



LUND
UNIVERSITY

EDAP15: Program Analysis

DATAFLOW ANALYSIS 3

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Welcome back!

Some Administrative:

- ▶ Quiz deadlines: some slack if you missed a deadline
 - ▶ 8 days buffer (cumulative across all quizzes), not counting weekends
- ▶ Automatic feedback with “hidden tests” for lab 1 online now
- ▶ Let us know if htis helps; may offer for lab 3, too

Questions?

Lecture Overview

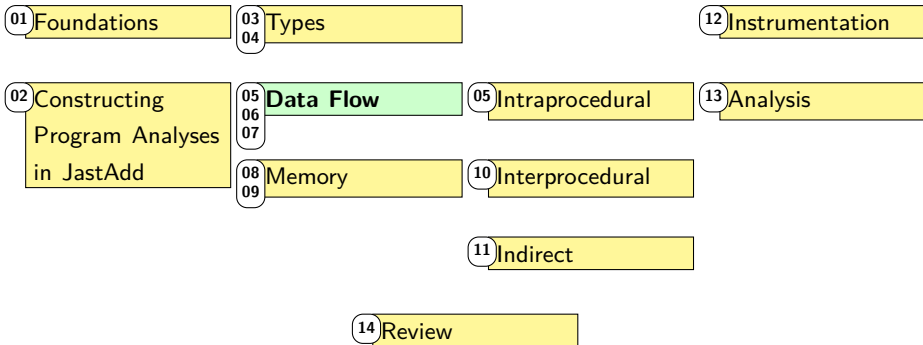
Foundations

Static Analysis

Dynamic
Analysis

Properties

Control Flow

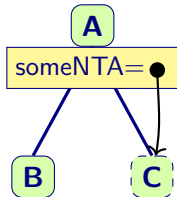


Non-Terminal Attributes

JastAdd

```
syn nta C AnyNode.someNTA() = new C(this);
```

- ▶ AST node as attribute
- ▶ Reifying implicit constructs (making them explicit in AST)
 - ▶ Built-in types
 - ▶ Built-in functions, constants
 - ▶ “Null Objects”, handle missing declarations ($\Rightarrow$ EDAN65)



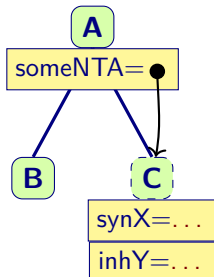
Beware

- ▶ NTAs must be **fresh** objects
- ▶ AST will be inconsistent if you re-use nodes
JastAdd does not check for this!

Non-Terminal Attributes

JastAdd

```
syn nta C AnyNode.someNTA() = new C(this);
```



- ▶ NTA may have attributes
- ▶ Owner node must provide inherited attributes

`A.someNTA().inhY() = ...`

Summary

- ▶ Nonterminal Attributes (NTAs):
 - ▶ “Synthetic” AST node
 - ▶ Useful e.g. for CFG nodes that have no AST equivalent
 - ▶ Need to be *fresh*
 - ▶ Need to be owned by exactly one parent
- ▶ Function like normal AST nodes
 - ▶ Can define / inherit attributes
 - ▶ Can participate in collection attributes

Building CFGs

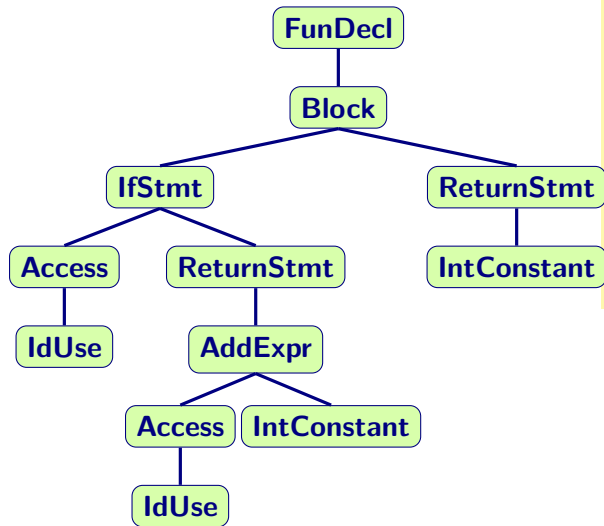
▶ 1. CFG separate from AST

- ▶ AST $\Rightarrow$ CFG translation
- ▶ Often simplified
- ▶ *Advantages:*
 - ▶ Fewer CFG node types
 - ▶ Analyses can communicate results by transforming CFG (Remove unreachable CFG nodes etc.)
 - ▶ *Common in compiler mid-ends/back-ends*

