile of the system. An even more interesting change in the usage profile occurs when we make two changes within the matrix  $\hat{Q}$ , to the transition probabilities  $\hat{p}_{kl}$  and  $\hat{p}_{km}$ ,

$$\delta_{kl} = \hat{p}'_{kl} - \hat{p}_{kl} = \hat{p}_{km} - \hat{p}'_{km} = -\delta_{km} \quad (13)$$

then the formulas become even longer, but for small delta the resulting change in system reliability can be very well approximated by

$$\delta_{Rel} = \delta_{kl} \times v_{1k} \times (s_l - s_m) \quad (14)$$

Or in other words, when changing the transition probabilities from a state  $k$  to the states  $l$  and  $m$ , the resulting change in reliability can be approximated by the product of the size of the original change to the transition probabilities, multiplied by the number of expected visits to state  $k$ , multiplied by the difference between the chance of overall success starting from the states  $l$  and  $m$ . This is equal to the results found in formulas (11) and (12), when we consider that the chance of overall success starting from the states  $f$  and  $t$  is respectively 0 and 1.

This also means that the reliability is most sensitive to changes in the transition probability  $\hat{p}_{kl}$ , from the state  $k$  that has the higher number of expected visits, to the state  $l$  with the highest total chance of leading to success.

### 3.3 Statistical Sensitivity

In this section we look for the effect of a large number of small changes or uncertainties in the usage profile on the system's overall reliability. To model these changes we will assume that every transition probability  $\hat{p}_{ij}$  in the system changes with a  $\delta_{ij}$ . Further we will assume that all the  $\delta_{ij}$  have the same distribution with mean 0 and variation  $\text{Var}_\delta$ .

$$E(\delta_{ij}) = 0 \quad \text{Var}(\delta_{ij}) = \text{Var}_\delta \quad (15)$$

Because we can not change one transition probability alone, for every change  $\delta_{ij}$  we will select a random transition probability  $\hat{p}_{ik}$  on row  $i$  that will undergo the opposite change  $\delta_{ik} = -\delta_{ij}$ . This will guarantee us that the sum of the transition probabilities from each state is always equal to 1.

From formula 14 we know that the resulting change in reliability will have a distribution with the following mean and variance:

$$E(\delta_{Rel}) = E(\delta_{ij}) \times v_{1i} \times (s_j - s_k) = 0 \quad (16)$$

$$\text{Var}(\delta_{Rel}) = \text{Var}(Rel) = \text{Var}_\delta \times v_{1i}^2 \times (s_j - s_k)^2 \quad (17)$$

Let  $\text{Var}_{diffS}$  be the variation of the difference between the total chance to reach terminal state  $t$  for two randomly selected states

$$\text{Var}_{diffS} = \text{Var}(s_i - s_j), \quad i, j = 1, \dots, n \quad (18)$$

Then the total change in system reliability resulting from randomly changing all the elements  $\hat{p}_{ij}$  as described above, will be the sum of all the changes resulting from  $n$  changes on each of the  $n$  rows. And therefore the total change in reliability will have a distribution with the following variance:

$$\begin{aligned} \text{Var}(Rel) &= \sum_{i=1}^n n \times \text{Var}_\delta \times \text{Var}_{diffS} \times v_{1i}^2 \\ &= n \times \text{Var}_\delta \times \text{Var}_{diffS} \times \|V_1\|^2 \end{aligned} \quad (19)$$

Because the total change in reliability is the sum of a large number independent changes, the distribution will be close to a normal distribution with mean 0 and a variance as defined in equation (19). Thus, for the standard deviation  $\sigma(Rel)$  of the reliability the following holds:

$$\sum V_1 \leq \frac{\sigma(Rel)}{\sigma_\delta \times \sigma_{diffS}} \leq n \times \max(V_1) \quad (20)$$

In other words, the standard deviation of the total system reliability under small changes divided by the size of the changes and divided by the standard deviation of the difference between the total chance of success between two random states, lies between the total number of expected periods of the system and  $n$  times the maximal number of expected visits to any state.

