



LUND
UNIVERSITY

EDAP15: Program Analysis

INTERPROCEDURAL ANALYSIS

Christoph Reichenbach



Welcome back!

- ▶ Lab 2: Bugfix pushed
- ▶ Lab 3: Simplified (but still two parts)
- ▶ Office hours:
 - ▶ **Wednesdays** 14:30–15:30
 - ▶ **Thursdays** 13:15–14:00

Questions?

Lecture Overview

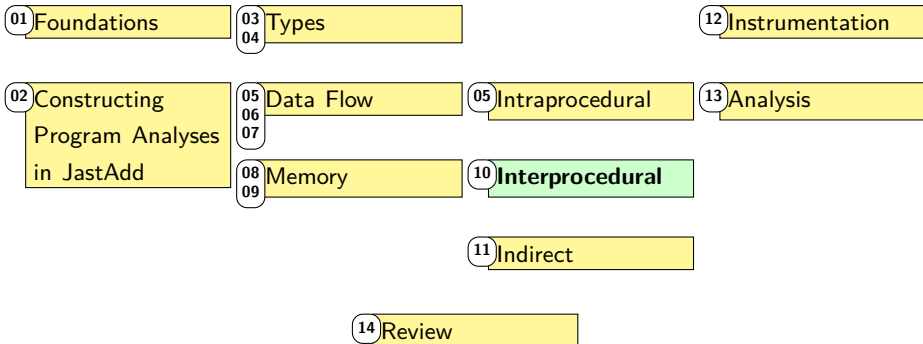
Foundations

Static Analysis

Dynamic
Analysis

Properties

Control Flow



What about subroutines?

Teal

```
var x := max(0, 5);  
print(10 / x); // Division by zero?
```

- Understanding code usually requires understanding subroutines like `max`

Inter- vs. Intra-Procedural Analysis

- ▶ **Intra**procedural: Within one procedure
 - ▶ Data flow analysis so far
- ▶ **Inter**procedural: Across multiple procedures
 - ▶ Type Analysis, especially. with polymorphic type inference

Limitations of Intra-Procedural Analysis

Teal-0

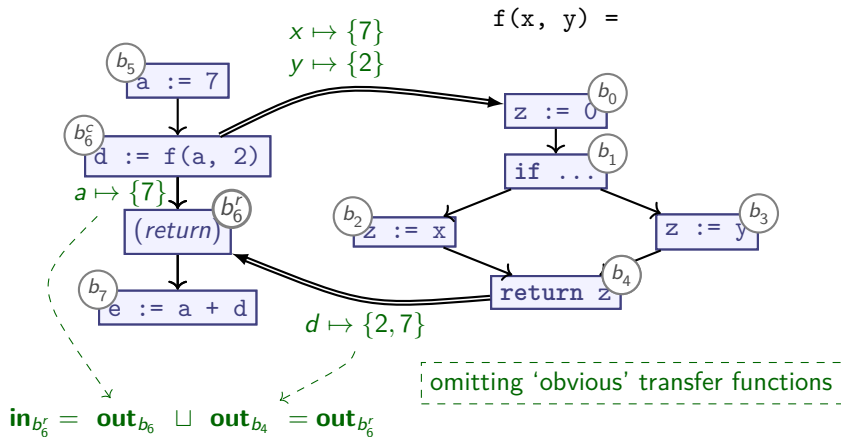
```
a := 7;  
d := f(a, 2);  
e := a + d;
```

Teal-0

```
fun f(x, y) = {  
    var z := 0;  
    if x > y {  
        z := x;  
    } else {  
        z := y;  
    }  
    return z;  
}
```

How can we compute Constant Propagation here?

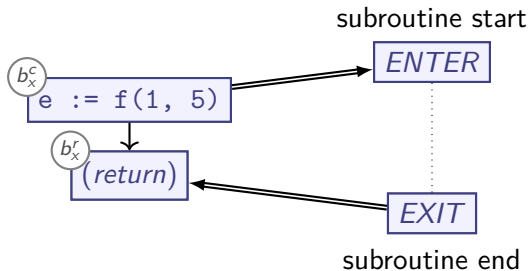
A Naïve Inter-Procedural Analysis



► **out_{b₇}:** $e \mapsto \{9, 14\}$

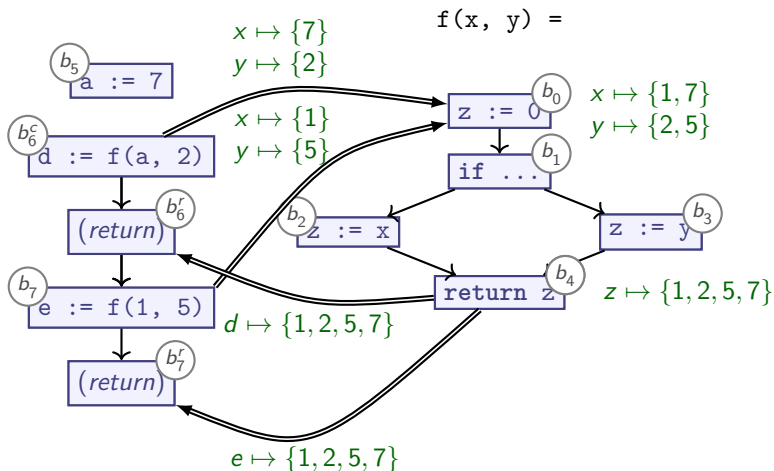
Works rather straightforwardly!

Inter-Procedural Control Flow Graph



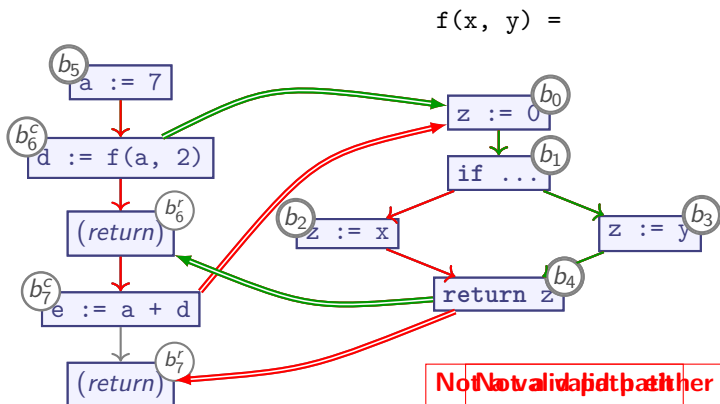
- ▶ Split call sites b_x into *call* (b_x^c) and *return* (b_x^r) nodes
- ▶ Intra-procedural edge $b_x^c \rightarrow b_x^r$ carries environment/store
- ▶ Inter-procedural edge ($\Rightarrow$):
 - ▶ Call site $\Rightarrow$ callee: substitutes parameters
 - ▶ Call site $\Leftarrow$ return: substitutes result
 - ▶ Otherwise like intra-procedural data flow edge

A Naïve Inter-Procedural Analysis



Imprecision!

Valid Paths



► $[b_5, b_6^c, b_0, b_1, b_3, b_4, b_6^r]$

Context-sensitive interprocedural analyses consider only valid paths

Summary

- ▶ **Intraprocedural** Analysis:

- ▶ Considers one subroutine at a time
- ▶ Calls to other subroutines treated as “worst-case” (e.g., $\top$ for dataflow analysis)

- ▶ **Interprocedural** Analysis:

- ▶ Analyses calls to subroutines
- ▶ For Dataflow analysis: uses **Interprocedural CFG** (ICFG)
 - ▶ ICFG represents subroutine calls as two nodes:
call and **return**
 - ▶ Special Call/Return edges caller $\Leftrightarrow$ callee
 - ▶ Naïve interpretation of ICFG call/return edges “spills” analysis results across call sites

Interprocedural Data Flow Analysis

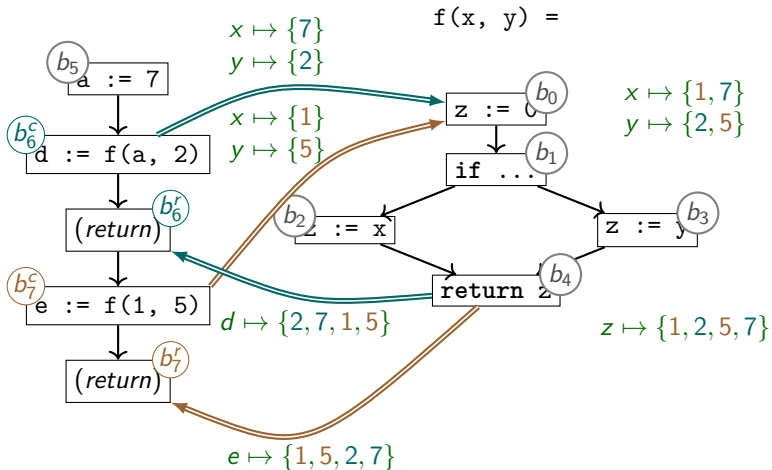
- ▶ **Call-site insensitive**

- ▶ Use same abstraction for each call site
- ▶ Examples for dataflow analysis:
 - ▶ Treat ICFG call/return edges like “regular” call/return edges
 - ▶ Use same transfer function everywhere (e.g., for builtin functions)

