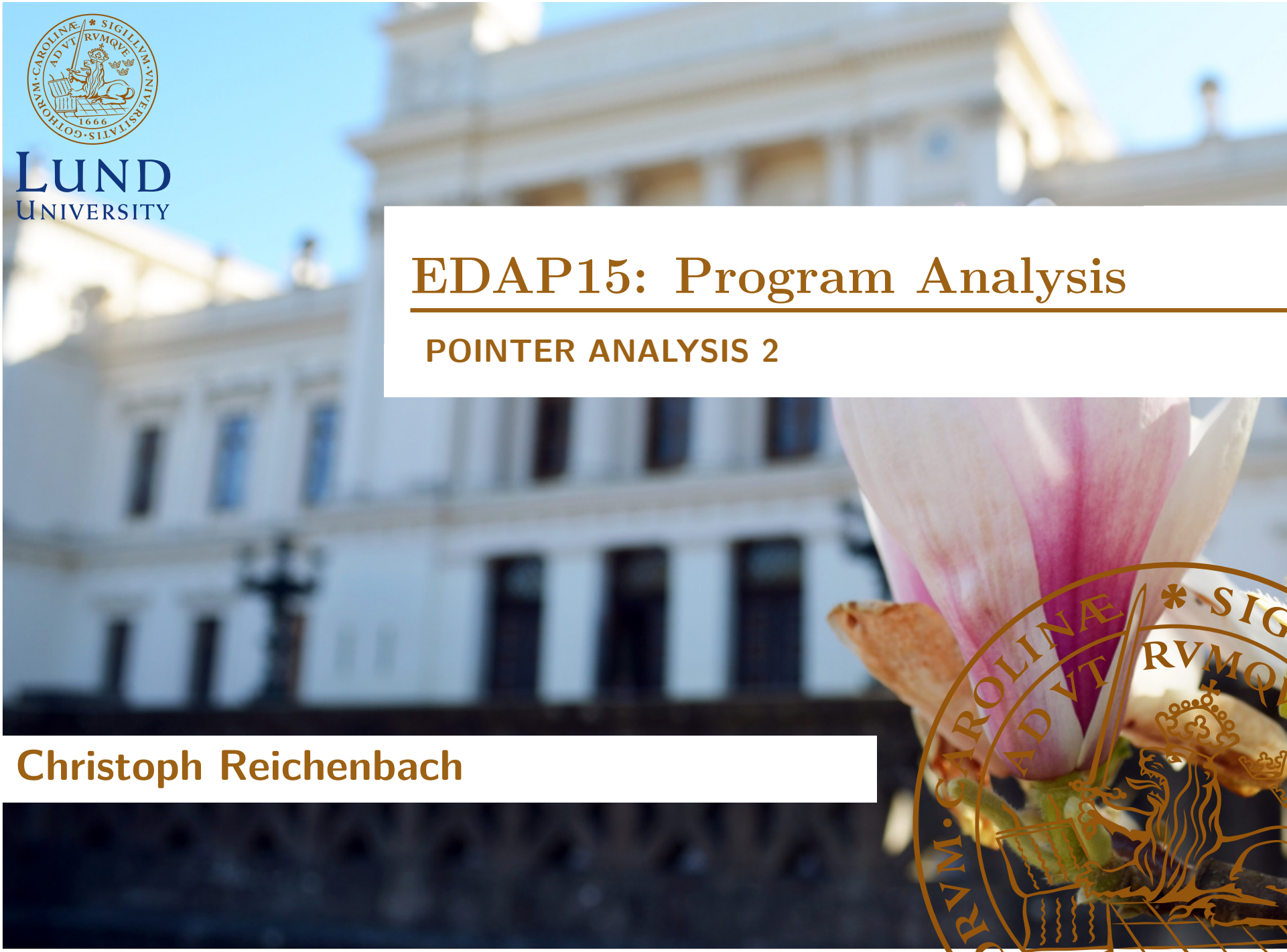


EDAP15: Program Analysis

POINTER ANALYSIS 2

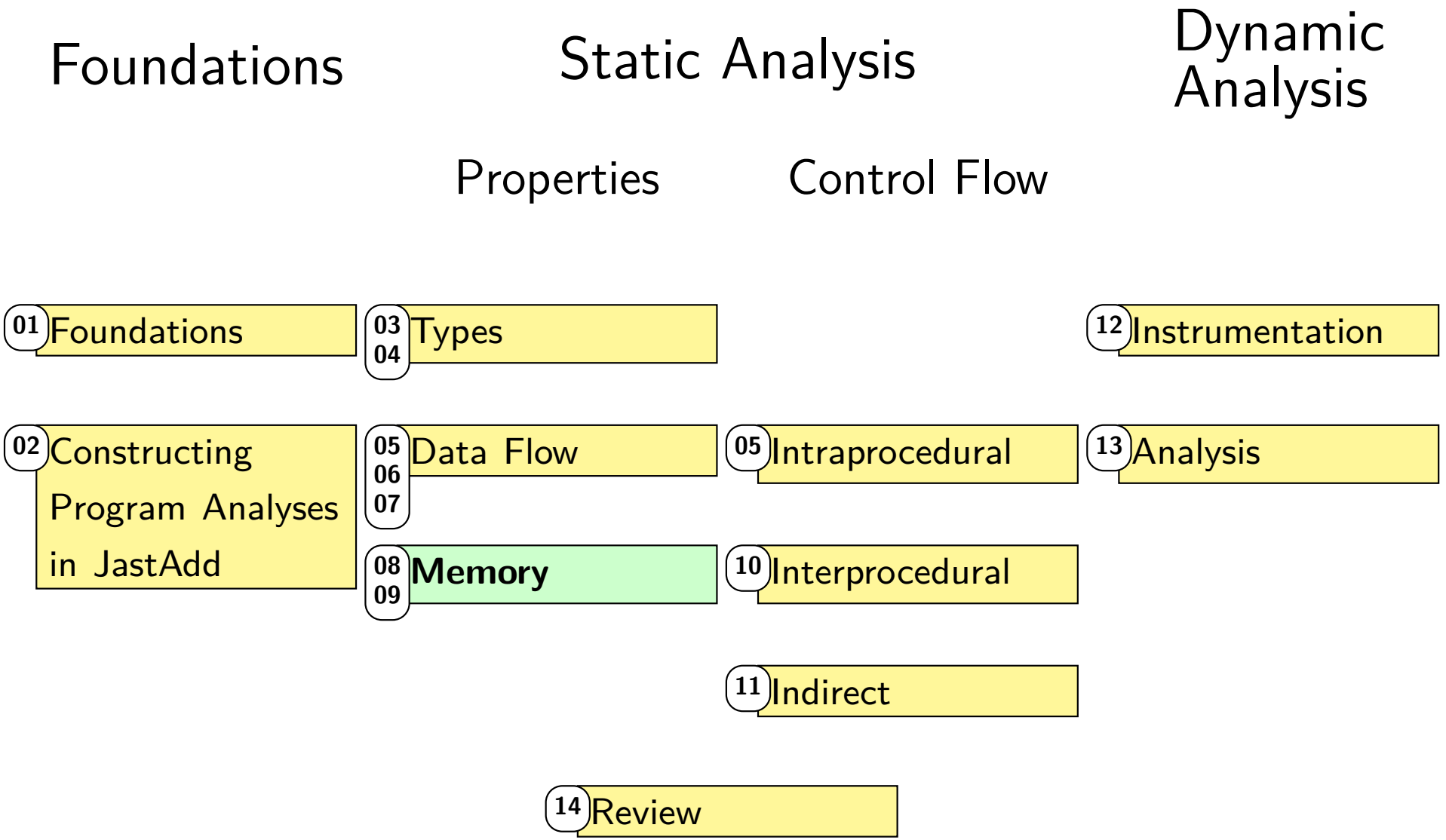
Christoph Reichenbach

Welcome back!

- Student representative?
- Homework 2 (first solo homework) up
- No office hours today

Questions?