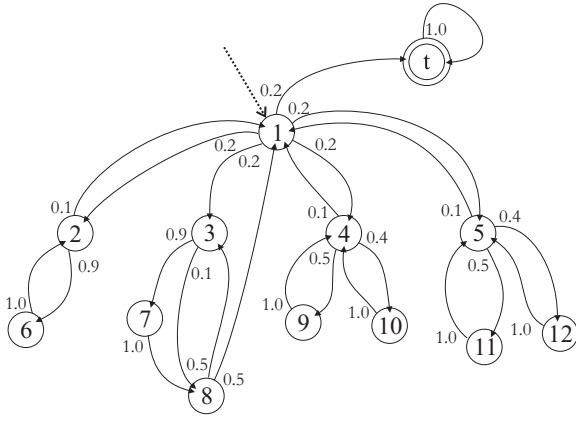
Equation 19 also shows that to decrease the sensitivity of the reliability to usage profile changes, it is important to decrease the differences between the overall success rates of the different states.

When we also want to account for random changes in the transition probabilities in the vectors  $T$  and  $F$ , equation (18) for  $\text{Var}_{diffS}$  can simply be extended to include the states  $t$  and  $f$  (in either the index  $i$  or  $j$ , but not in both) with  $s_t = 1$  and  $s_f = 0$ . This will of course seriously increase  $\text{Var}_{diffS}$  and therefore also the variation on the reliability.

### 3.4 Limitations

It is important to understand that equation (19) only holds under a number of assumptions. First of all the changes to the system's transition probabilities have to be sufficiently small, for equation 14 to be a good approximation of their effect on the system's reliability. When the changes become too large the structural changes in the system model will become more important and influence the reliability of the system.

Secondly this model also assumes that positive and negative changes are equally likely for each of the transition probabilities. In practical examples this is rarely true. For example, most systems will contain a large number of transition probabilities equal to zero to indicate impossible transitions. The error on this transition probability can only be negative or zero. When we apply random changes to a system with many zeroes in its transition matrix, but without altering the zeroes, then the expected change of the reliability will no longer be zero but be biased towards a negative or positive change depending on the structure of the system. And also the estimate of the variation from equation (19) will be



**Figure 1: Example system.** Each directed arc, labelled with its transition probability, indicates that control passes from one component to another in the direction of the arrow.

less exact. However, in most cases, it will still be a good approximation of the variation of the reliability or at least a good indication of the magnitude of the expected reliability change.

#### 4. EXAMPLE SYSTEM

In this section we will apply the theory from Section 3 to an example system from Poore et al. [14]. The transition graph of the faultless system can be seen in Figure 1. The transition matrix and different statistics of the different states can be found in Tables 1 and 2.

The reliability of the whole system is  $s(1) = 0.9899$  and the expected number of periods is  $\sum v_{1i} = 67.056$ .

##### 4.1 Maximum sensitivity

From Table 2 we can see that the state with the most expected visits is state 5, closely followed by the states 2 and 4, the least visited state is state 7. This means the system is most sensitive to changes to the transition probabilities from these states. The state with the highest overall chance on success is state 1, the state with the lowest overall chance on success is state 3.

In Table 3 we can see the real effect of some transition probability changes to the overall reliability. We see that the predicted value corresponds well to the real value in all cases. For the first example the predicted value is exact, since there are no changes to the dynamic properties of the system. For the second example we see a very large change with also an error on the prediction of about 10%. This can be explained because the large change to the dynamics of the system that this change causes. A change of 0.01 is a big change for a value of  $\hat{p}_{51} = 0.10$ .

Overall we can see that the maximum change in reliability is about 9 times bigger than the original change to the transition probabilities. But when we do not change the values  $r(i)$  and only make changes inside the matrix  $\hat{Q}$ , then the resulting change in reliability is only 2% of the original change to the transition probabilities because of the small

differences between the overall success rates of the different states. And in that case the difference between the predicted and the real value is a lot smaller, only for very big changes the predicted value deviates a little from the real value.

##### 4.2 Statistical Sensitivity

To check the statistical sensitivity of the system's reliability to many small changes in the transition probabilities, we will conduct a Monte Carlo simulation where we make a small random change to each of the 144 transition probabilities in the matrix  $\hat{Q}$ , as described in Section 3.3. To predict the effect on the system's reliability we first calculate

$$\text{Var}_{diff} = \text{Var}(s_i - s_j) = 5.8610^{-4}$$

and

$$\|V_1\| = 22.10 .$$

With equation (19) we can now predict the standard deviation of the systems reliability for 144 random changes with  $\sigma_\delta = 0.005$ :

$$\sigma(Rel) = 2,24 \times 10^{-4}.$$

The resulting reliability change from 1000 random simulations, plotted on normal probability paper, can be seen in Figure 2. As you can see the results of the experiments fit extremely well with the predicted changes indicated by the dashed line. The measured standard deviation equals  $2.32 \times 10^{-4}$  while the measure average equals  $1.09 \times 10^{-6}$ , which is very close to the predicted 0.