- ▶ **Call-site sensitive**

- ▶ Use different abstractions at different call sites

Call-Site Insensitive Analysis



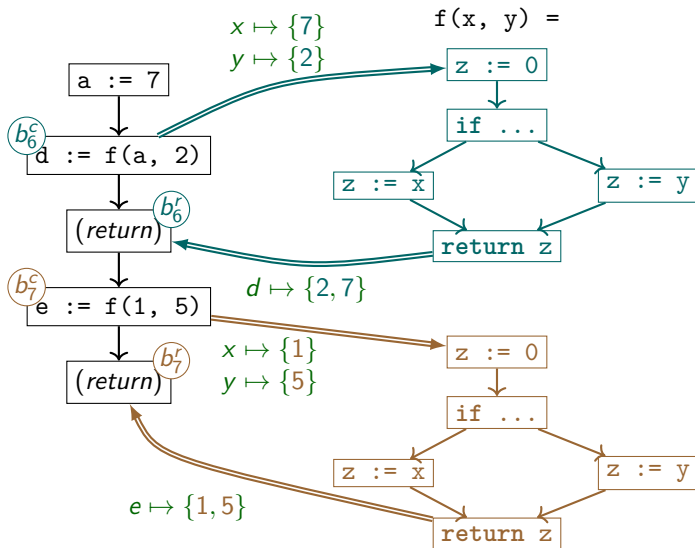
Call-site insensitive: analysis merges all callers to $f()$

Precise Interprocedural Dataflow

► Precision via one of:

- 1 **Inlining** or **AST cloning**
- 2 Call Strings
- 3 Procedure Summaries

Inlining

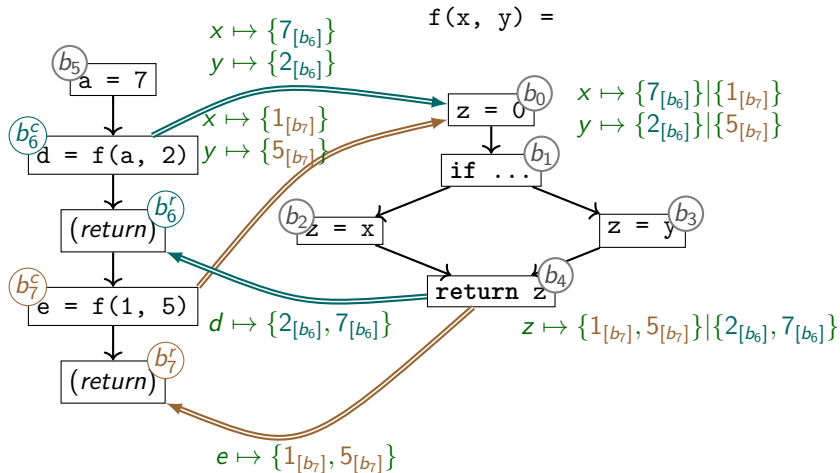


Clone subroutine IRs for each *calling context*

Precise Interprocedural Dataflow

- ▶ Precision via one of:
 - 1 Inlining or AST cloning
 - 2 **Call Strings**
 - 3 Procedure Summaries

Call Strings of Length 1



Degrees of Call-Site Sensitivity

- ▶ We used *call strings* to make call sites explicit:
 - ▶ $[b_6]$ in $2_{[b_6]}$
- ▶ “Strings” because this idea generalises:
 - ▶ Can keep track of *multiple* callers
 - ▶ Example: 2-call-site sensitivity: $[b_0, b_6]$ vs $[b_1, b_6]$

Teal

```
fun g(y: int): int = { return y }  
fun f(x: int): int = {  
    return g(x) // b6  
        + g(5); // b7  
}  
...  
f(1); // b0  
f(2); // b1
```

Must bound length of call strings to ensure termination

Summary

Strategies for call-site sensitive analysis:

► **Inlining**

- Copy subroutine bodies for each caller
- Performance cost
- Recursion: fall back to T

► **Call Strings**

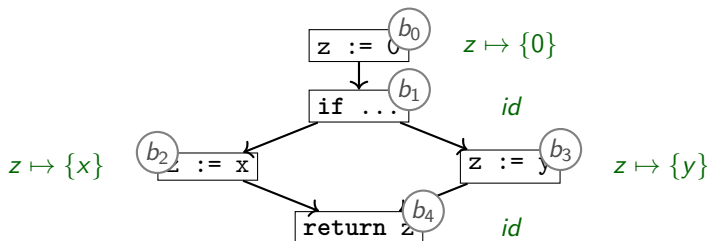
- Call string length:
 - Unbounded: Maximum precision, may not terminate with recursion
 - Bounded to length k : k degrees of call site sensitivity (speed/precision trade-off)

Precise Interprocedural Dataflow

- ▶ Precision via one of:
 - 1 Inlining or AST cloning
 - 2 Call Strings
 - 3 **Procedure Summaries**

Summarising Procedures

$f(x, y) =$



► **Compose transfer functions:**

- $trans_{b_0} \circ trans_{b_1} = [z \mapsto 0]$
- $trans_{b_0} \circ trans_{b_1} \circ trans_{b_2} = [z \mapsto \{x\}]$
- $trans_{b_0} \circ trans_{b_1} \circ trans_{b_3} = [z \mapsto \{y\}]$
- $trans_{b_0} \circ trans_{b_1} \circ (trans_{b_2} \sqcup trans_{b_3}) = [z \mapsto \{x, y\}]$
- $trans_{b_0} \circ trans_{b_1} \circ (trans_{b_2} \sqcup trans_{b_3}) \circ trans_{b_4} = [z \mapsto \{x, y\}]$

Procedure Summaries vs Recursion

f calls g calls h calls f

- ▶ Requires additional analysis to identify who calls whom
- ▶ Compute summaries of mutually recursive functions together
- ▶ Recursive call edges analogous to loops
- ▶ Loops/recursion require fixpoint computation over function composition!

Procedure Summaries

- ▶ Composing transfer functions yields a combined transfer function for $f()$:

$$trans_f = [\mathbf{return} \mapsto \{x, y\}]$$

- ▶ Use $trans_f$ as transfer function for $f()$, discard f 's body
- ▶ **Opportunities:**
 - ▶ Can yield compact subroutine descriptions
 - ▶ Can speed up call site analysis dramatically
- ▶ **Challenges:**
 - ▶ More complex to implement
 - ▶ Loops / recursion are challenging
- ▶ **Limitations:**
 - ▶ Requires suitable representation for summary
 - ▶ Requires mechanism for abstracting and applying summary
 - ▶ Worst cases:
 - ▶ $trans_f$ is symbolic expression more complex than f itself

Procedure Summaries for Dataflow

- ▶ Procedure Summaries *can* be as precise as inlining/call strings
- ... *but only for Distributive Frameworks*
 - ▶ Algorithm for Gen/Kill analyses: IFDS
 - ▶ Algorithm for other analyses: IDE

Summary

Making interprocedural dataflow precise:

- ▶ **Call-site sensitive approaches:**

- ▶ *Inlining*
- ▶ *Call strings*

- ▶ **Call-site insensitive approaches:**

- ▶ *Procedure Summaries*
 - ▶ Precise + compact summaries only possible for distributive frameworks

Procedure Summaries with Gen/Kill-sets

- ▶ Special case: Gen/Kill sets
- ▶ Transfer functions always consist of
 - ▶ Gen-set
 - ▶ Kill-set
- ▶ *No symbolic reasoning needed:*
 - ▶ Compose gen-sets, kill-sets, receive combined gen/kill-set for subroutine
- ▶ Maybe there are other such cases?

Greta Yorsh, Eran Yahav, Satish Chandra: “Generating Precise and Concise Procedure Summaries”, in Principles of Programming Languages 2008

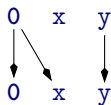
Procedure Summaries for Dataflow

- ▶ Procedure Summaries *can* be as precise as inlining/call strings
- ... *but only for Distributive Frameworks*
 - ▶ Algorithm for Gen/Kill analyses: IFDS
 - ▶ Algorithm for other analyses: IDE

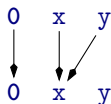
Representation Relations

Example procedure summary representation:

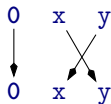
```
x := null;  
y := y;
```



```
if x != y {  
  x := y;  
}  
y := 1;
```



```
{ t := x  
  x := y  
  y := t }
```



'May be null' analysis

- ▶ $c \rightarrow d$:
if $P(c) \in \mathbf{in}_b$ then $P(d) \in \mathbf{out}_b$
- ▶ Representation Relations relate $\mathbf{in}_b$ and $\mathbf{out}_b$ variables $\mathcal{V}$
- ▶ $R \subseteq (\mathcal{V} \cup \{0\}) \times (\mathcal{V} \cup \{0\})$
- ▶ if $\langle 0, X \rangle \in R$:
 X always 'may be null' in $\mathbf{out}_b$
- ▶ if $\langle Y, X \rangle \in R$:
If Y 'may be null' in $\mathbf{in}_b$:
 $\Rightarrow X$ 'may be null' in $\mathbf{out}_b$

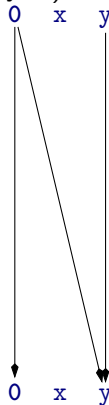
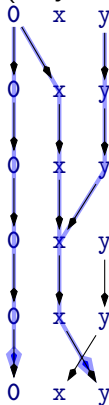
Composing Representation Relations