▶ 2. CFG is part of AST

- ▶ Some AST nodes are also CFG nodes
- ▶ *Advantages:*
 - ▶ No translation needed
 - ▶ *Common in compiler front-ends and IDEs*
 - ▶ **Teal:** Uses JastAdd's IntraCFG framework

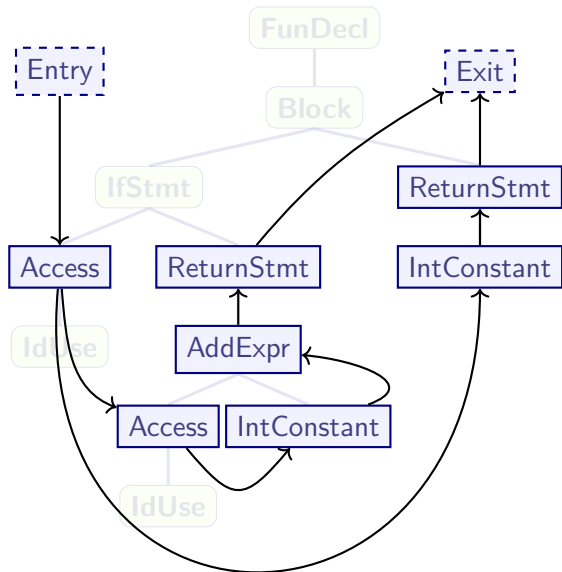
CFGs on the AST



Teal

```
fun f(x) = {  
    if x {  
        return x + 1;  
    }  
    return 0;  
}
```


CFGs on the AST



- ▶ Some **ASTNode**s are **CFGNodes**
 - ▶ For example, **Teal**:
 - ▶ All **Expr**, some **Stmt**
 - ▶ Special NTAs: **Entry**, **Exit**,...
- ▶ **CFGNode.succ()** attribute for CFG successors: **N1** → **N2**

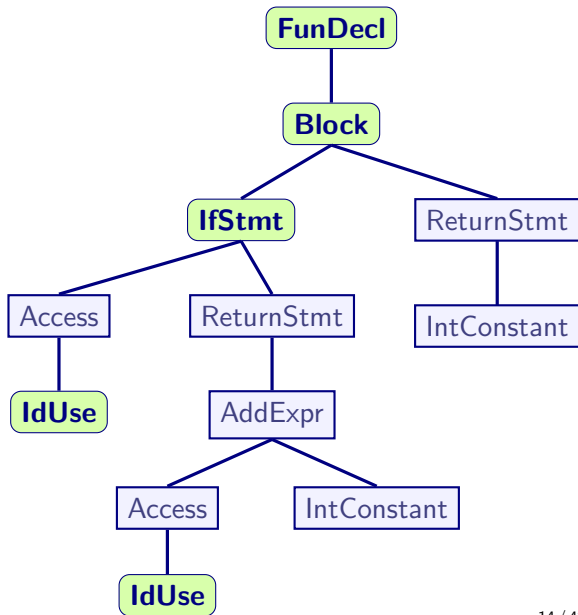
Teal

```
fun f(x) = {
  if x {
    return x + 1;
  }
  return 0;
}
```

Constructing CFGs on the AST (1/2)

- Categorise AST nodes by role in CFG

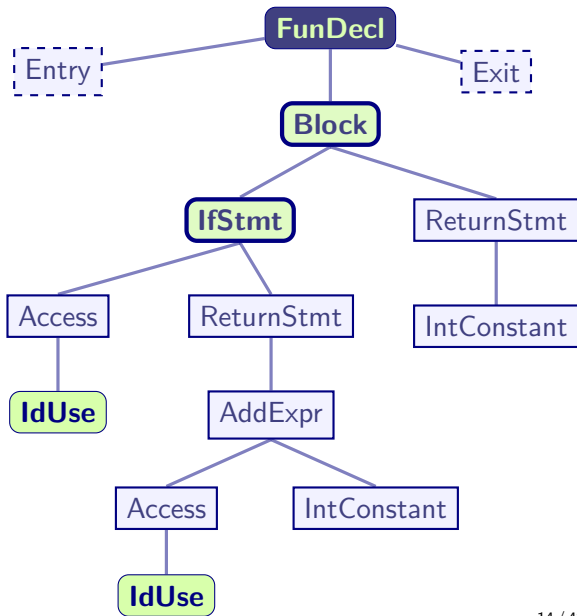
- **CFGNode**: part of CFG



Constructing CFGs on the AST (1/2)