##### 4.3 Limitations

As we discussed in Section 3.4, the formulas that predict the statistical change in reliability become less precise when the  $\sigma_\delta$  increases. For example in Figure 3 we can see what happens when we increase  $\sigma_\delta$  to 0.01. For 90% of the experiments, our predictions are still quite accurate, but we also notice a large group of outliers where the random changes have a large influence on the systems dynamic behaviour with a large change in the reliability as a consequence. Of course the number of outliers will increase even more as  $\sigma_\delta$  increases.

Also, it is important to notice that in the simulation in Figure 1 both positive and negative changes were allowed to all transition probabilities. This means that the changed transition probability matrix  $\hat{P}'$  will contain some slightly negative elements which is practically impossible. When we change the simulation and only allow positive changes to the zeros in the transition probability matrix we get the results shown in Figure 4. It is immediately clear that the average change in reliability is no longer 0, almost in all the experiments the change in reliability is negative. From 1000 random experiments, we find that the average change in reliability is  $-5.76 \times 10^{-4}$ . This is due to the fact that forcing positive changes onto the zeros causes automatic negative changes to the positive values already present in the transition probability matrix. Column 1 of this matrix contains the most non-zero elements and will therefore be the most affected by these negative changes. Unfortunately state 1 is the state with the highest overall success rate. Therefore those negative changes in column 1 will have a negative effect on the reliability. From the data in Figure 4 we find an

|           |          |          |          |          |          |          |          |          |          |           |           |           |          |
|-----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|-----------|-----------|----------|
|           | <i>1</i> | <i>2</i> | <i>3</i> | <i>4</i> | <i>5</i> | <i>6</i> | <i>7</i> | <i>8</i> | <i>9</i> | <i>10</i> | <i>11</i> | <i>12</i> | <i>t</i> |
| <i>1</i>  | 0        | 0.2      | 0.2      | 0.2      | 0.2      | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0.2      |
| <i>2</i>  | 0.1      | 0        | 0        | 0        | 0        | 0.9      | 0        | 0        | 0        | 0         | 0         | 0         | 0        |
| <i>3</i>  | 0        | 0        | 0        | 0        | 0        | 0        | 0.9      | 0.1      | 0        | 0         | 0         | 0         | 0        |
| <i>4</i>  | 0.1      | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0.5      | 0.4       | 0         | 0         | 0        |
| <i>5</i>  | 0.1      | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0         | 0.5       | 0.4       | 0        |
| <i>6</i>  | 0        | 1.0      | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0        |
| <i>7</i>  | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 1.0      | 0        | 0         | 0         | 0         | 0        |
| <i>8</i>  | 0.5      | 0        | 0.5      | 0        | 0        | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0        |
| <i>9</i>  | 0        | 0        | 0        | 1.0      | 0        | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0        |
| <i>10</i> | 0        | 0        | 0        | 1.0      | 0        | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0        |
| <i>11</i> | 0        | 0        | 0        | 0        | 1.0      | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0        |
| <i>12</i> | 0        | 0        | 0        | 0        | 1.0      | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0        |
| <i>t</i>  | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 1        |

Table 1: The transition matrix  $\mathbf{P}$  of the faultless example system

| <i>i</i>  | Reliability $r_i$ | Expected number of visits $v_{1i}$ | Overall chance on success $s_i$ | $(s_1 - s_i)/10^{-3}$ |
|-----------|-------------------|------------------------------------|---------------------------------|-----------------------|
| <i>1</i>  | 0.9990            | 4.9546                             | 0.9899                          | 0                     |
| <i>2</i>  | 0.9999            | 9.8895                             | 0.9888                          | 1.0779                |
| <i>3</i>  | 0.9990            | 1.9778                             | 0.9879                          | 2.0154                |
| <i>4</i>  | 0.9999            | 9.8899                             | 0.9889                          | 1.0384                |
| <i>5</i>  | 0.9999            | 9.8903                             | 0.9889                          | 0.9890                |
| <i>6</i>  | 1                 | 8.8996                             | 0.9888                          | 1.0878                |
| <i>7</i>  | 1                 | 1.7783                             | 0.9889                          | 1.0275                |
| <i>8</i>  | 1                 | 1.9758                             | 0.9889                          | 1.0176                |
| <i>9</i>  | 1                 | 4.9444                             | 0.9889                          | 1.0483                |
| <i>10</i> | 1                 | 3.9555                             | 0.9889                          | 1.0384                |
| <i>11</i> | 1                 | 4.9447                             | 0.9889                          | 0.9890                |
| <i>12</i> | 1                 | 3.9557                             | 0.9889                          | 0.9890                |