Representation Relations (*may be null analysis*):

```
x := null;
y := y;
```

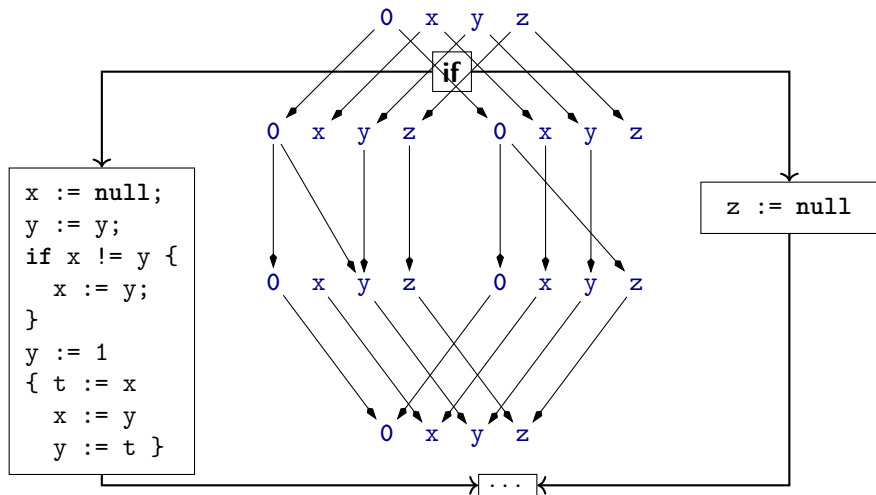
```
if x != y {
  x := y;
}
```

```
{ t := x;
  x := y;
  y := t; }
```

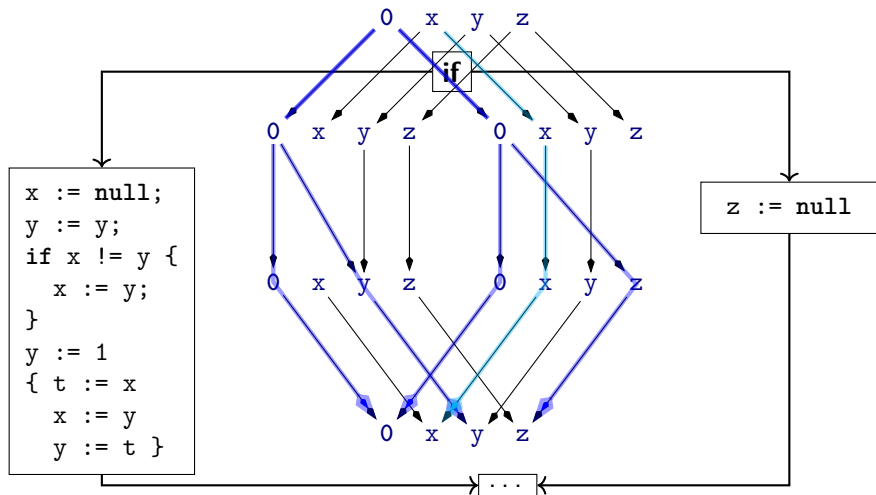


Composed representation relations are again representation relations

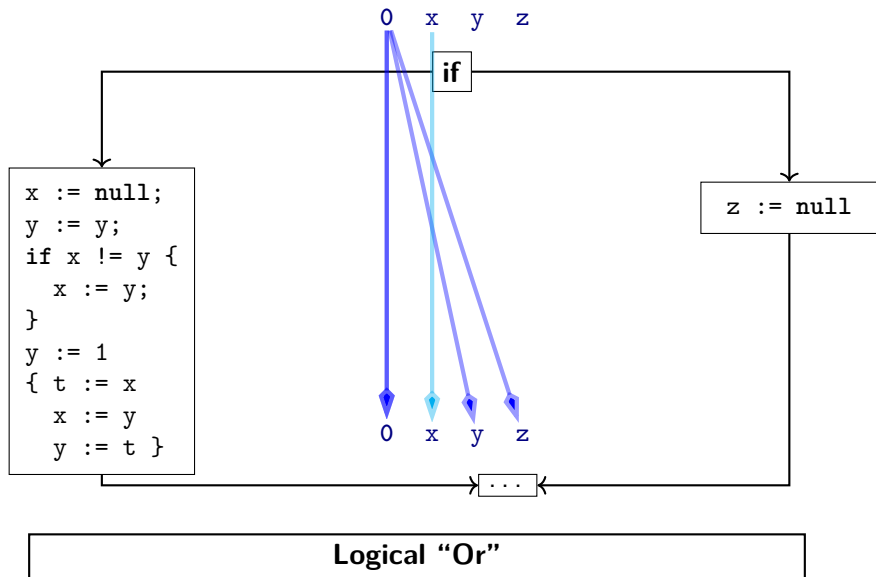
Joining Control-Flow Paths



Joining Control-Flow Paths



Joining Control-Flow Paths



Dataflow via Graph Reachability

$$n = \langle b, v \rangle$$

- ▶ Assume binary lattice $(\{\top, \perp\}, \sqsubseteq, \sqcap, \sqcup)$
 - ▶ $\top \sqcup y = \top = x \sqcup \top$ and $\perp \sqcup \perp = \perp$
 - ▶ Typical for 'May' analysis ($P(x) = \text{'x may be null'}$)
- ▶ Encode Dataflow problem as *Graph-Reachability*
- ▶ Graph nodes $n = \langle b, v \rangle$
 - ▶ b : CFG node
 - ▶ v : Variable or $\mathbf{0}$
 - ▶ $\mathbf{0}$: $\langle b_1, \mathbf{0} \rangle \rightarrow \langle b_2, y \rangle$: $P(y)$ at b_2 holds always
 - ▶ Variable: $\langle b_1, x \rangle \rightarrow \langle b_2, y \rangle$: $P(x)$ at $b_1 \implies P(y)$ at b_2

Dataflow via Graph Reachability

$$n = \langle b, v \rangle$$

- ▶ Assume binary lattice $(\{\top, \perp\}, \sqsubseteq, \sqcap, \sqcup)$
 - ▶ $\top \sqcup y = \top = x \sqcup \top$ and $\perp \sqcup \perp = \perp$
 - ▶ Typical for 'May' analysis ($P(x) = \text{'x may be null'}$)
 - ▶ Equivalently for 'Must' analysis:
'x must be null' = not ('x may be non-null')
- ▶ Encode Dataflow problem as *Graph-Reachability*
- ▶ Graph nodes $n = \langle b, v \rangle$
 - ▶ b : CFG node
 - ▶ v : Variable or **0**
 - ▶ **0**: $\langle b_1, \mathbf{0} \rangle \rightarrow \langle b_2, y \rangle$: $P(y)$ at b_2 holds always
 - ▶ Variable: $\langle b_1, x \rangle \rightarrow \langle b_2, y \rangle$: $P(x)$ at $b_1 \implies P(y)$ at b_2

A Dataflow Worklist Algorithm: IFDS

- ▶ Call-site sensitive interprocedural data flow algorithm
- ▶ IFDS = (Interprocedural **F**inite **D**istributive **S**ubset problems)
- ▶ ‘Exploded Supergraph’: $G^\# = (N^\#, E^\#)$
 - ▶ $N^\# = N_{\text{CFG}} \times (\mathcal{V} \cup \{0\})$
 - ▶ Plus parameter/return call edges
- ▶ Property-of-interest holds if reachable from $\langle b_{\text{main}}^s, \mathbf{0} \rangle$
 - ▶ b_{main}^s is CFG *ENTER* node of main entry point
- ▶ **Key ideas:**
 - ▶ Worklist-based
 - ▶ Construct Representation Relations on demand
 - ▶ Construct ‘Exploded Supergraph’
 - ▶ CFG of all functions $\times \mathcal{V} \cup \{\mathbf{0}\}$

IFDS Datastructures

Instead of $\langle\langle b_0, v_0 \rangle, \langle b_3, v_0 \rangle\rangle$ we also write:

$$\langle b_0, v_0 \rangle \rightarrow \langle b_3, v_0 \rangle$$

WORKLIST edge

$$\langle b_0, v_0 \rangle \dashrightarrow \langle b_3, v_0 \rangle$$



PATHEDGE edge

All WORKLIST edges are also PATHEDGE edges

Result of our analysis

$N^\sharp$ -edge



SUMMARYINST

Generated from summary nodes
Otherwise equivalent to $N^\sharp$ -edges

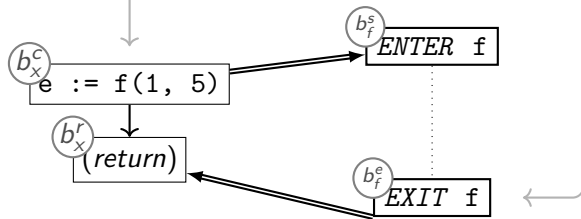
IFDS Strategy

- ▶ Algorithm distinguishes between three types of nodes:

- ▶ Exit nodes (b_f^e)

- ▶ Call nodes (b_x^c)

- ▶ Other nodes



On-demand processing

```
Procedure propagate( $n_1 \rightarrow n_2$ ):  
begin  
  if  $n_1 \rightarrow n_2 \in \text{PATHEDGE}$  then  
    return  
   $\text{PATHEDGE} := \text{PATHEDGE} \cup \{n_1 \rightarrow n_2\}$   
   $\text{WORKLIST} := \text{WORKLIST} \cup \{n_1 \rightarrow n_2\}$   
end
```