- Categorise AST nodes by role in CFG

- **CFGNode**: part of CFG
- **CFGRoot**: start/end of CFG with **Entry** / **Exit** NTAs (e.g., FunDecl)
- **CFGSupport**: not part of CFG but influence CFG edges



Constructing CFGs on the AST (2/2)

```
interface CFGNode extends CFGSupport;
```

```
→ syn Set CFGSupport.firstNodes()
```

```
→ inh Set CFGSupport.nextNodes()
```

```
AddExpr.firstNodes() = getLeft().firstNodes()
```

```
AddExpr.getLeft().nextNodes() = getRight().firstNodes()
```

```
AddExpr.getRight().nextNodes() = new Set({this})
```

```
BlockStmt.firstNodes() = getStmt(0).firstNodes()
```

```
BlockStmt.getStmt(i).nextNodes() =
```

```
    if (i < size): getStmt(i+1).firstNodes()
```

```
    else: this.nextNodes()
```

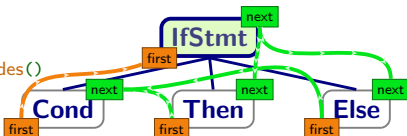
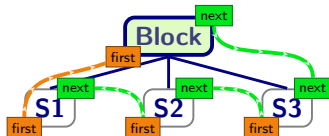
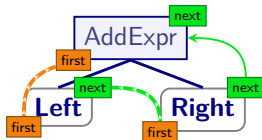
```
IfStmt.firstNodes() = getCond().firstNodes()
```

```
IfStmt.getCond().nextNodes() =
```

```
    getThen().firstNodes() ∪ getElse().firstNodes()
```

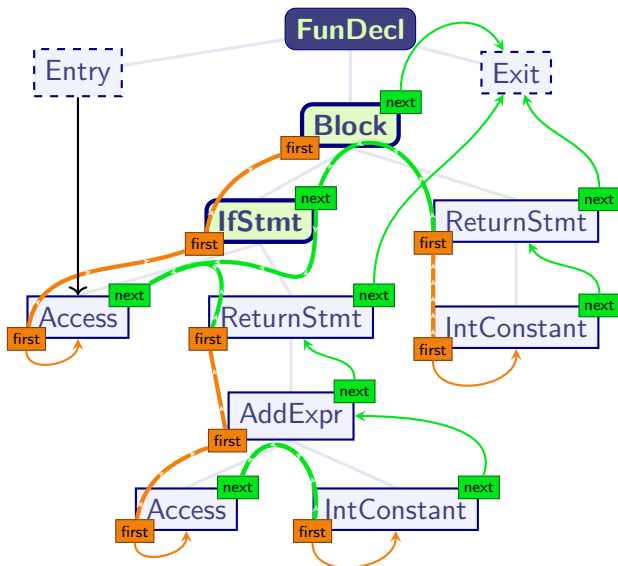
```
IfStmt.getThen().nextNodes() = this.nextNodes()
```

```
IfStmt.getElse().nextNodes() = this.nextNodes()
```



Constructing CFGs on the AST (1/2)

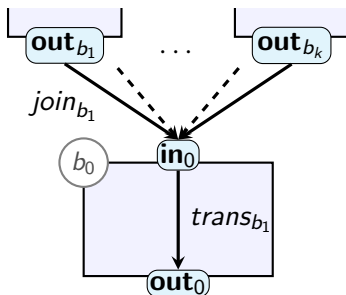
- ▶ Categorise AST nodes by role in CFG
 - ▶ **CFGNode**: part of CFG
 - ▶ **CFGRoot**: start/end of CFG with **Entry** / **Exit** NTAs (e.g., FunDecl)
 - ▶ **CFGSupport**: not part of CFG but influence CFG edges
 - ▶ **ASTNode**s: can ignore
 - ▶ Construct edges
 - ▶ For each subtree: first CFGNodes in subtree?
 - ▶ For each CFGNode: next CFGNodes after self?
- `succ()`
→ `firstNodes()`
→ `nextNodes()`