Table 2: Values of the state reliability  $r_i$ , the number of expected visits  $v_{1i}$ , the overall chance of success  $s_i$  and the difference between the overall chances of success  $(s_1 - s_i)/10^{-3}$  for the different states

| $\hat{p}_{kl} \xrightarrow{-\delta_{kl} = \delta_{km}} \hat{p}_{km}$ | real $\delta_{Rel}/10^{-3}$ | predicted $\delta_{Rel}/10^{-3}$ |
|--|-----------------------------|----------------------------------|
| $\hat{p}_{1t} \xrightarrow{0.01} \hat{p}_{1f}$                       | -49.5                       | -49.5                            |
| $\hat{p}_{51} \xrightarrow{0.01} \hat{p}_{5f}$                       | -89.1                       | -97.9                            |
| $\hat{p}_{13} \xrightarrow{0.01} \hat{p}_{11}$                       | 0.100                       | 0.100                            |
| $\hat{p}_{78} \xrightarrow{0.01} \hat{p}_{73}$                       | -0.018                      | -0.018                           |
| $\hat{p}_{51} \xrightarrow{0.01} \hat{p}_{53}$                       | -0.199                      | -0.199                           |
| $\hat{p}_{51} \xrightarrow{0.05} \hat{p}_{53}$                       | -0.996                      | -0.997                           |
| $\hat{p}_{51} \xrightarrow{0.001} \hat{p}_{53}$                      | -0.020                      | -0.020                           |

Table 3: Some real values compared with some calculated values for the change in reliability for different changes in the transition probabilities