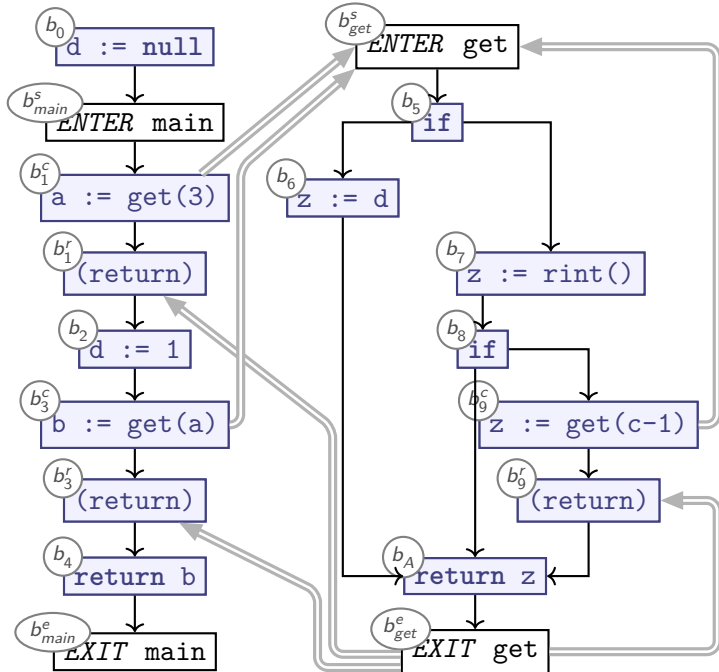
Running Example

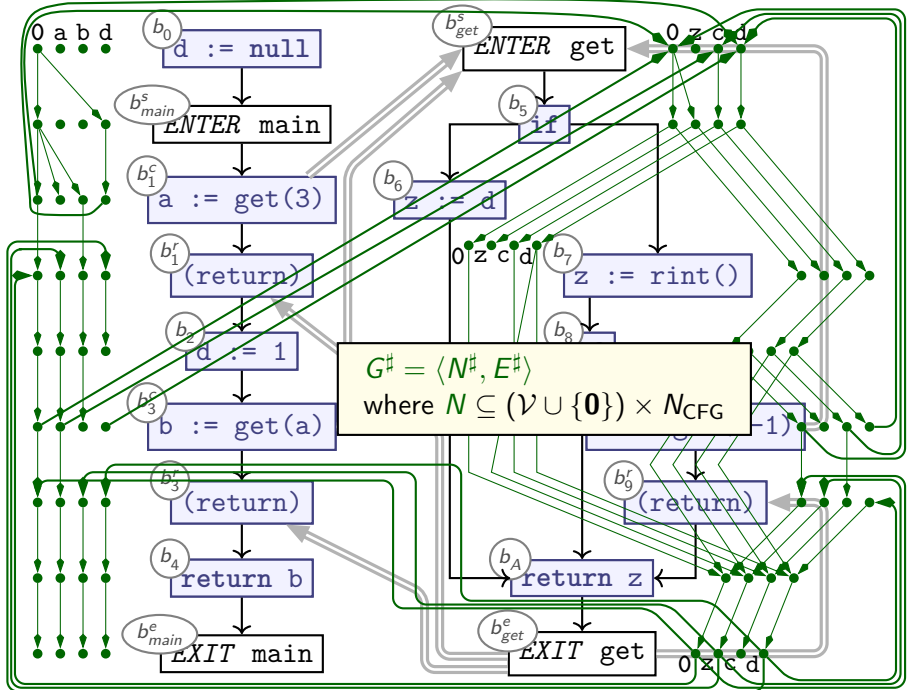
Teal-0: *main()*

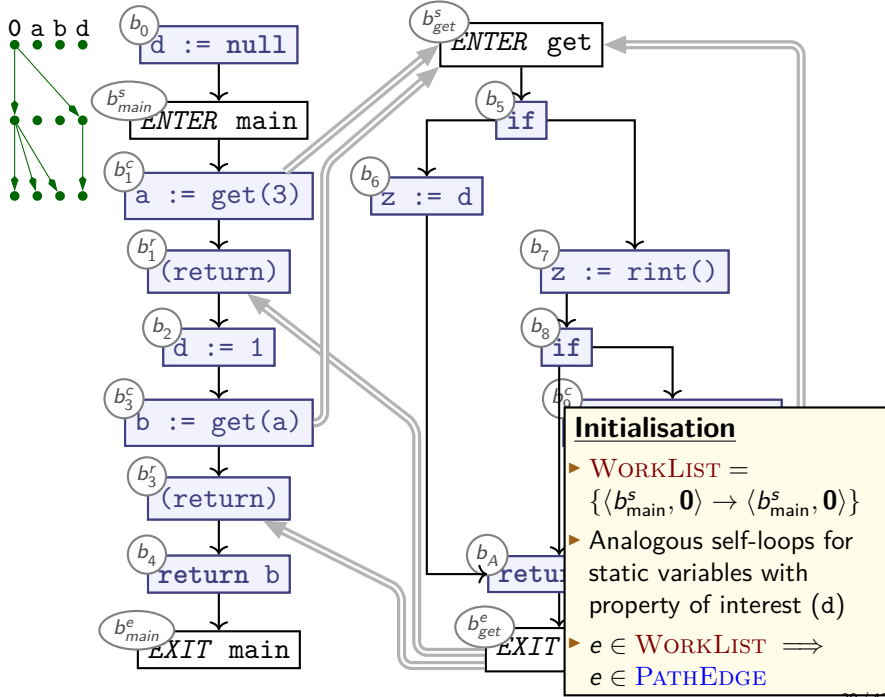
```
var default := null;
fun main() = {
  var a := get(3);
  default := 1;
  var b := get(3);
  return b;
}
```

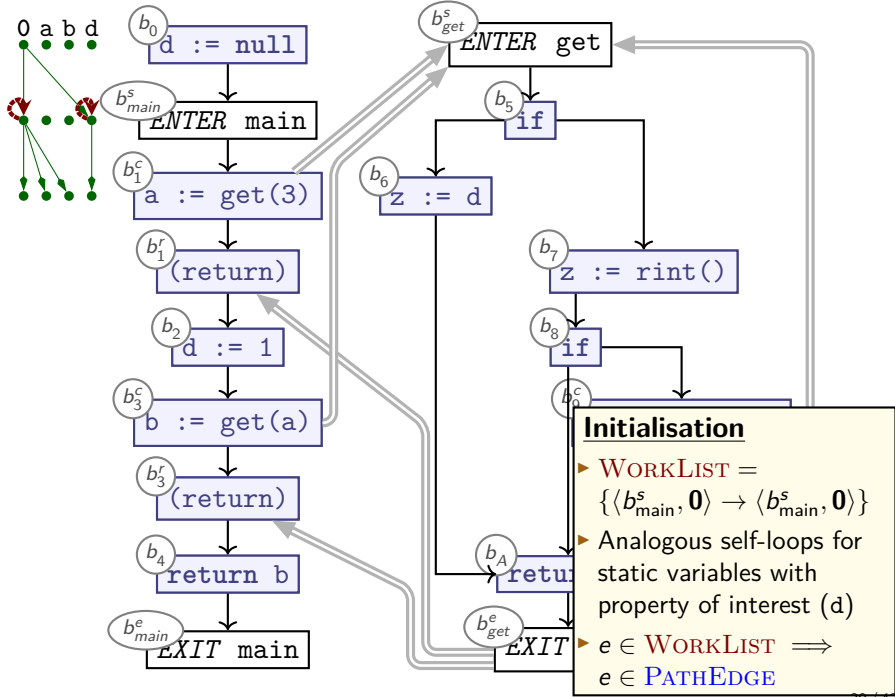
Teal-0: *get()*

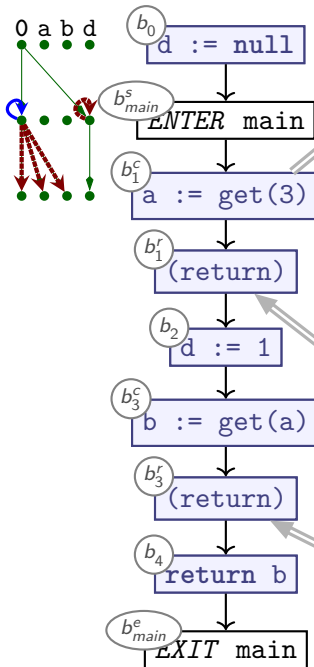
```
fun get(c) = {
  if c == 0 {
    z := default;
  } else {
    z := read_int();
    if z < 0 {
      z := get(c - 1);
    }
  }
  return z;
}
```