Summary

- ▶ CFG can be separate or overlaid on AST
- ▶ Teal uses an overlay CFG
- ▶ CFGNodes are:
 - ▶ ASTNodes that participate in CFG
 - ▶ Some NTAs:
 - ▶ **Entry**: Subprogram start
 - ▶ **Exit**: Subprogram end
 - ▶ Others can be useful e.g. for exception handling
- ▶ Constructing CFG with IntraCFG:
 - ▶ **firstNodes**: For this *subtree*, which CFGNodes execute first?
synthesised attribute
 - ▶ **nextNodes**: For this *CFGNode*, which CFGNodes execute next?
inherited attribute

Implementing Data Flow Analysis



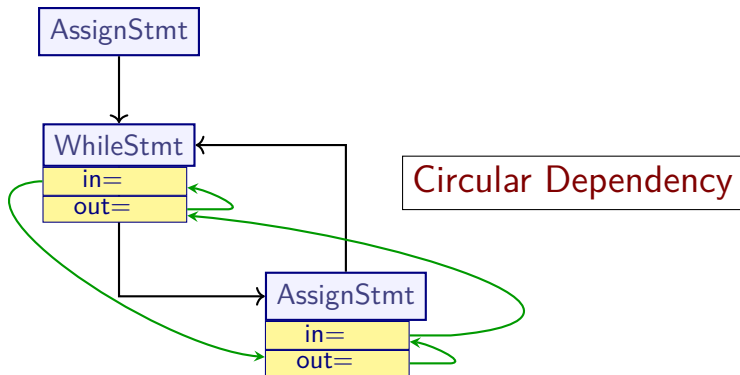
JastAdd

```
syn Lattice CFGNode.in() {  
    Lattice r =  $\perp$ ;  
    for (CFGNode b: pred()) {  
        r = r  $\sqcup$  b.out();  
    }  
    return r;  
}  
  
syn Lattice CFGNode.out() {  
    return trans(in());  
}
```

JastAdd

```
// Default: trans() is no-op  
syn Lattice CFGNode.trans(Lattice v) = v;  
  
// Specialised transfer function  
syn Lattice AssignStmt.trans(Lattice v) = ...
```

Fixpoints and Reference Attributes



This solution is *not well-defined*

Fixpoints from Circular Attributes

JastAdd

```
syn Lattice CFGNode.out() circular [Lattice.bottom()];
```

↑
Lattice

↖
⊥

► Circular Attributes

- JastAdd allows circular dependency *if explicitly declared*

- `CFGNode.out()` can now recursively call itself

$v_1 = \perp$ = `CFGNode.out()` at iteration 1

v_2 = `CFGNode.out()` at iteration 2

...

v_k = `CFGNode.out()` at iteration k

- Iterates until $v_{k-1}.equals(v_k)$

Beware

- Your lattice must have a correct `equals()` method
- You must be in a monotone framework

*JastAdd can **not** check for this!*

Implementation Strategy

- Definitions for analysis a on lattice $\mathcal{L}$:

Attribute	Forward	Backward
$\mathcal{L} \ a \text{In}()$	$\sqcup \{p. a \text{Out}() \mid p \in \text{pred}()\}$	$a \text{Transfer}(a \text{Out}())$
$\mathcal{L} \ a \text{Transfer}(\mathcal{L})$	transfer fn	transfer fn
$\mathcal{L} \ a \text{Out}():$	$a \text{Transfer}(a \text{In}())$	$\sqcup \{p. a \text{In}() \mid p \in \text{succ}()\}$

- Not necessary to declare all attributes as circular
 - Only one attribute in each cycle need be circular
- ... but can declare circular if unsure

Combining with Expressions

- ▶ Data flow analysis resolves *flow-sensitive* information
- ▶ May still combine with simple synthesised attributes

Example: Constant propagation / folding

- ▶ computing `Expr.constantValue()`
- ▶ `Access.constantValue()  $\Leftarrow$  constantOut()`
- ▶ but `AddExpr.constantValue()` will not typically use `constantOut()` / `constantIn()` directly
- ▶ uses `getLeft().constantValue()` / `getRight().constantValue()`

Summary

- ▶ Attributes that depend on themselves:
Usually $\implies$ *AG not well-defined*
- ▶ **Circular** attributes are exception
 - ▶ JastAdd suppresses recursion check
 - ▶ Repeated evaluation
 - ▶ Evaluation stops once current result `.equals()` last result
- ▶ It is up to attribute definition to guarantee termination!
 - ▶ Monotone framework
 - ▶ *Finite lattice height*
 - ▶ (Or *widening*, later today)