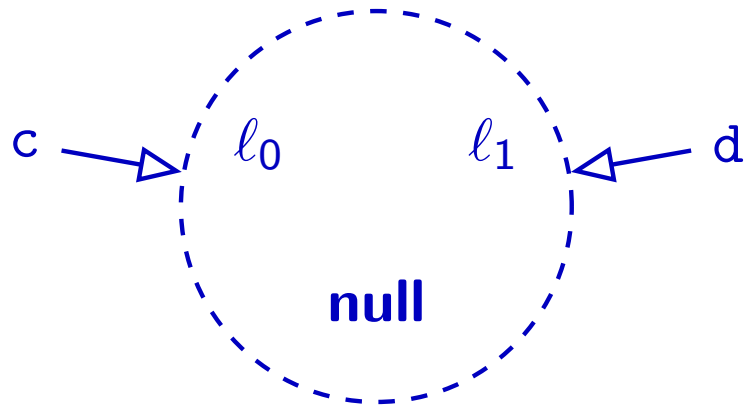
Lecture Overview



Alias Analysis in Practice (1/2)

Teal

```
var c := new_{ℓ0}();
var d := new_{ℓ1}();
if ...  {
   c := null;
} else {
   d := null;
}
```



$c \overset{alias}{=\!=} d$

null as unique memory location: Imprecision!

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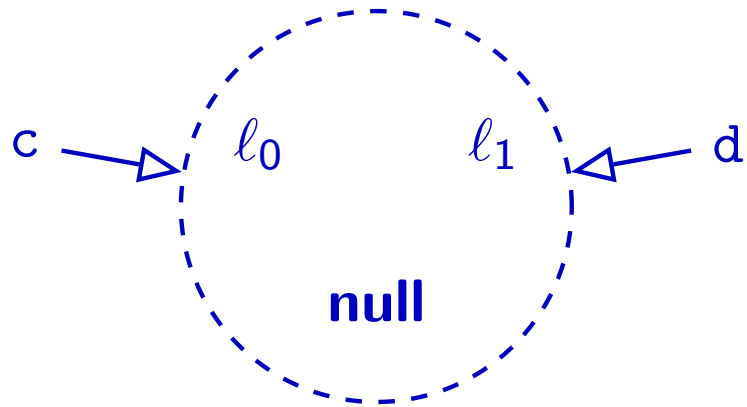

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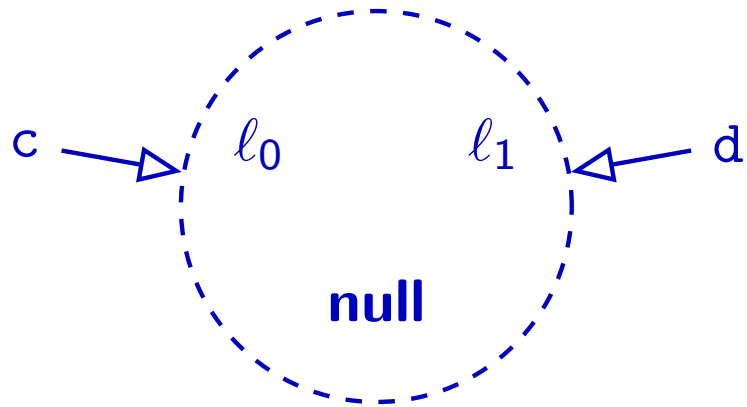

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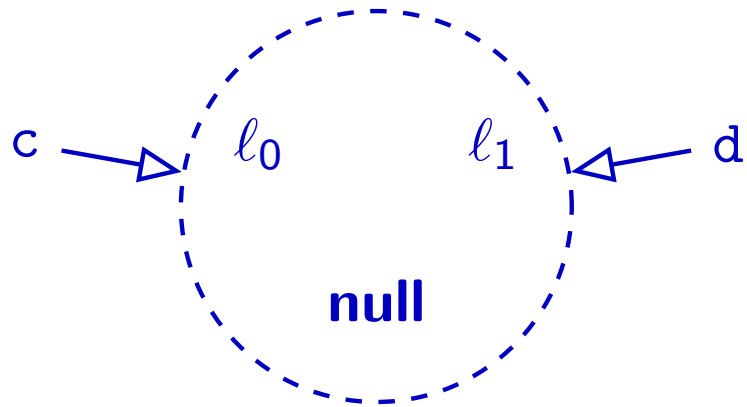
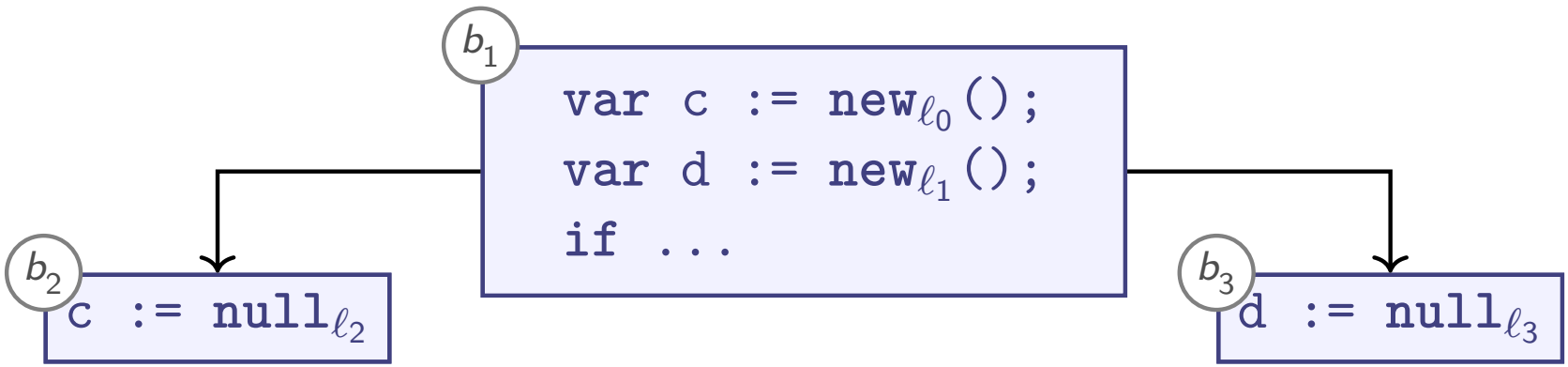


$$c \overset{alias}{=\!=} d$$

null as unique memory location: Imprecision!

Representing Null Pointers



1. One unique **null**

2. Many **null**s (More precise, takes up extra memory)

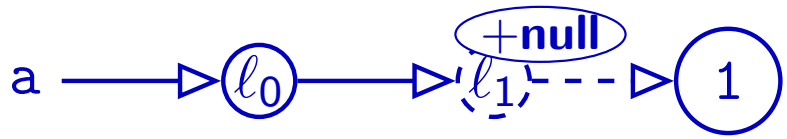
3. Nullness flags (Also more precise, minimal extra memory, but more complex analysis code)

Alias Analysis in Practice (2/2)

Teal

```
var a := new_ℓ0 XY();
a.x := new_ℓ1 XY();
a.x.x := 1;
a.y := null;

print(a.x.x);
// null dereference?
```



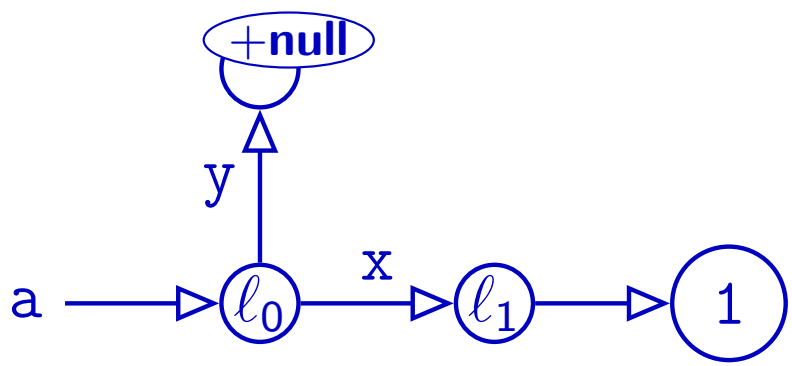
$$\texttt{a.x} \overset{alias}{=\!=} \textbf{null} \overset{alias}{=\!=} \texttt{a.y}$$

Field Sensitivity

- So far, we have merged all fields:

$$\texttt{a.x} \overset{\textit{alias}}{=\!=} \texttt{a.}\square \overset{\textit{alias}}{=\!=} \texttt{a.y}$$

- Points-to analysis so far *field insensitive*
- Analogous for array indices
- A *field-sensitive* analysis would distinguish:



Summary

- Practical points to analysis usually wants to represent **null**
  - Single global **null** may reduce precision (unification-based analysis)
- Simple program analyses are **field insensitive**:

$$\texttt{a.x} \overset{\textit{alias}}{=\!=} \texttt{a.}\square \overset{\textit{alias}}{=\!=} \texttt{a.y}$$

- **Field-sensitive** analyses improve precision by distinguishing fields along points-to edges:

$$\texttt{a.x} \overset{\cancel{\textit{alias}}}{=\!=} \texttt{a.y}$$

- Analogously for **Index-sensitive** analyses (for array indices)

Dataflow-Based Points-To Analysis

$$\begin{array}{lcl} G_{\mathsf{AHG}} & = & \langle \overline{L}, \rightarrow\!\!\triangleright \rangle \\ (\rightarrow\!\!\triangleright) & \subseteq & \overline{L} \times \overline{L} \end{array}$$

- Points-To via Dataflow:
- Lattice over set of edges between memory locations $\overline{L}$
- $\sqcup = \cup$
- $\sqsubseteq = \subseteq$

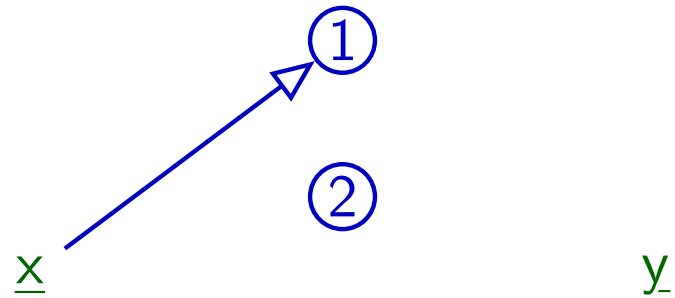
Example: Allocation and Update

Teal

```
    var x := new1();
⇒   var y := new2();

    if ... {
      y := new5();
    }

    x := new7();
    y := x;
```



Case	**Transfer Function**	
$\texttt{x := new}_\ell\texttt{()}$	$\mathit{trans}_1(\rightarrow) = (\rightarrow)$	$\cup\ \{\ x \rightarrow \ell\ \}$

Slight abuse of notation: writing $x \rightarrow \ell$ for $\langle x, \ell \rangle$

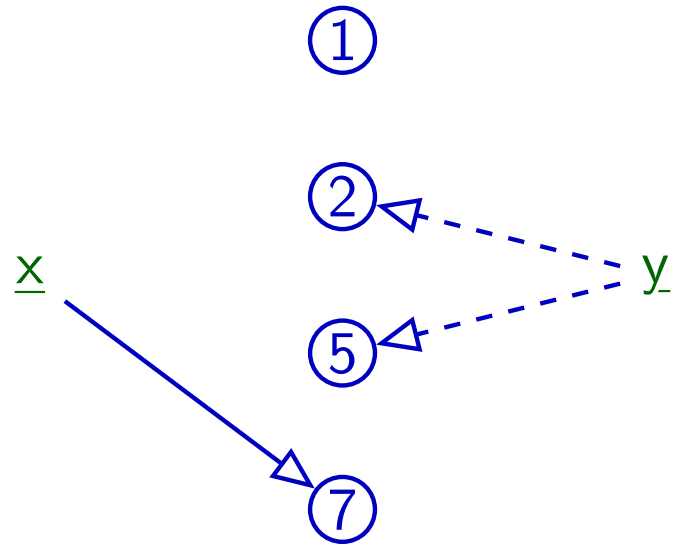
Example: Allocation and Update

Teal

```
    var x := new1();
    var y := new2();

    if ... {
      y := new5();
    }

⇒ x := new7();
    y := x;
```



Remove all $\underline{x}\rightarrow\ell$ for any $\ell \in \overline{L}$

Case	**Transfer Function**
$\underline{x}$:= new$_\ell$()	$trans_1(\rightarrow) = (\rightarrow) \setminus (\{\underline{x}\} \times \overline{L}) \cup \{\, \underline{x}\rightarrow\ell \,\}$

Slight abuse of notation: writing $x\rightarrow\ell$ for $\langle x, \ell \rangle$

Example: Allocation and Update

Teal

```
    var x := new1();
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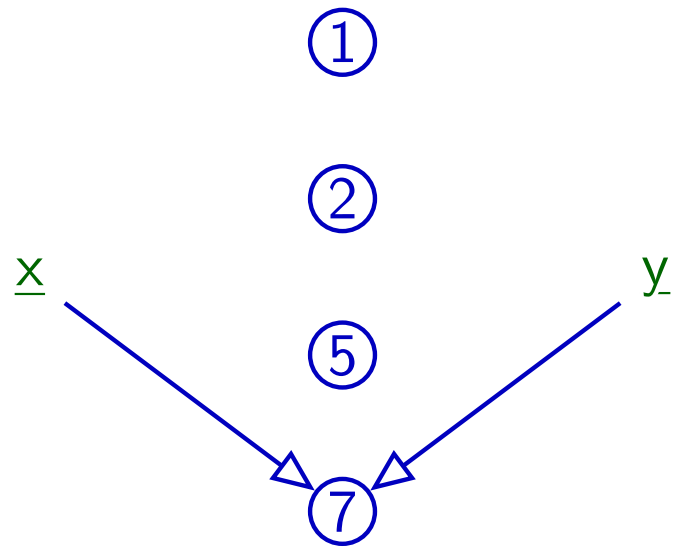
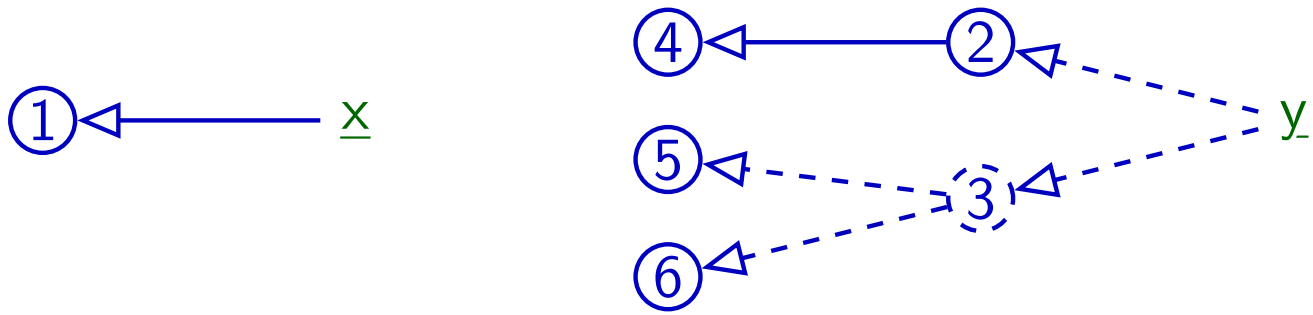


Case	**Transfer Function**
$\mathtt{x} := \mathtt{new}_\ell()$	$trans_1(\rightarrow) = (\rightarrow) \setminus (\{\mathtt{x}\} \times \overline{L}) \cup \{ \mathtt{x} \rightarrow \ell \}$
$\mathtt{x} := \mathtt{y};$	$trans_2(\rightarrow) = (\rightarrow) \setminus (\{\mathtt{x}\} \times \overline{L}) \cup \{ \mathtt{x} \rightarrow \ell \mid \mathtt{y} \rightarrow \ell \}$

Slight abuse of notation: writing $x \rightarrow \ell$ for $\langle x, \ell \rangle$

Dereferencing Read

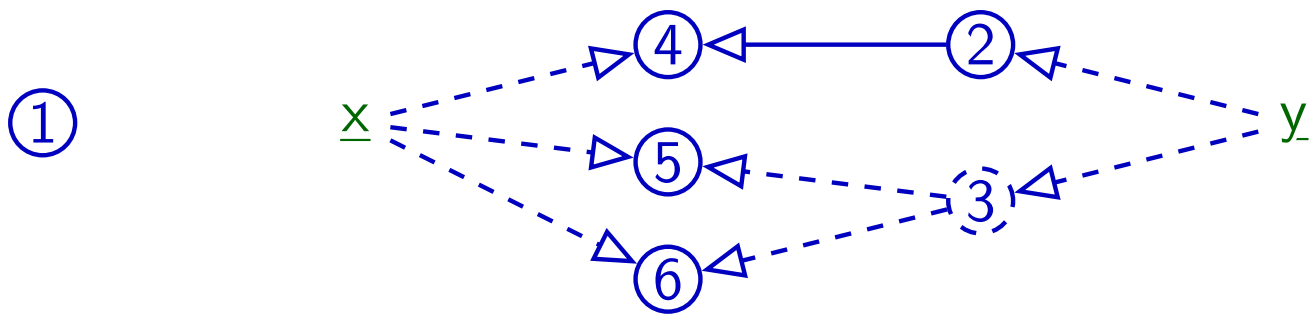
Before:



Teal