Procedure propagate($n_1 \rightarrow n_2$):

begin

if $n_1 \rightarrow n_2 \in \text{PATHEDGE}$ **then**

return

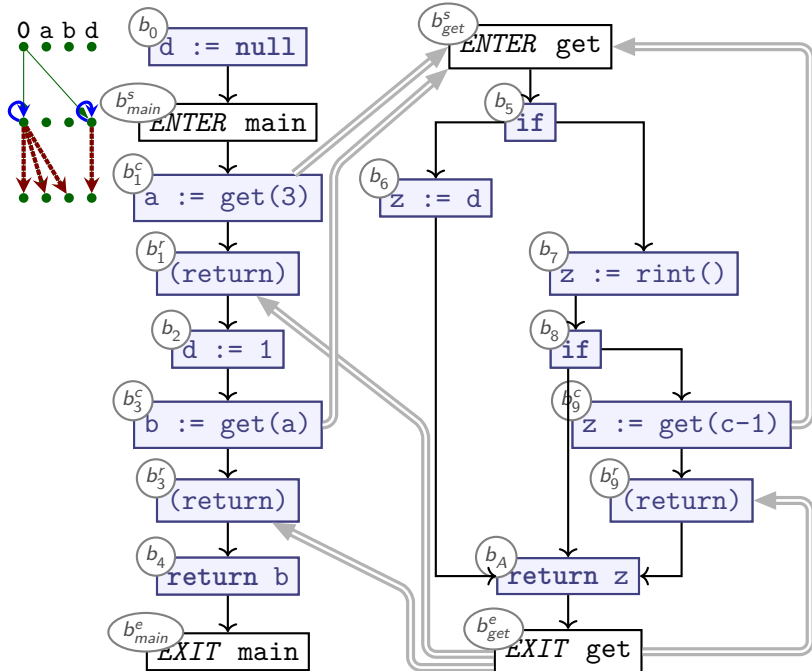
$\text{PATHEDGE} := \text{PATHEDGE} \cup \{n_1 \rightarrow n_2\}$

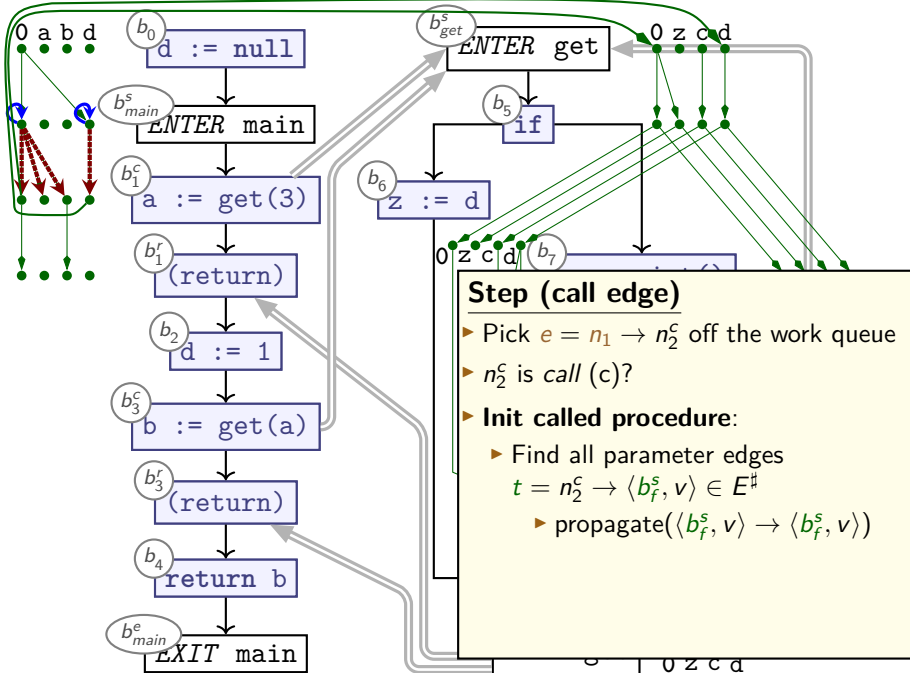
$\text{WORKLIST} := \text{WORKLIST} \cup \{n_1 \rightarrow n_2\}$

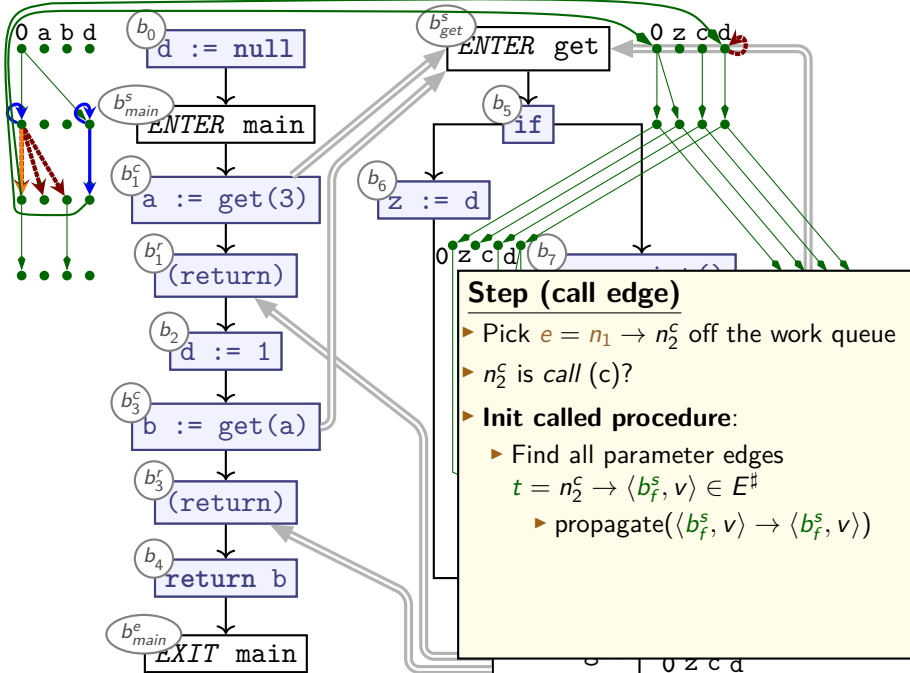
end

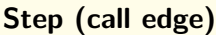
Step (regular edge)

- ▶ Pick e off the work queue
 $e = n_1 \rightarrow n_2$
- ▶ n_2 neither call (c) nor exit (e)?
- ▶ Find all $n_2 \rightarrow n_3$
propagate($n_1 \rightarrow n_3$)
- ▶ Remove e from WORKLIST
- ▶ e remains in PATHEDGE