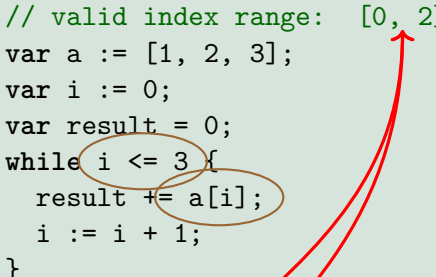
Dimensions of Data Flow

- ▶ Data Flow analysis is versatile:
- ▶ Can adjusting:
 - ▶ Lattice and transfer functions
 - ▶ Data representation
 - ▶ Specialised lattices
 - ▶ CFG history awareness
 - ▶ Analysis integration (later!)
 - ▶ Treatment of subroutine calls (later!)

Numerical Domains

Teal

```
// valid index range: [0, 2]
var a := [1, 2, 3];
var i := 0;
var result = 0;
while i <= 3 {
  result += a[i];
  i := i + 1;
}
```



- ▶ Bug: i may be 3, and out of bounds for a
- ▶ Analysis: Compute bounding intervals $[min, max]$
 - ▶ **Interval Abstract Domain**
- ▶ $\llbracket i \rrbracket = [0, 3]$

Numerical Domains

Teal

```
var a := [1, 2, 3];  
var i := 0;  
var  $\llbracket i \rrbracket = [0, 2]$  new array[int](3);  
while  $i < 3$  {  
    var j := 0;  
    var c  $\llbracket j \rrbracket = [0, 2]$   
    while  $j < 3 - i$  {  
        c := c + a[i + j];  
         $\llbracket i + j \rrbracket = [0, 4]$   
        j := j + 1;  
    }  
    result[i] := c;  
    i := i + 1;  
}
```

Out of bounds?

- ▶ Guarantee: $j < 3 - i$
 $\implies j + i < 3$
- ▶ Array access is safe!
- ▶ Analysis must capture relations between variables
 - ▶ **Octagon Abstract Domain**

Numerical Domains

- ▶ **Interval Abstract Domain**

- ▶ Constraints: $x \in [min_x, max_x]$

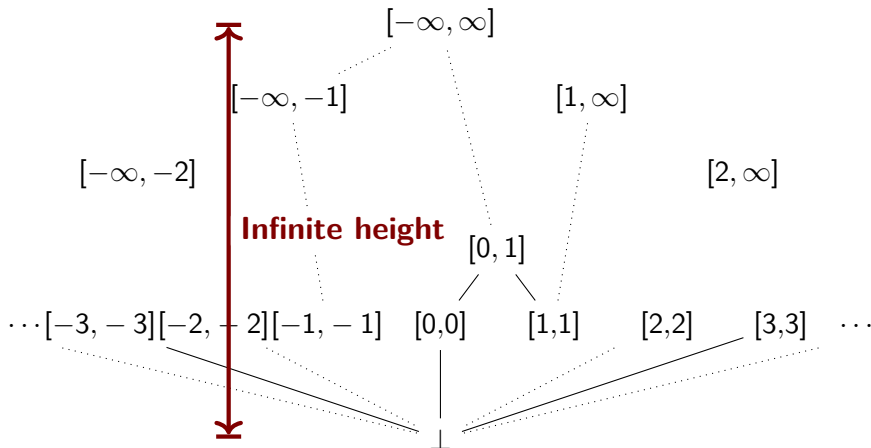
- ▶ **Octagon Abstract Domain**

- ▶ Constraints: $\pm x \pm y \leq c$
- ▶ (x, y variables, c constant number)

- ▶ **Polyhedra Abstract Domain**

- ▶ $c_1x_1 + c_2x_2 + \dots + c_nx_n \leq c_0$
- ▶ $c_1x_1 + c_2x_2 + \dots + c_nx_n = c_0$
- ▶ Increasingly powerful, increasingly expensive to analyse