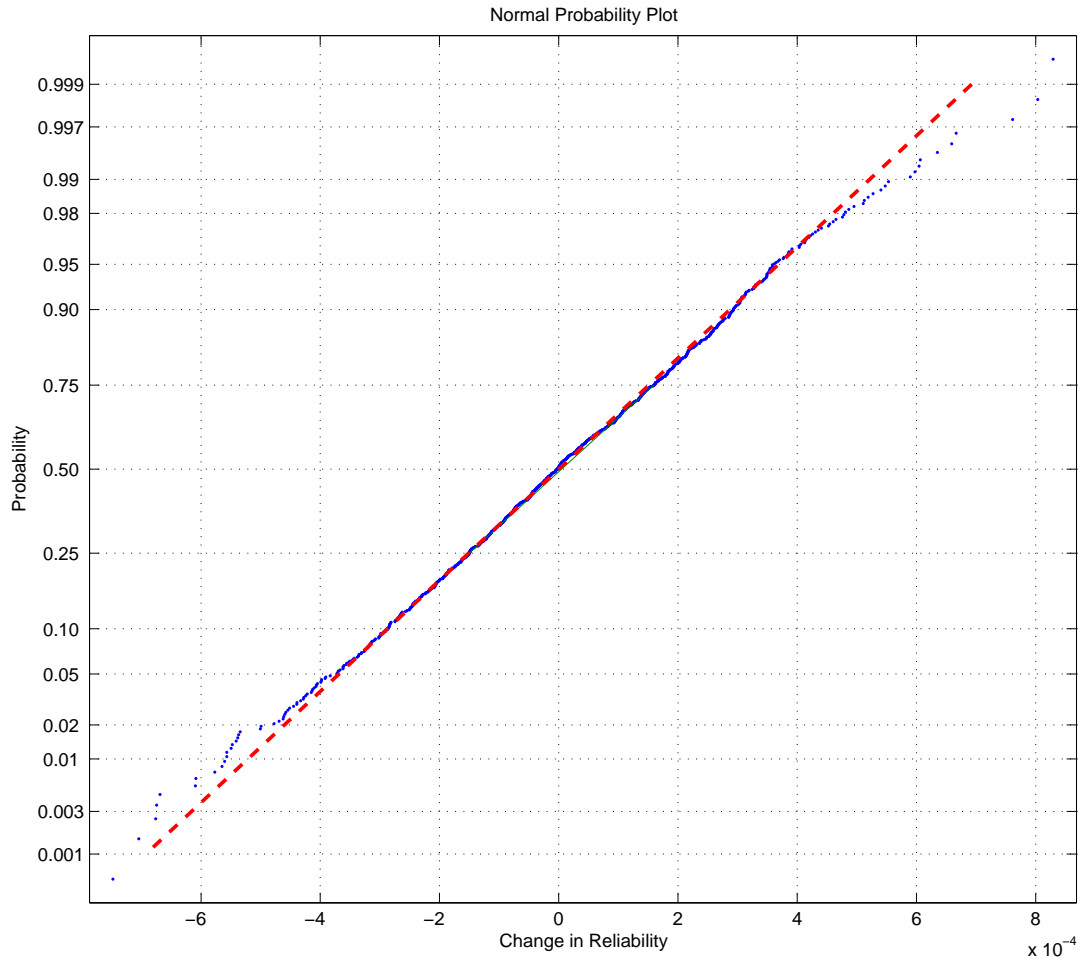


Figure 2: Cumulative normal plot of the predicted (*dashed line*) and experimental (*1000 dots*) statistical sensitivity to 144 random changes with  $\sigma_\delta = 0.005$  from 1000 experiments

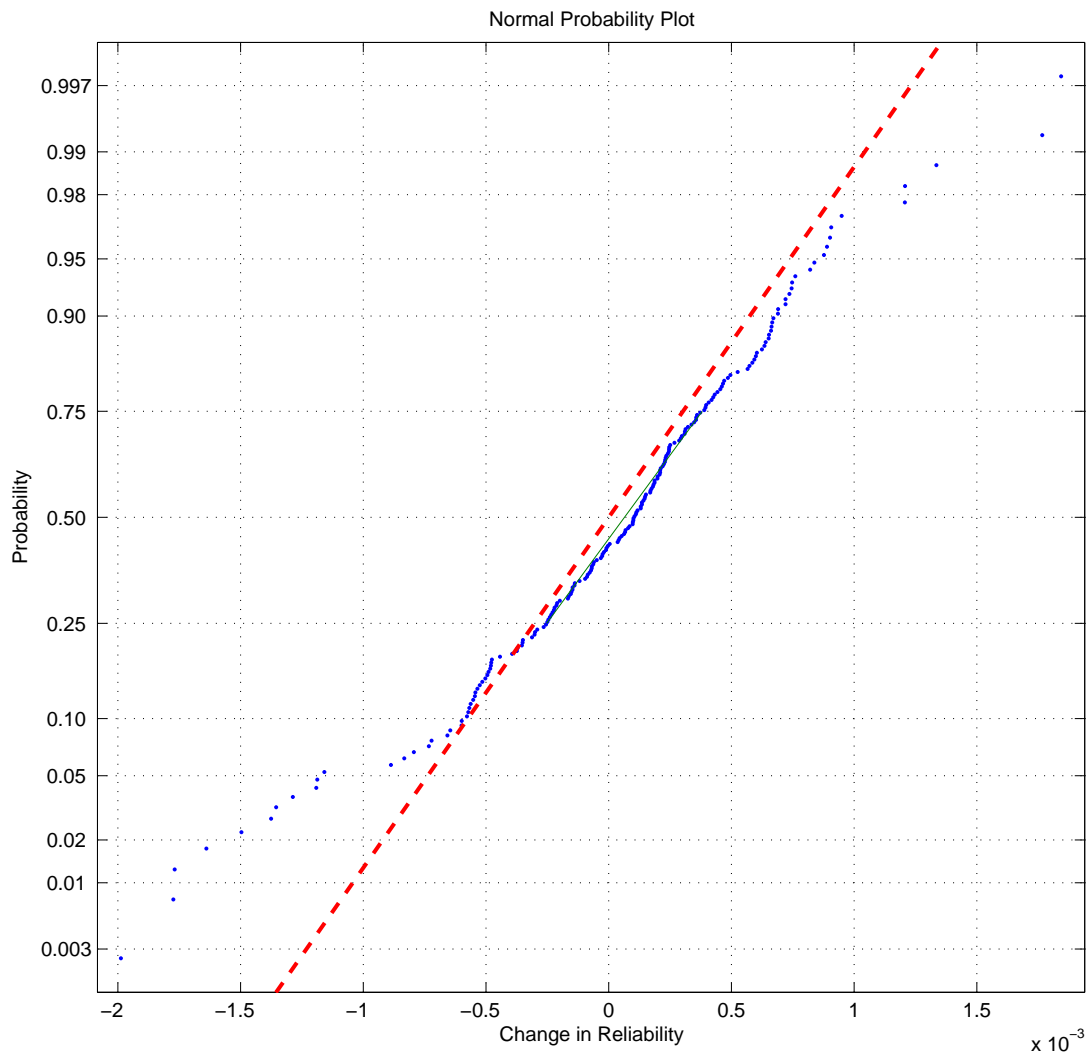
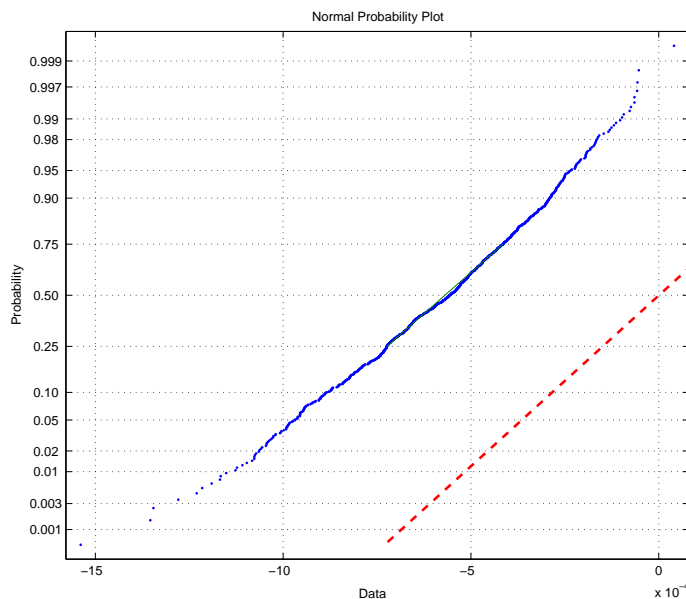