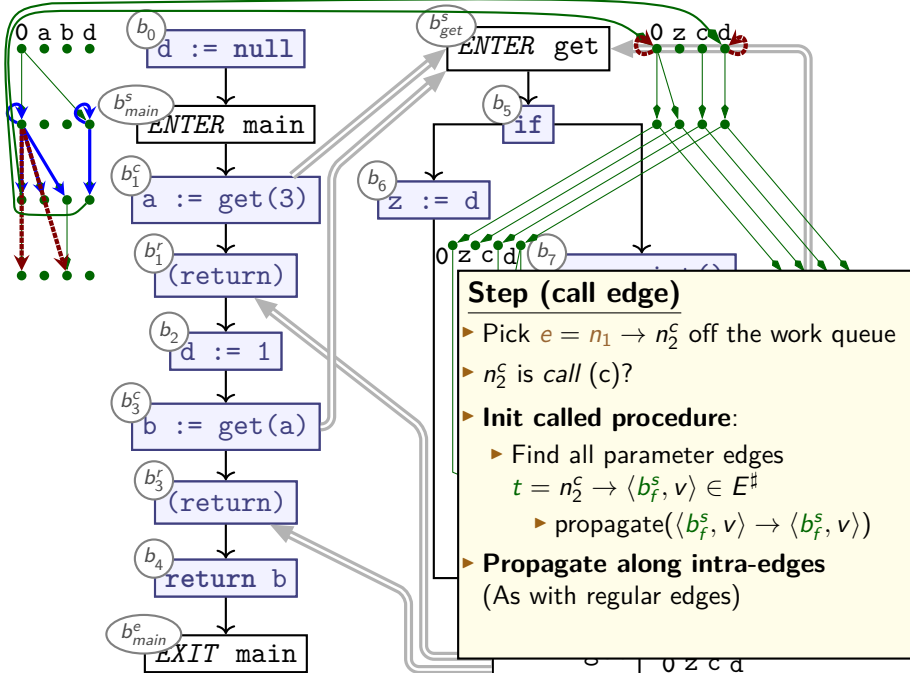


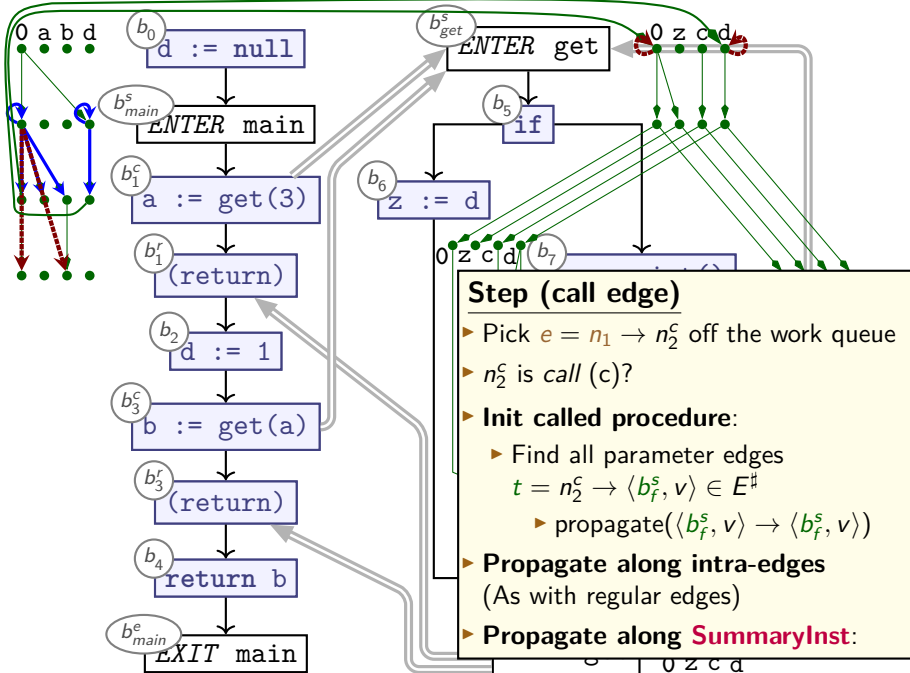


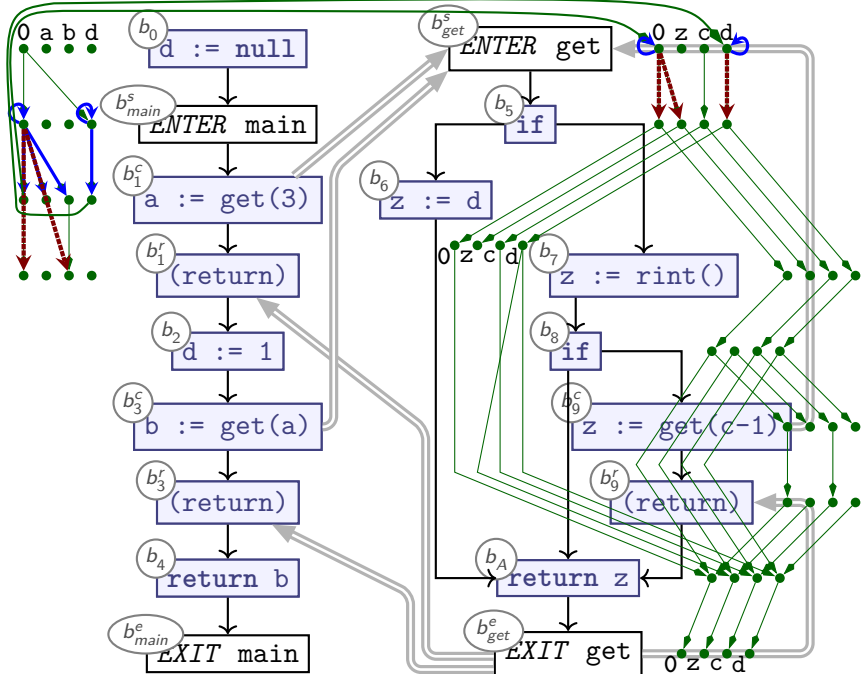


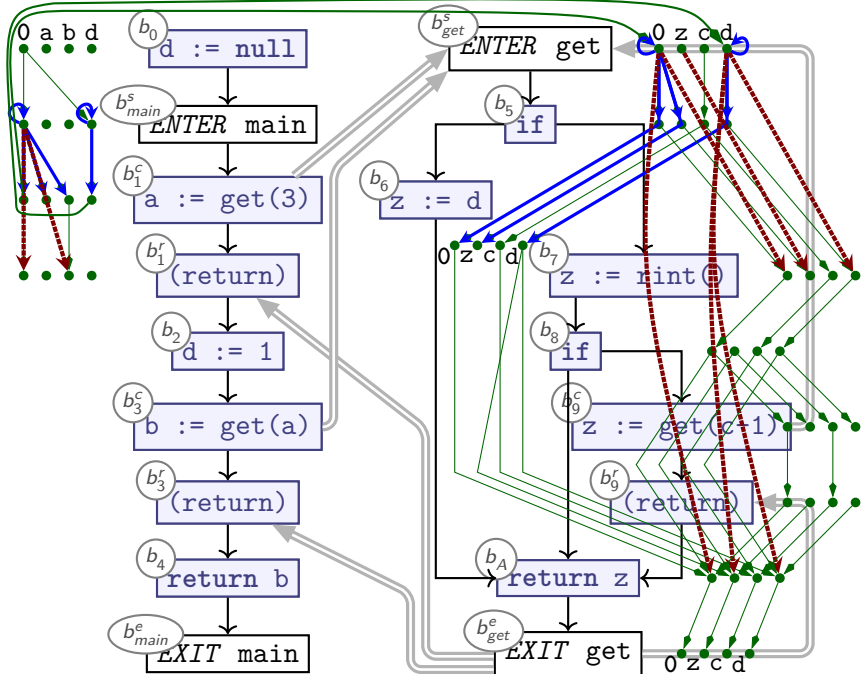


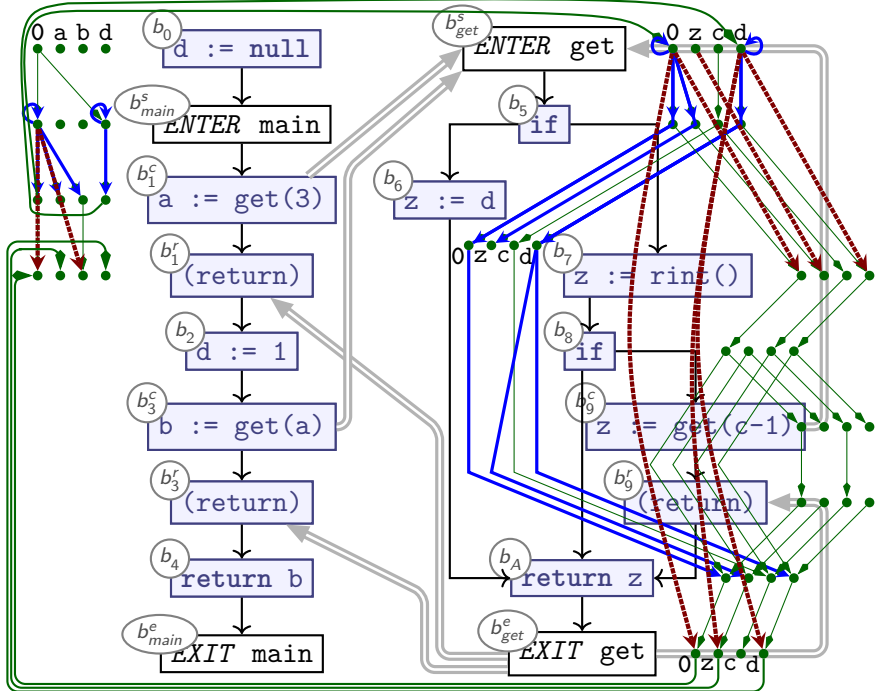
- | | | | | |
|---|---|---|---|---|
| 0 | 0 | 2 | c | d |
|---|---|---|---|---|

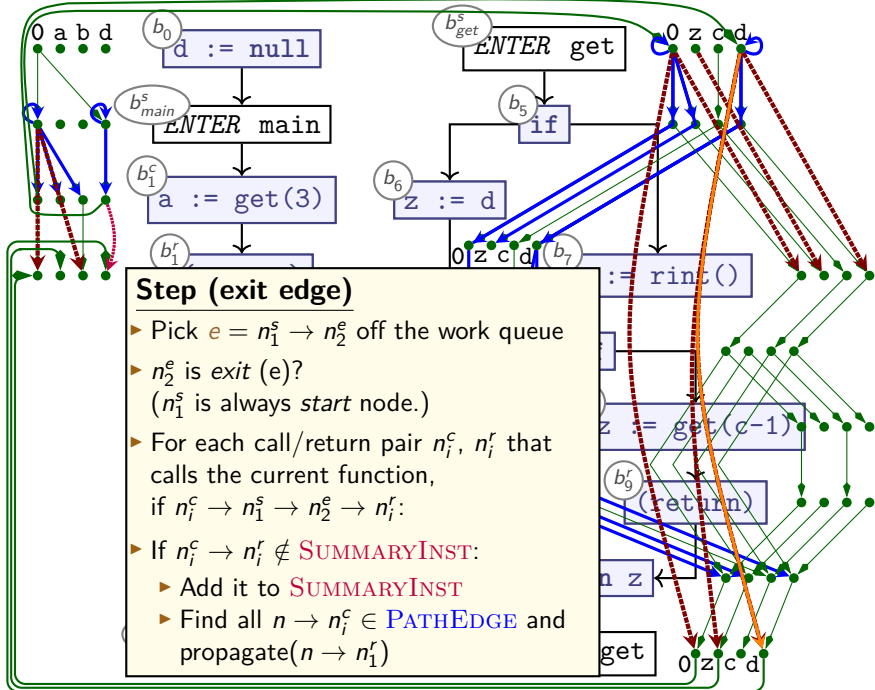


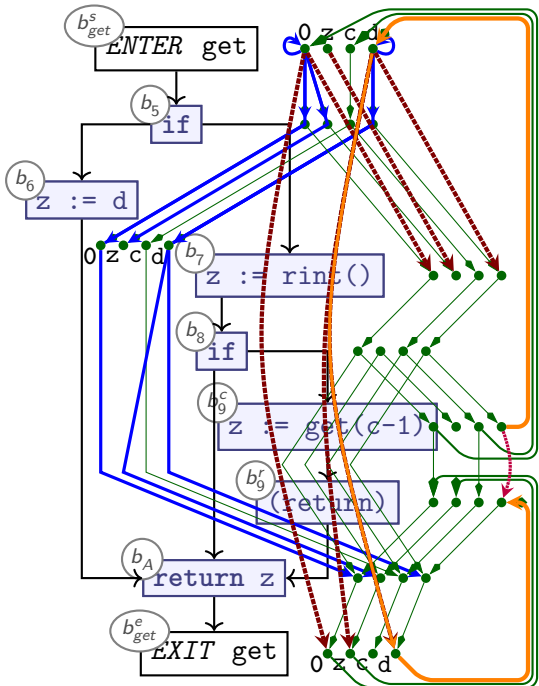
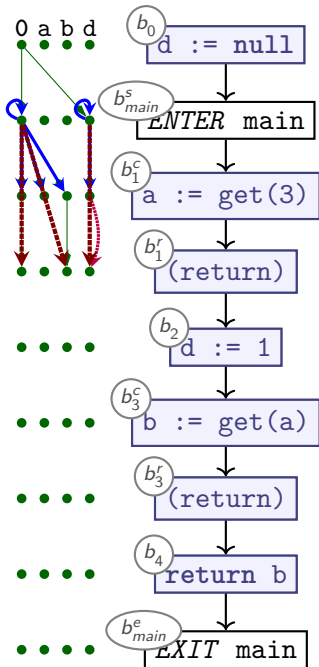