```
x := y.f;
```

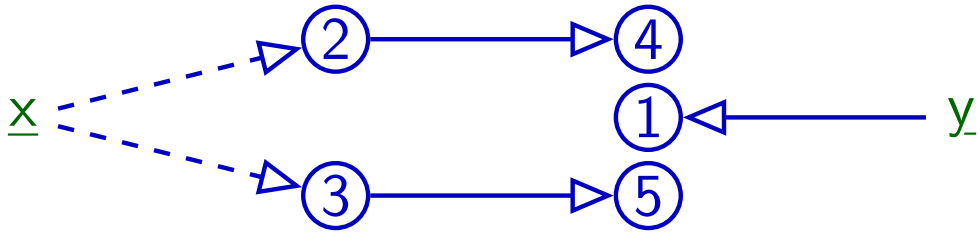
After:



Case	**Transfer Function**
$x := y.\square;$	$trans_3(\rightarrow) = (\rightarrow) \setminus (\{x\} \times \overline{L}) \cup \{\, x \rightarrow \ell' \mid \exists \ell.\ y \rightarrow \ell \rightarrow \ell' \}$

Dereferencing Write

Before:

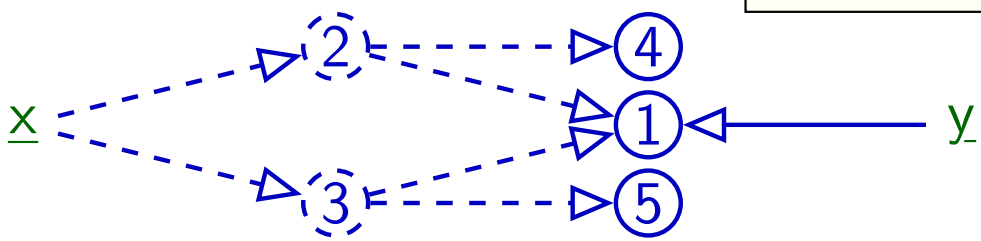


Teal

```
x.f := y;
```

Cannot remove edges: We don't know if it was 2 or 3 that was changed!

After:



Case	**Transfer Function**
$\underline{x}.\square := \underline{y};$	$trans_4(\rightarrow) = (\rightarrow) \setminus (\{\underline{x}\} \times \overline{L}) \cup \{\, \ell \rightarrow \ell' \mid \underline{x} \rightarrow \ell, \underline{y} \rightarrow \ell' \}$

Weak vs Strong Update?

- **Strong update**:
  - Can remove "obsolete" information
  - So far our default
- **Weak update**:
  - Cannot remove "obsolete" information
  - Observed with dereferencing write
- Dereferencing write *can* be strong if x can point to only one location

Teal

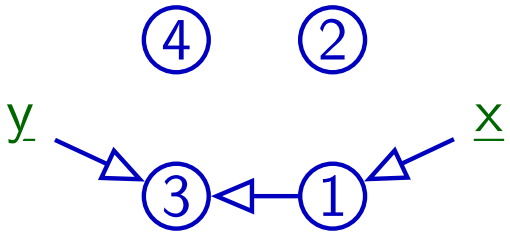
```
x.f := y;
```

Dataflow-Based Points-To Analysis

$$
\begin{array}{lcl}
G_{\mathsf{AHG}} & = & \langle \overline{L}, \rightarrow\!\triangleright \rangle \\
(\rightarrow\!\triangleright) & \subseteq & \overline{L} \times \overline{L}
\end{array}
$$

$\underline{\mathtt{x}}$ `:=` $\mathtt{new}_\ell()$	$\mathit{trans}_1(\rightarrow\!\triangleright) = (\rightarrow\!\triangleright) \setminus (\{\underline{\mathtt{x}}\} \times \overline{L}) \cup \{\ \underline{\mathtt{x}} \rightarrow\!\triangleright \ell\ \}$
$\underline{\mathtt{x}}$ `:=` $\underline{\mathtt{y}}$`;`	$\mathit{trans}_2(\rightarrow\!\triangleright) = (\rightarrow\!\triangleright) \setminus (\{\underline{\mathtt{x}}\} \times \overline{L}) \cup \{\ \underline{\mathtt{x}} \rightarrow\!\triangleright \ell \mid \underline{\mathtt{y}} \rightarrow\!\triangleright \ell\}$
$\underline{\mathtt{x}}$ `:=` $\underline{\mathtt{y}}$`.`$\square$`;`	$\mathit{trans}_3(\rightarrow\!\triangleright) = (\rightarrow\!\triangleright) \setminus (\{\underline{\mathtt{x}}\} \times \overline{L}) \cup \{\ \underline{\mathtt{x}} \rightarrow\!\triangleright \ell' \mid \exists \ell.\ \underline{\mathtt{y}} \rightarrow\!\triangleright \ell \rightarrow\!\triangleright \ell'\}$
$\underline{\mathtt{x}}$`.`$\square$ `:=` $\underline{\mathtt{y}}$`;`	$\mathit{trans}_4(\rightarrow\!\triangleright) = (\rightarrow\!\triangleright) \cup \{\ \ell \rightarrow\!\triangleright \ell' \mid \underline{\mathtt{x}} \rightarrow\!\triangleright \ell, \underline{\mathtt{y}} \rightarrow\!\triangleright \ell'\}$

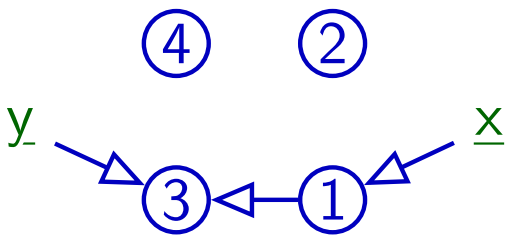
Distributivity?



Teal

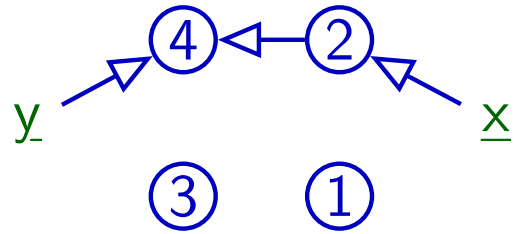
```
x.f := y;
```

trans



Transfer function *trans* does not change either graph in this case

trans



Teal

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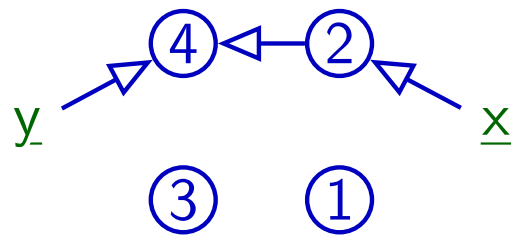

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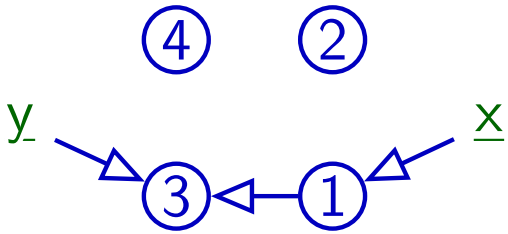

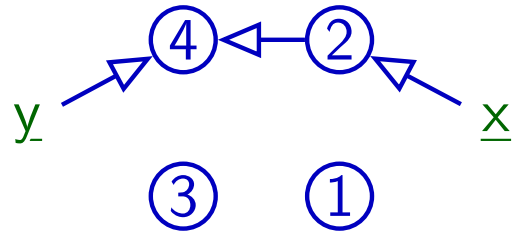

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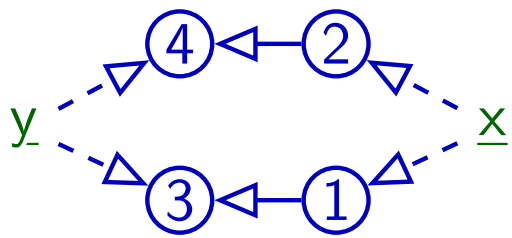

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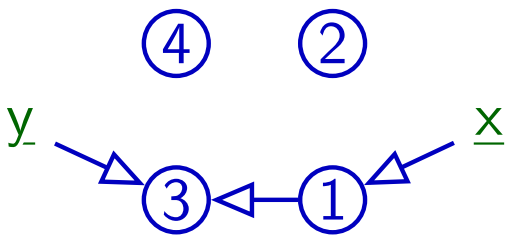
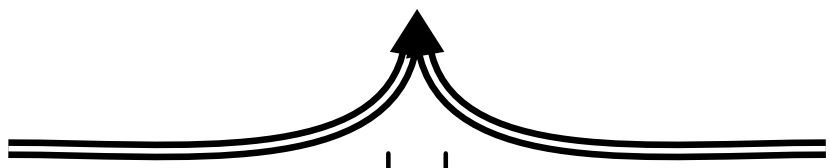
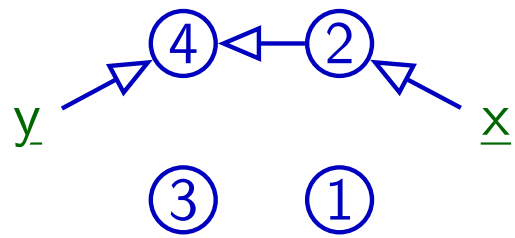
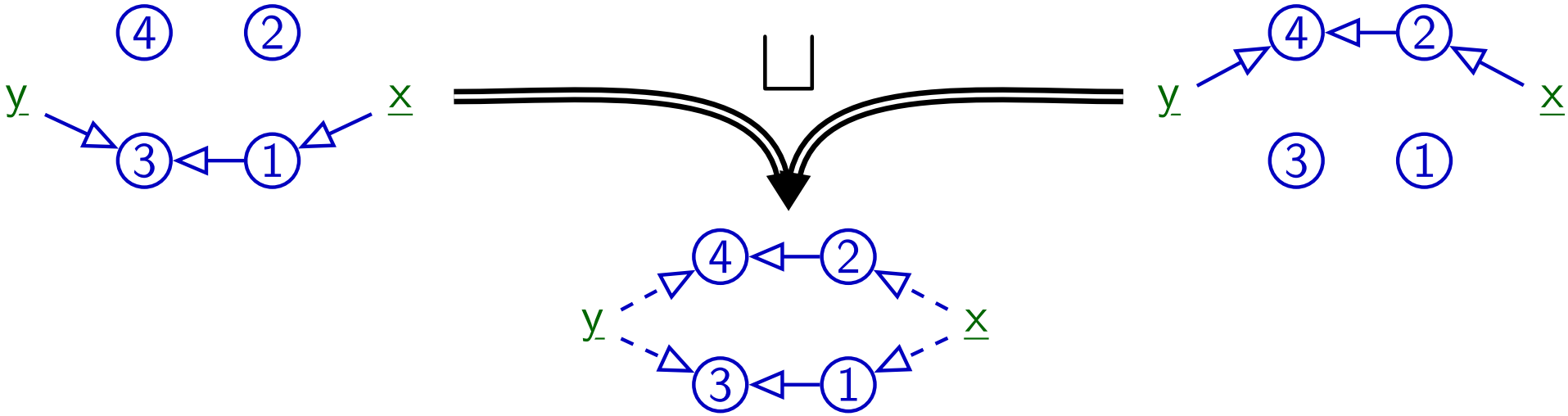




Result for join *after* transfer



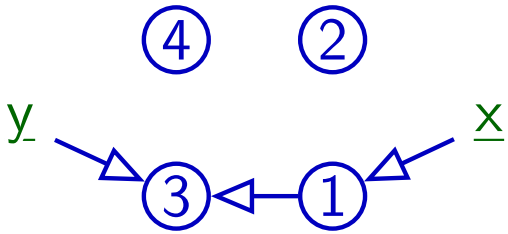
Distributivity?

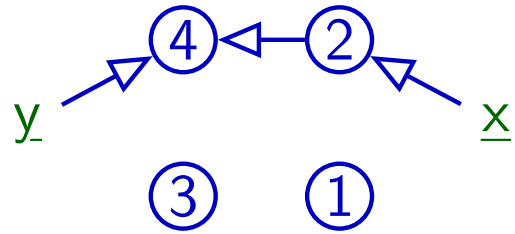


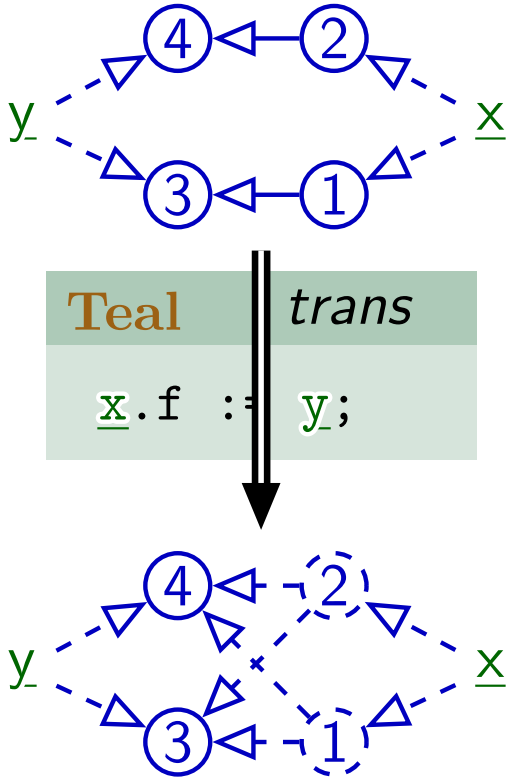
Teal