Figure 3: Cumulative normal plot of the predicted (*dashed line*) and experimental (*200 dots*) statistical sensitivity to 144 random changes with  $\sigma_\delta = 0.01$  from 200 experiments





**Figure 4:** Cumulative normal plot of the predicted (*dashed line*) and experimental (*1000 dots*) statistical sensitivity to 144 random changes with  $\sigma_\delta = 0.005$  from 1000 experiments, where negative changes to the zeros in  $\hat{P}$  are not allowed. Although the estimate of the variation is still quite accurate, the average is clearly no longer 0, but significantly negative.

experimental standard deviation of  $2.27 \times 10^{-4}$ , which is still very close to the predicted standard deviation of  $2.24 \times 10^{-4}$ .

When we would simply not allow any changes to the zeros in the transition probability matrix, considering those transitions as absolutely impossible, then the variation of the system's reliability would of course be lower than the predicted value, since there are a lot less changes made to the transition probability matrix which is usually quite sparse. But at least then the average change in reliability would of course still be zero, since for every change the opposite change is equally likely again.

## 5. SUMMARY AND FUTURE WORK

In this paper we have made a first quantitative study of the sensitivity of the reliability estimate to changes in the usage profile with the help of Markov models. With the theory described here, it is possible to make a good estimate of the effect of one or many small changes in the usage profile on the reliability of a system. Further the theory also makes it possible to easily find the transitions and states to which the reliability is most sensitive and to identify measures that can be taken to reduce this sensitivity.

When comparing the theory with some experimental results, the results are very good, taking into account a number of limitations described at the end of the paper.

Further work will be done on finding similar results for the alternative Markov model used in [18], and on improving the theory to take into account the limitations posed by system's with sparse transition probability matrices. Also we will be looking more into the exact nature of the changes

and uncertainties in the usage profile of different systems, with a special focus on crisis situations. A very important goal is also to apply this sensitivity analysis to a real industrial system to further investigate the applicability of these results.

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