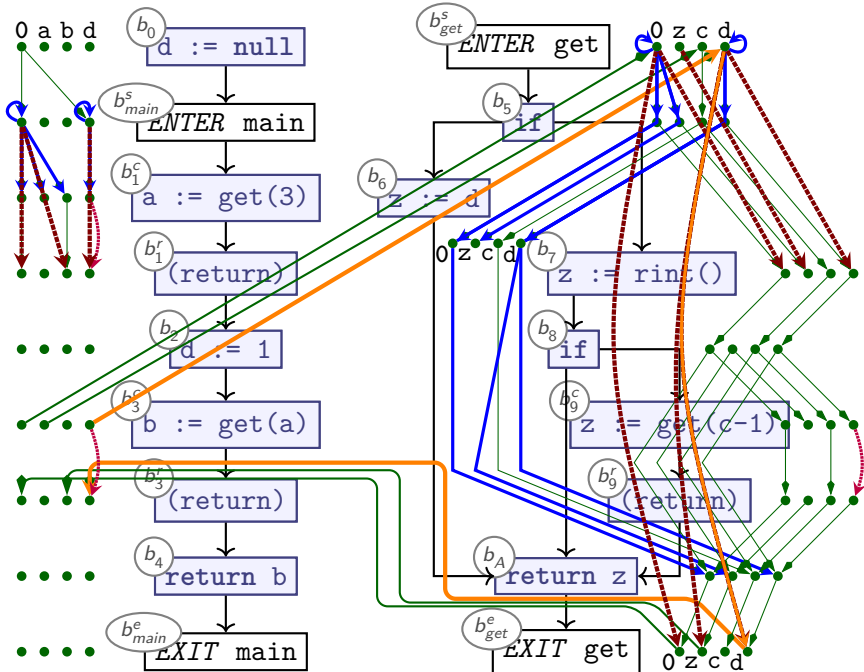


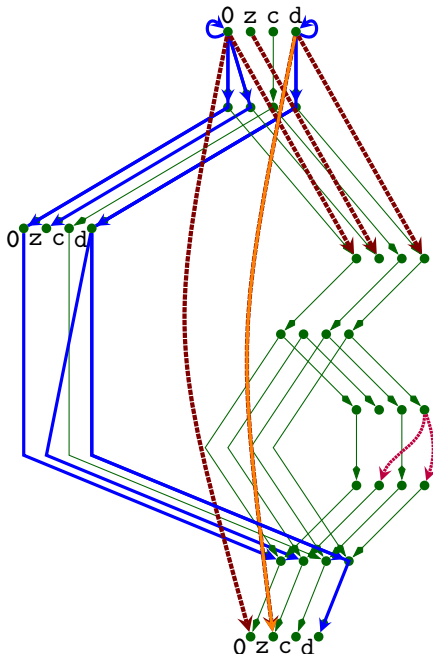
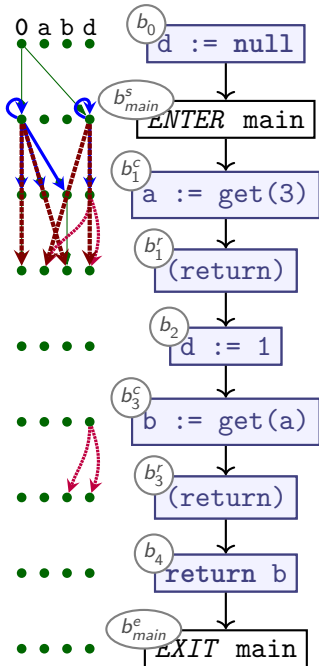


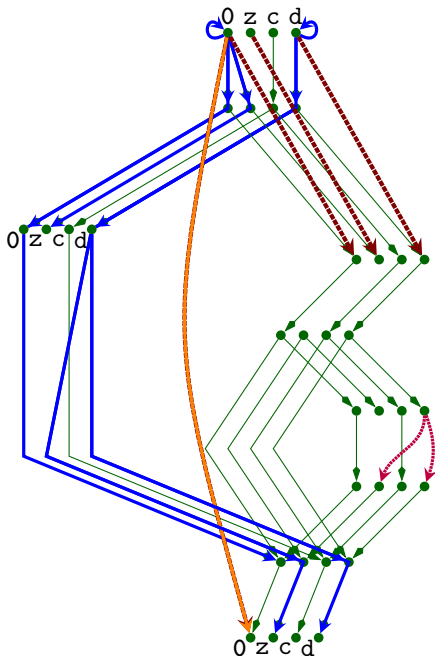
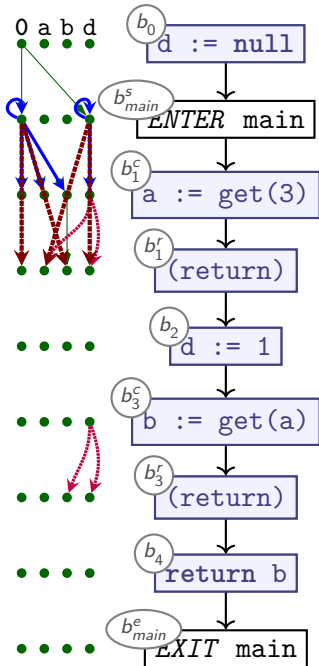


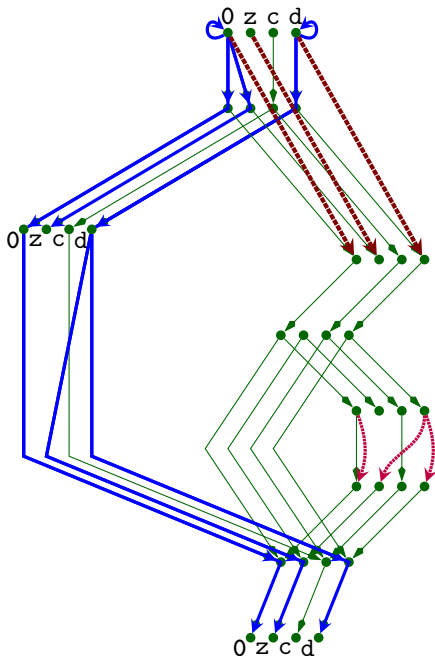
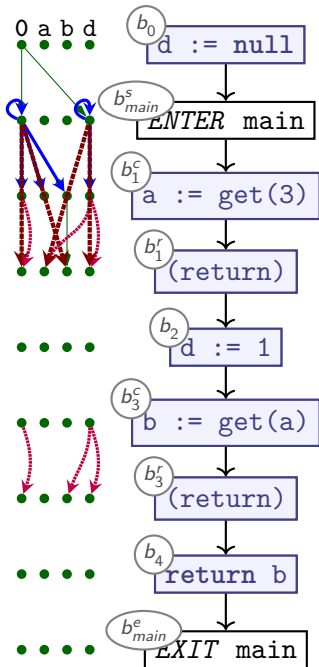