```
x.f := y;
```

Distributivity?







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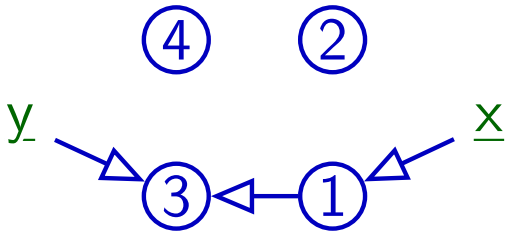
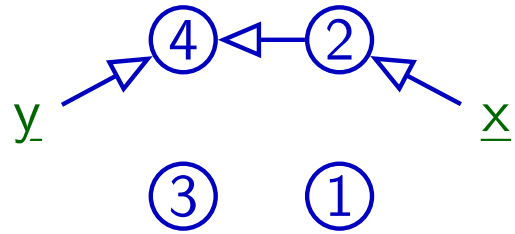
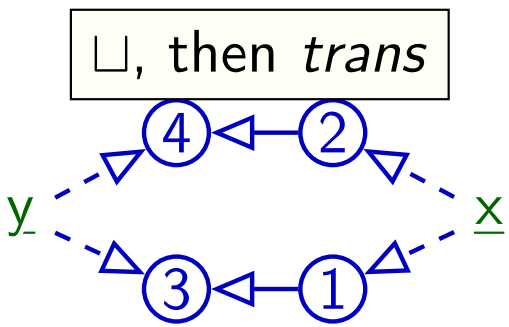
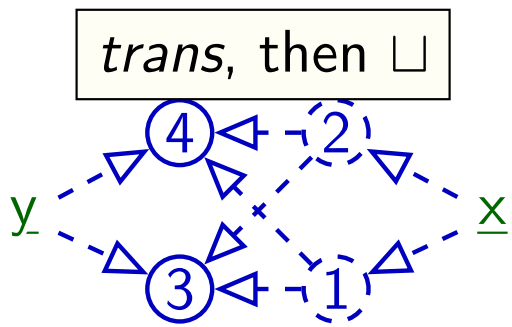








Different results $\Longrightarrow$ not distributive!

Summary

- Flow-sensitive points-to analysis is possible but expensive
- **Weak updates** add new points-to relationship options
  - Don't remove existing options
- **Strong updates** add but also remove points-to relationship options
  - More precise than weak updates
  - Only possible if updated pointer is unambiguous
- *Not Distributive*

Andersen's Points-To Analysis

- Asymptotic performance is $O(n^3)$
- More precise than Steensgaard's analysis
- *Subset-based* (a.k.a. *inclusion-based*)
- $\Longrightarrow$ Flow-insensitive but *directed*
- Popular as basis for current points-to analyses

L. Andersen, "Program Analysis and Specialization for the C Programming Language", PhD. thesis, DIKU report 94/19, 1994

Collecting Constraints

- Collect constraints, resolve as needed
- For each statement in program, we record:
  - If **Referencing** (`x := new`$_{\ell_i}$ `A()`):

$$\ell_i \in pts(\mathtt{x}) \qquad (x \rightarrowtriangle \ell_i)$$

  - If **Aliasing** (`x := y`):

$$pts(\mathtt{x}) \supseteq pts(\mathtt{y})$$

  - If **Dereferencing read** (`x := y.`$\square$):

$$pts(\mathtt{x}) \supseteq pts(\mathtt{y}.\square)$$

  - If **Dereferencing write** (`x.`$\square$ `:= y`):

$$pts(\mathtt{x}.\square) \supseteq pts(\mathtt{y})$$

Solving Constraints

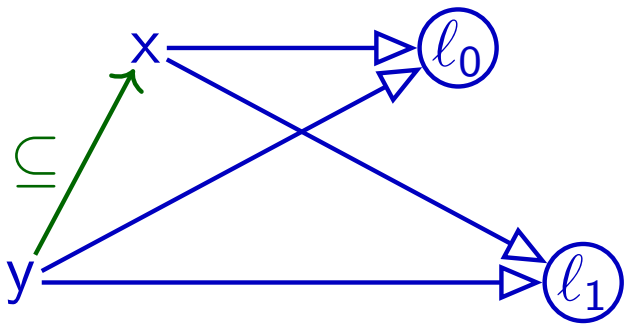
1. Fact extraction:
   - Initial points-to sets: $\ell \in pts(x)$, meaning $\ell \leftarrow x$
   - Constraints:
     - $pts(x) \supseteq pts(y)$
     - $pts(x) \supseteq pts(y.\square)$
     - $pts(x.\square) \supseteq pts(y)$

Subset Constraints (1/2)

- Solving $pts(x) \supseteq pts(y)$