Interval Domain

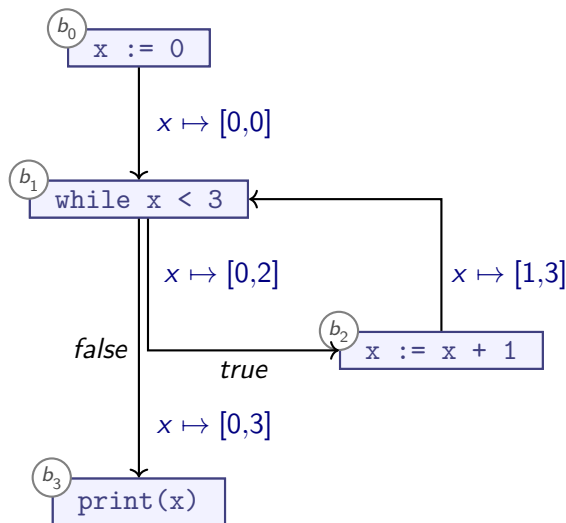


- ▶ $\top = [-\infty, \infty]$
- ▶ $[l_1, r_1] \sqcup [l_2, r_2] = [\min(l_1, l_2), \max(r_1, r_2)]$

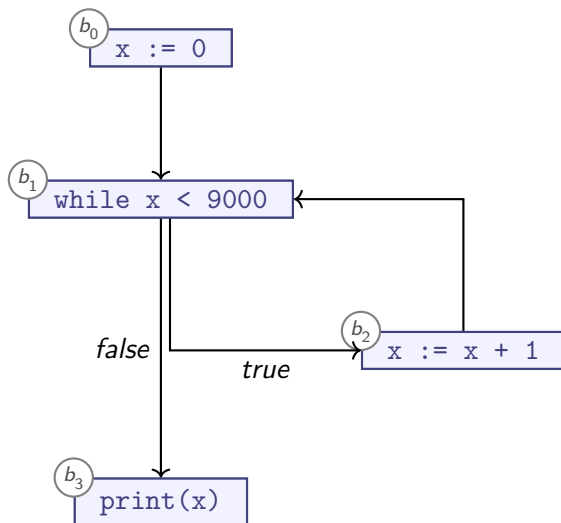
Summary

- ▶ Numerical Abstract Domains capture linear relations between variables and constants
 - ▶ **Interval Abstract Domain:** $x \in [min_x, max_x]$
 - ▶ Octagon Abstract Domain: $\pm x \pm y \leq c$
 - ▶ Polyhedra Abstract Domain: Arbitrary linear relationships
- ▶ Infinite Domain height: No termination guarantee with our current tools

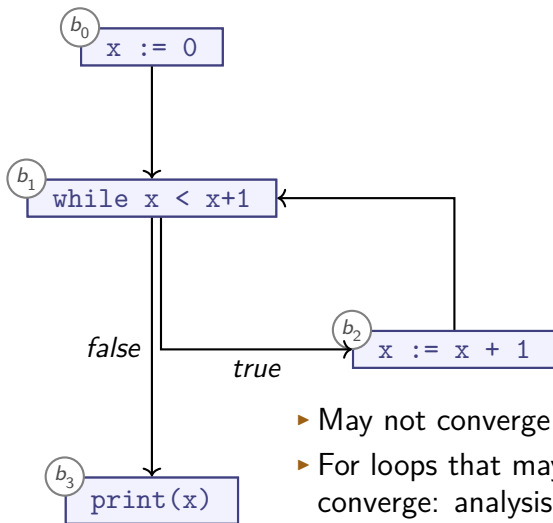
Applying the Interval Domain



Applying the Interval Domain

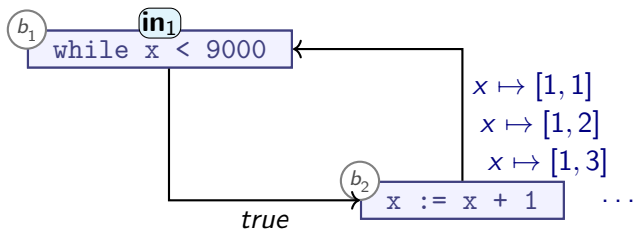


Applying the Interval Domain



- ▶ May not converge
- ▶ For loops that may take long to converge: analysis is slow

Widening



- Inefficient: no reason to assume 2, 3, ... will help us converge
- Detection: when updating in_1 :
 - Check if we have converged
 - Otherwise, **widen**

$$v_1 \nabla v_2 = \begin{cases} v_1 & \iff v_1 = v_2 \\ \mathbf{widen}(v_1 \sqcup v_2) & \iff v_1 \neq v_2 \end{cases}$$

- For a suitable **widen** function

Widening Functions

- ▶ For convergence: satisfy Ascending Chain Condition on:

$$v_{i+1} = \mathbf{widen}(v_i)$$

- ▶ Suitable functions for Interval Domain?