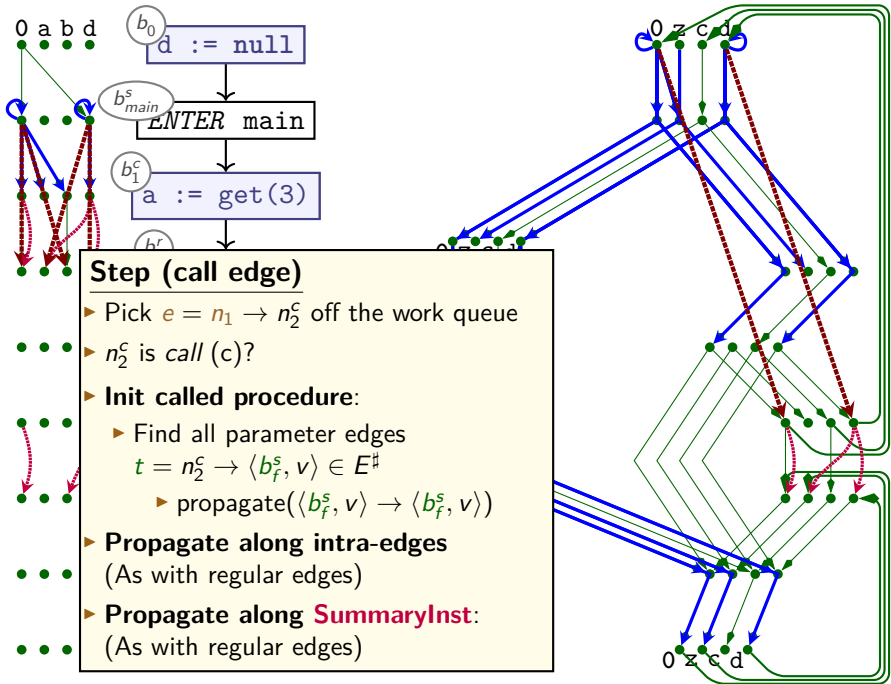


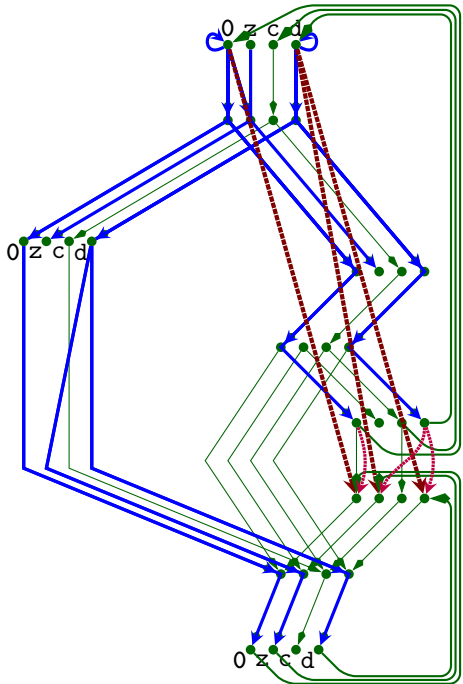
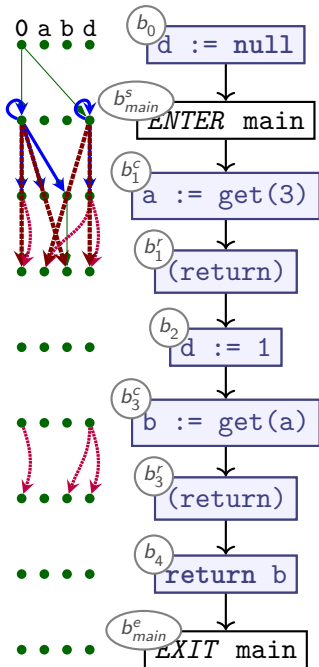


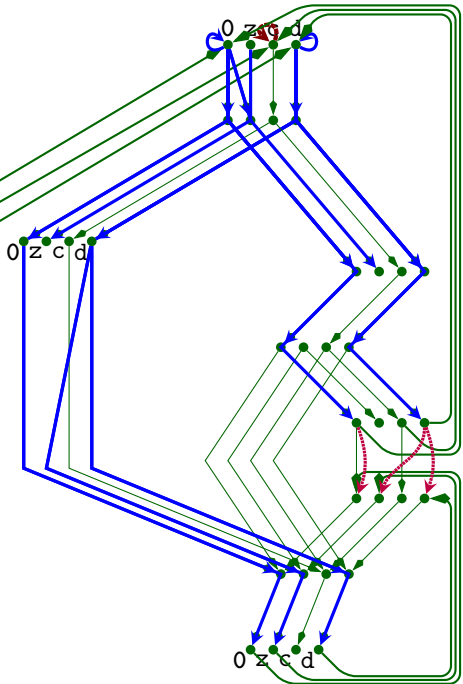
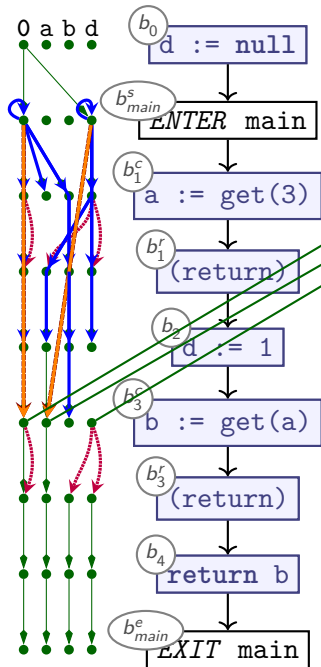


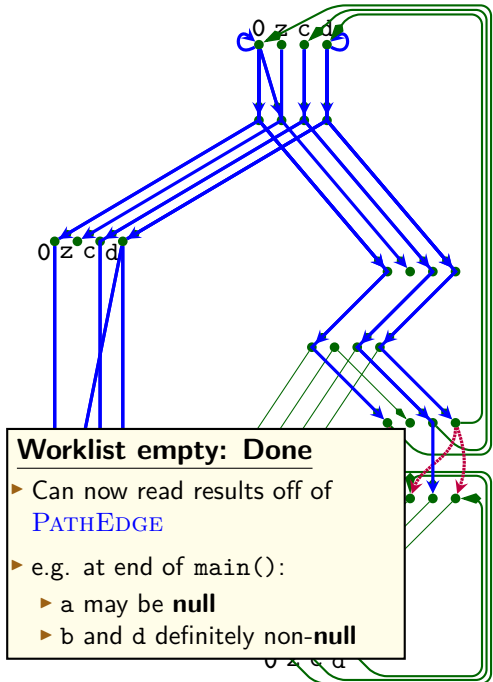
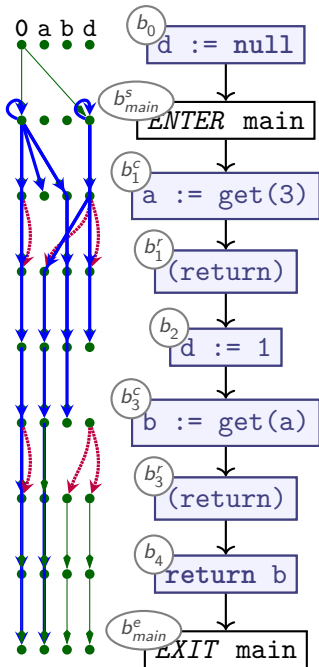












Worklist empty: Done

- Can now read results off of **PATHEDGE**
- e.g. at end of `main()`:
 - a may be **null**
 - b and d definitely non-**null**

The IFDS Algorithm: Initialisation and Propagation)

Procedure Init():

begin

WORKLIST := **PATHEDGE** := $\emptyset$

propagate($\langle b_{\text{main}}^s, \mathbf{0} \rangle \rightarrow \langle b_{\text{main}}^s, \mathbf{0} \rangle$)

ForwardTabulate()

end

Procedure propagate($n_1 \rightarrow n_2$):

begin

if $n_1 \rightarrow n_2 \in \mathbf{PATHEDGE}$ **then**

return

PATHEDGE := **PATHEDGE** $\cup \{n_1 \rightarrow n_2\}$

WORKLIST := **WORKLIST** $\cup \{n_1 \rightarrow n_2\}$

end

IFDS: Forward Tabulation

Procedure ForwardTabulate():

begin

while $n_0 \rightarrow n_1 \in \text{WORKLIST}$ **do**

WorkList := **WorkList** $\setminus \{n_0 \rightarrow n_1\}$

$\langle b_0, v_0 \rangle = n_0$; $\langle b_1, v_1 \rangle = n_1$

if b_1 is neither *Call* nor *Exit* node **then**

foreach $n_1 \rightarrow n_2 \in E^\#$:

propagate($n_0 \rightarrow n_2$)

else if b_1 is *Call* node **then begin**

foreach call edge $n_1 \rightarrow n_2 \in E^\#$:

propagate($n_2 \rightarrow n_2$)

foreach non-call edge $n_1 \rightarrow n_2 \in E^\# \cup \text{SUMMARYINST}$:

propagate($n_0 \rightarrow n_2$)

end else if b_1 is *Exit* node **then begin**

foreach caller/return node pair b_i^c, b_i^r that calls b_0 **and** vars v_0, v_1 **do**

$n_s = \langle b_i^c, v_0 \rangle$; $n_r = \langle b_i^r, v_1 \rangle$

if $\{n_s \rightarrow n_0, n_0 \rightarrow n_1, n_1 \rightarrow n_r\} \subseteq E^\#$ **and not** $n_s \rightarrow n_r \in \text{SUMMARYINST}$ **then**

SUMMARYINST := **SUMMARYINST** $\cup \{n_s \rightarrow n_r\}$

foreach $n_z \rightarrow n_s \in \text{PATHEDGE}$:

propagate(n_z, n_r)

end done end done end

Summary: IFDS Algorithm

- ▶ Computes yes-or-no analysis on all variables
 - ▶ Original notion of 'variables' is slightly broader)
- ▶ Represents facts-of-interest as nodes $\langle b, v \rangle$:
 - ▶ b is node (basic block) in CFG
 - ▶ v is variable that we are interested in
- ▶ Uses
 - ▶ '*Exploded Supergraph*' $G^\#$
 - ▶ All CFGs in program in one graph
 - ▶ Plus interprocedural call edges
 - ▶ *Representation relations*
 - ▶ *Graph reachability*
 - ▶ *A worklist*
- ▶ Distinguishes between *Call* nodes, *Exit* nodes, others
- ▶ **Demand-driven**: only analyses what it needs
- ▶ **Whole-program analysis**
- ▶ **Computes Least Fixpoint on distributive frameworks**

Outlook

- ▶ More static analysis on Tuesday
- ▶ Lab 3 will go up today

<http://cs.lth.se/EDAP15>