```
y := new_ℓ0();
while ...  {
  x := y;
  y := new_ℓ1();

}
```

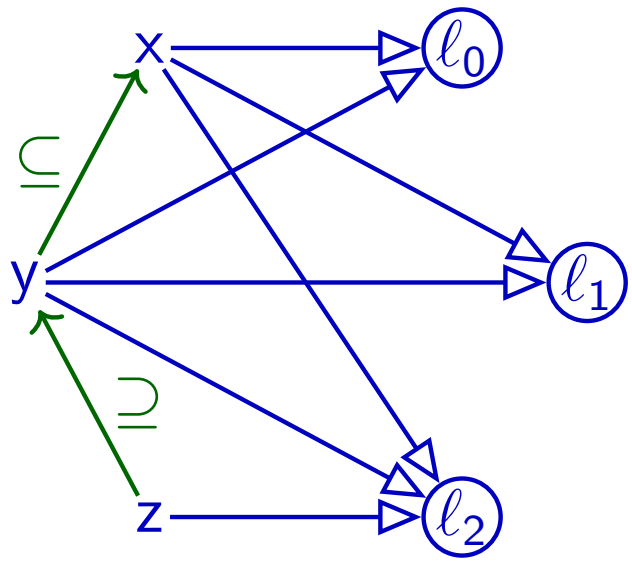


- $\ell \leftarrow y$ and $pts(x) \supseteq pts(y)$:
  $\Longrightarrow \ell \leftarrow x$
- *Flow insensitive*: can't distinguish before/after

Subset Constraints (1/2)

- Solving $pts(x) \supseteq pts(y)$

```
y := new_ℓ0();
while ... {
  x := y;
  y := new_ℓ1();
  z := new_ℓ2();
  if ... {
    y := z;
  }
}
```



- $\ell \leftarrow y$ and $pts(x) \supseteq pts(y)$:
  $\Longrightarrow$ $\ell \leftarrow x$
- *Flow insensitive*: can't distinguish before/after

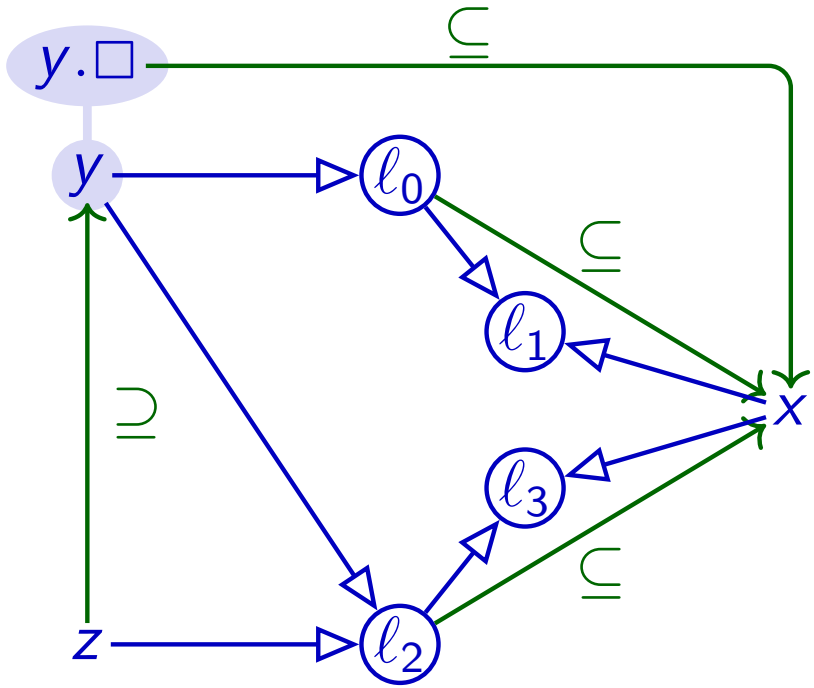
Solving one ($\supseteq$) can depend on all ($\leftarrow$) and ($\supseteq$) in program

Subset Constraints (2/2)

- Solving $pts(x) \supseteq pts(y.\square)$

```
y := new_ℓ0();
y.n := new_ℓ1();
z := new_ℓ2();
z.n := new_ℓ3();
if ... {
  y := z;
}
x := y.n;
```

Simplified presentation (omitting $(\supseteq)$ constraints)



- Recall:
  $\ell \leftarrow z$ and $pts(y) \supseteq pts(z)$:
  $\Longrightarrow \ell \leftarrow y$
- $\ell \leftarrow y$ and $pts(x) \supseteq pts(y.\square)$:
  $\Longrightarrow pts(()x) \supseteq pts(()\ell)$

Fresh Assignments to Fields

- Recall:

  `y.n := new`$_{\ell_1}$`();`
- No direct pattern for this code
- Can model as:

  `var tmp := new`$_{\ell_1}$`();`

  `y.n := tmp;`

Solving Constraints

1. Fact extraction:
   - Initial points-to sets: $\ell \in pts(x)$, meaning $\ell \leftarrow x$
   - Constraints:
     - $pts(x) \supseteq pts(y)$
     - $pts(x) \supseteq pts(y.\square)$
     - $pts(x.\square) \supseteq pts(y)$
2. Build directed *inclusion graph* $G_I = \langle MemLoc, E \rangle$
   - $x \leftarrow y$ represents $pts(x) \supseteq pts(y)$ ("$x := y$")
3. Expand and propagate along inclusion graph:
   - Propagate points-to sets along E:
     - $\ell \leftarrow y$ and $x \leftarrow y$:
       $\Longrightarrow \ell \leftarrow x$
     - $\ell \leftarrow y$ and $x \leftarrow y.\square$:
       $\Longrightarrow x \leftarrow \ell$
     - $\ell \leftarrow x$ and $x.\square \leftarrow y$:
       $\Longrightarrow \ell \leftarrow y$

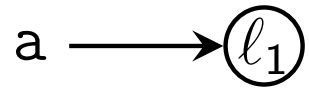
Example

```
⇒ x := new_ℓz     x⇀ℓz
  x := y          x←y
  x := y.□        x←y.□
  x.□ := y        x.□←y
```

Teal