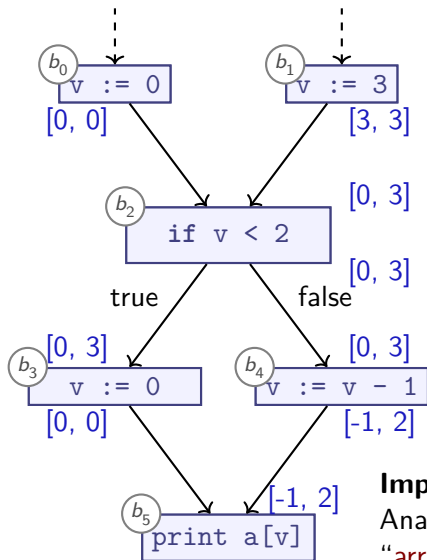
- ▶ $\mathbf{widen}_{\top}(v) = \top$
 - ▶ Very conservative
 - ▶ Ensures convergence
- ▶ $\mathbf{widen}_{10000}([l, r]) = [l - 10000, r + 10000]$
 - ▶ *No convergence*: still allows infinite ascending chain
- ▶ $\mathbf{widen}_{\mathcal{K}}([l, r]) = [\max(\{v \in \mathcal{K} \mid v < l\}), \min(\{v \in \mathcal{K} \mid v > r\})]$
 - ▶ Ensures convergence *iff* $\mathcal{K}$ is finite
 - ▶ Must pick “good” $\mathcal{K}$
 - ▶ Common strategy:
 $\mathcal{K} = \{-\infty, \infty\} \cup$ all numeric literals in program
Our example: $\mathcal{K} = \{-\infty, 0, 1, 9000, \infty\}$

```
var x := 0;
while x < 9000 {
    x := x + 1;
}
```

Summary

- ▶ **Widening** allows us to use infinite domains $\mathcal{L}$
 - ▶ Use **widen** function
 - ▶ **widen** must satisfy Ascending Chain Condition on $\mathcal{L}$
 - ▶ **widen**($\mathcal{L}$) generates finite lattice
 - ▶ Widening operator ∇ applies **widen** function iff needed
 - ▶ Approach:
 - 1 Before analysis runs: we design analysis on infinite-height lattice
 - 2 When analysis runs on concrete program:
 - ▶ **widen** constructs finite-height lattice specific to program
 - ▶ ∇ applies **widen** on demand
- MFP: When updating: $\mathbf{in}_i := \mathbf{in}_i \nabla \mathbf{out}_j$

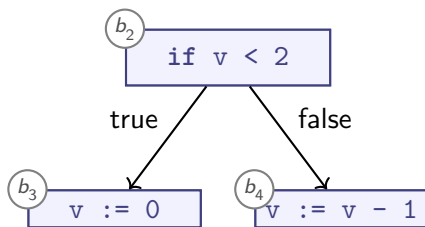
Conditionals



Imprecision can yield false positive
Analysis concludes:
“array index may be negative”

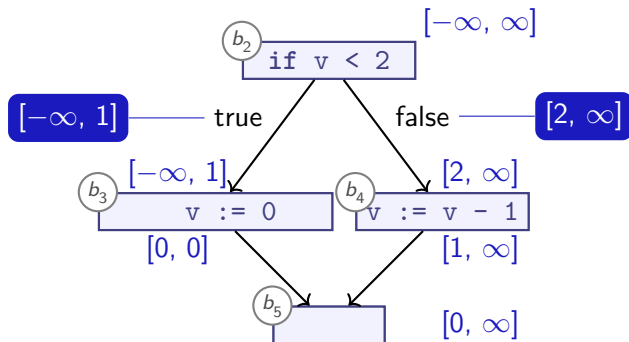
Handling Conditionals (1/2)

- ▶ So far: Did not make use of conditional predicate
 - ▶ true branch: only if $v < 2$
 $v \in [-\infty, 1]$
 - ▶ false branch: only if $\neg(v < 2)$
 $v \in [2, \infty]$