```
var a := new_ℓ1(); //⇐
var b := new_ℓ2();
a := new_ℓ3();
var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
q.n := b;
p := q;
var r := q.n;
```

- **Actual**:



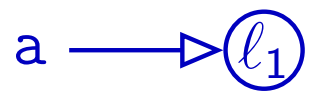
p

q

b

r

- **Andersen**:



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Example

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⇒ x := new_ℓz     x⇀ℓz
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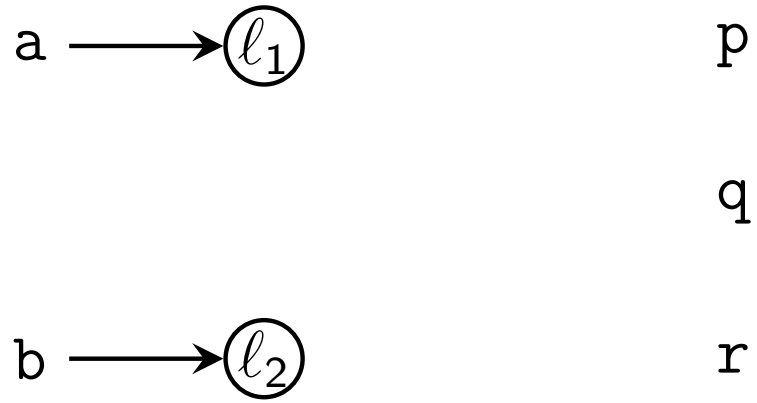
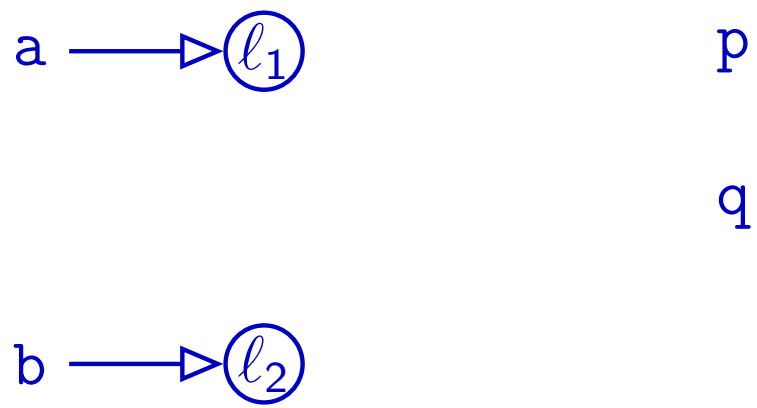


- **Andersen**:



Example

```
⇒ x := new_ℓz     x⇾ℓz
  x := y          x←y
  x := y.□        x←y.□
  x.□ := y        x.□←y
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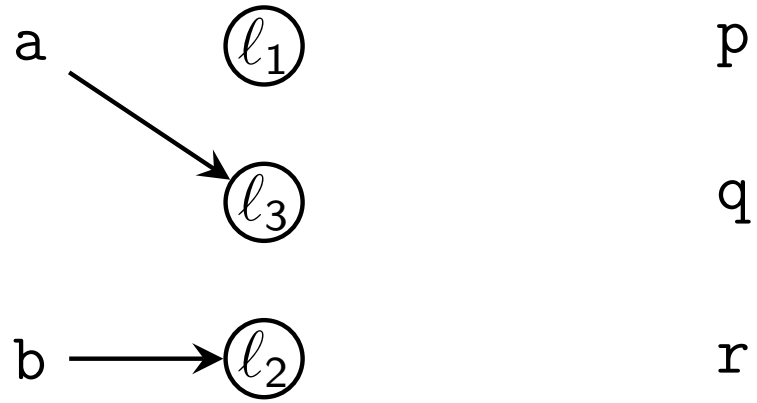
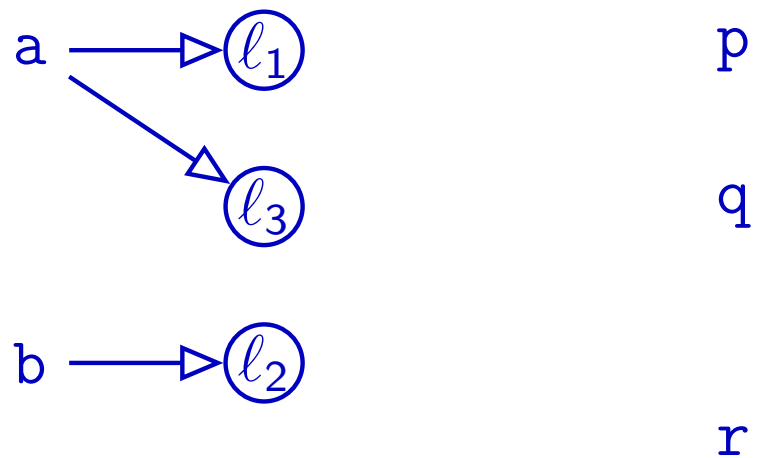


- **Andersen**:



Teal

```
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p := q;
var r := q.n;
```

Example

⇒	`x := new`ℓ_z	$x \rightarrowtriangle \ell_z$
	`x := y`	$x \leftarrow y$
	`x := y.`$\square$	$x \leftarrow y.\square$
	`x.`$\square$ `:= y`	$x.\square \leftarrow y$