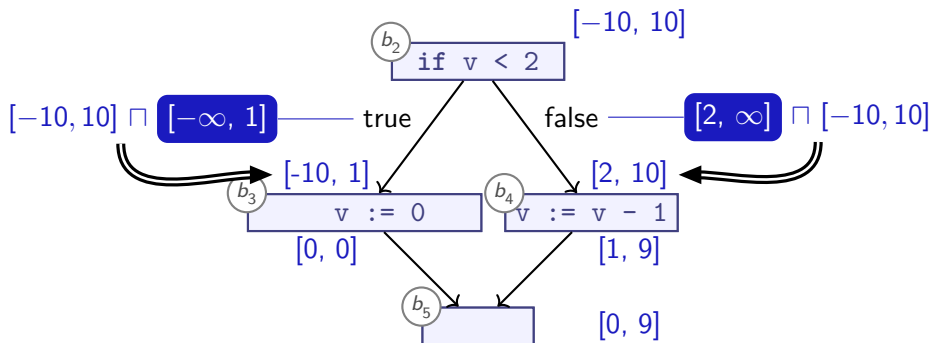
- ▶ **Control Sensitive** analysis utilises this information
 - ▶ Filter possible values

Handling Conditionals (2/2)



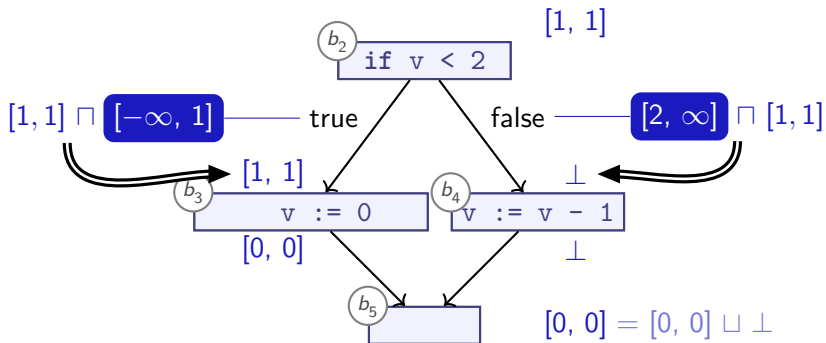
- *Idea*: Split interval for v for true/false branches
 - Analyse each branch with part of original interval
 - “Re-assemble” interval on join afterwards

Handling Conditionals (2/2)



- *Idea*: Split interval for v for true/false branches
 - Analyse each branch with part of original interval
 - “Re-assemble” interval on join afterwards
- If not $v \mapsto \top$:
 - **Filter** with lattice *meet*: $\sqcap$

Contradictions



Summary

- ▶ **Control sensitive** analysis considers conditionals:
 - ▶ May propagate different information along different edges:
 - ▶ **if** P :
 - ▶ Special transfer function for '**assert** P ' on 'true' edge
 - ▶ Special transfer function for '**assert** not P ' on 'false' edge
- ▶ More precise than **control insensitive** analysis
- ▶ Utilises *Lattice Meet* operation $\sqcap$

Intuition: $a \sqcap b$ "satisfy a **and** b "
 $a \sqcup b$ "satisfy a **or** b "
- ▶ $a \sqcap b = \perp$ can happen: *branch will never execute*

Lecture Overview

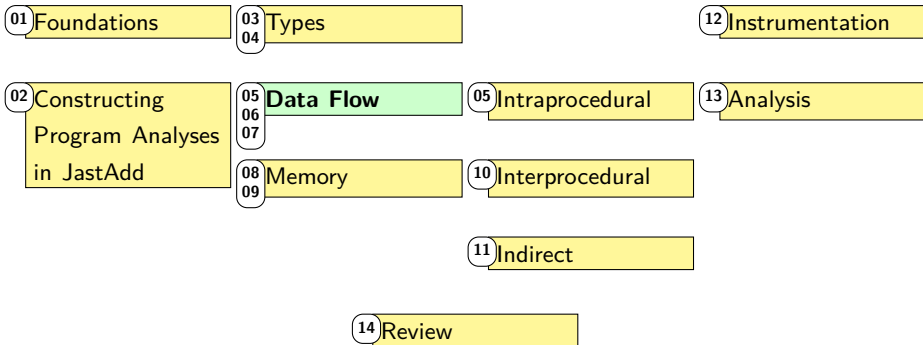
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Properties

Control Flow



Summary and Outlook

- ▶ Summary:
 - ▶ Non-Terminal Attributes
 - ▶ Building CFGs
 - ▶ Circular Attributes
 - ▶ Control Sensitivity & Path Sensitivity
 - ▶ Numerical Domains (esp. the Interval Domain)
 - ▶ Widening
- ▶ Next up: Analysing the Heap

<http://cs.lth.se/EDAP15>