Teal

```
var a := new_ℓ1();
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a := new_ℓ3();
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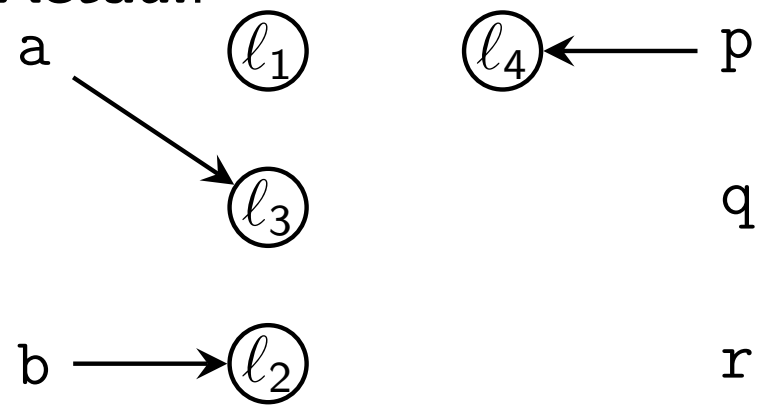
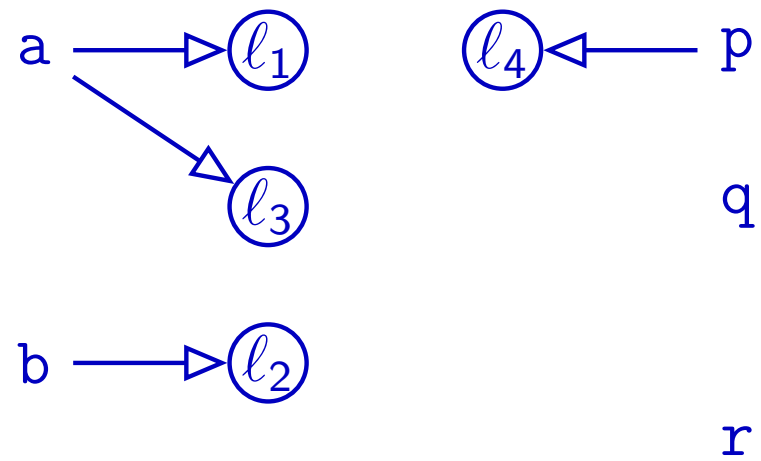


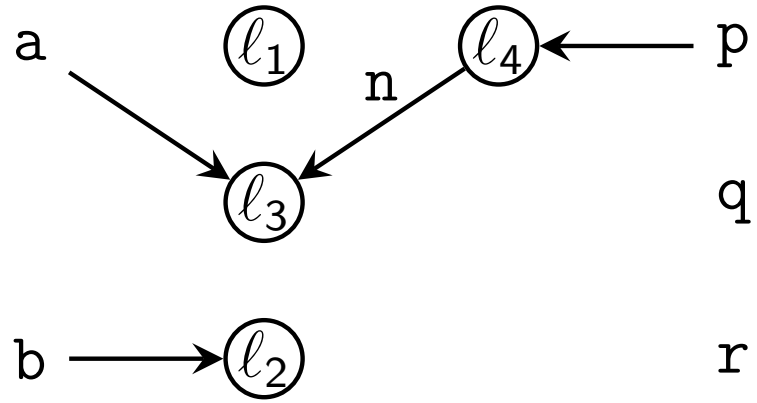
- **Andersen**:



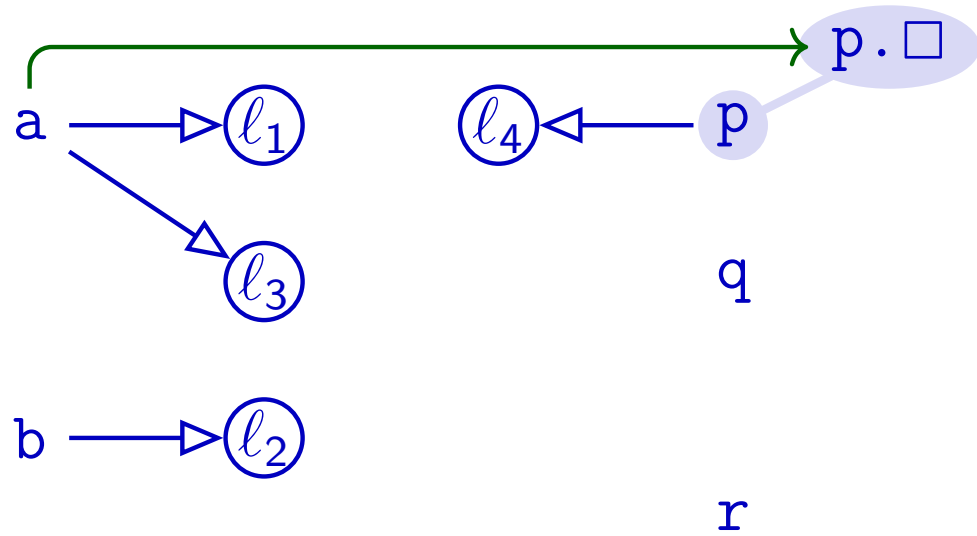
Example

`x := new`ℓ_z	$x \rightarrowtriangle \ell_z$
`x := y`	$x \leftarrow y$
`x := y.`$\square$	$x \leftarrow y.\square$
$\Rightarrow$ `x.`$\square$ `:= y`	$x.\square \leftarrow y$

- **Actual**:



- **Andersen**:



Teal

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p.n := a;          //⇐
var q := new_ℓ6();
q.n := b;
p := q;
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```

Example

⇒	`x := new`ℓ_z	$x \rightarrowtriangle \ell_z$
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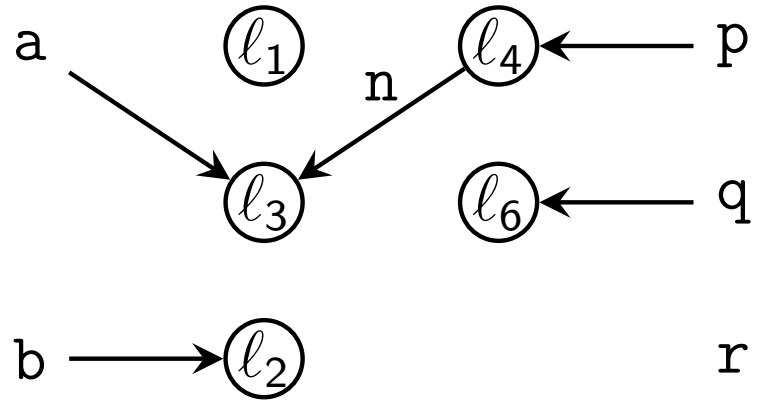

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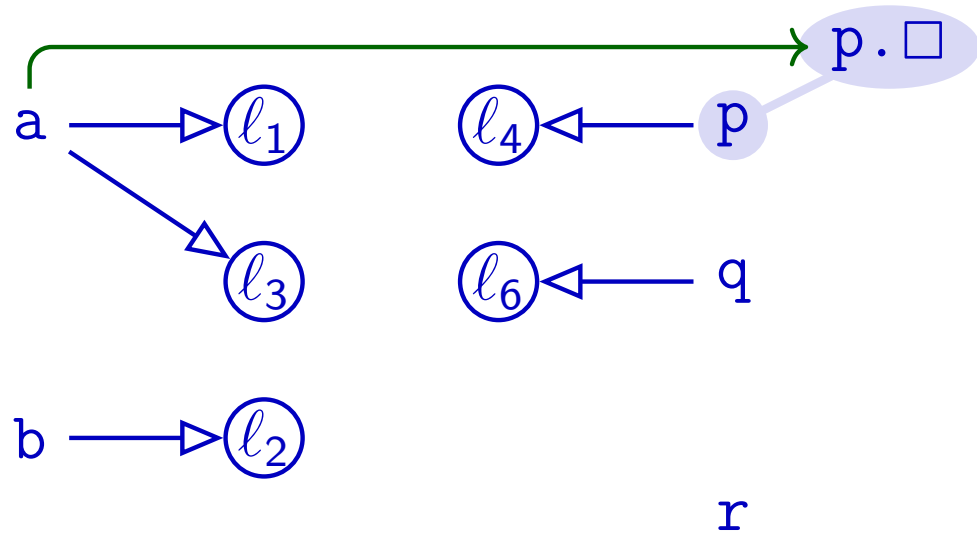

Example

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`x := y`	$x \leftarrow y$
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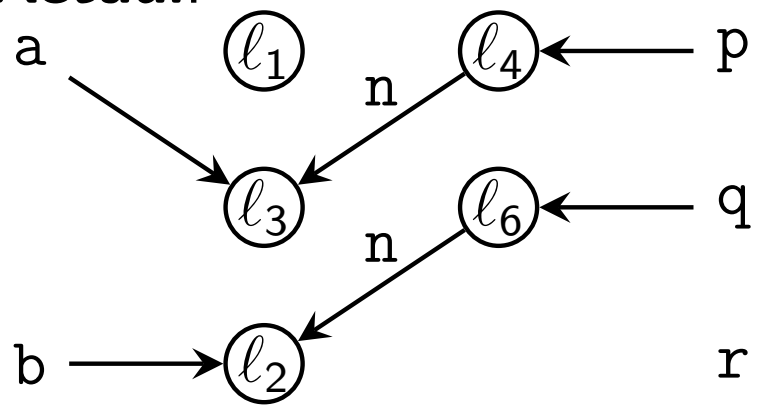

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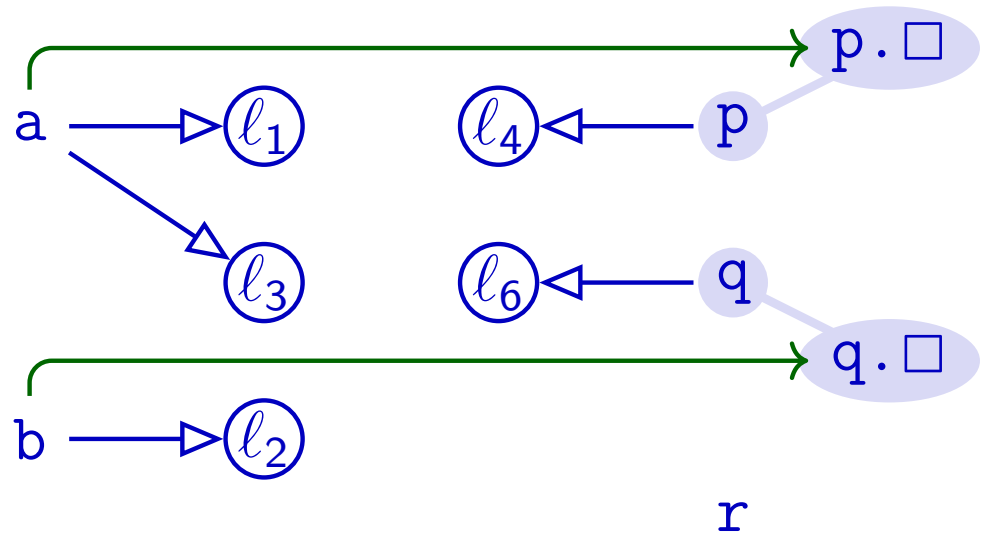

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q.n := b;          //⇐
p := q;
var r := q.n;
```

Example

	x := new$_{\ell_z}$	x$\rightarrowtriangle\ell_z$
$\Rightarrow$	x := y	x$\leftarrow$y
	x := y.$\square$	x$\leftarrow$y.$\square$
	x.$\square$:= y	x.$\square\leftarrow$y

Teal

```
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var q := new_ℓ6();
q.n := b;
p := q;              //⇐
var r := q.n;
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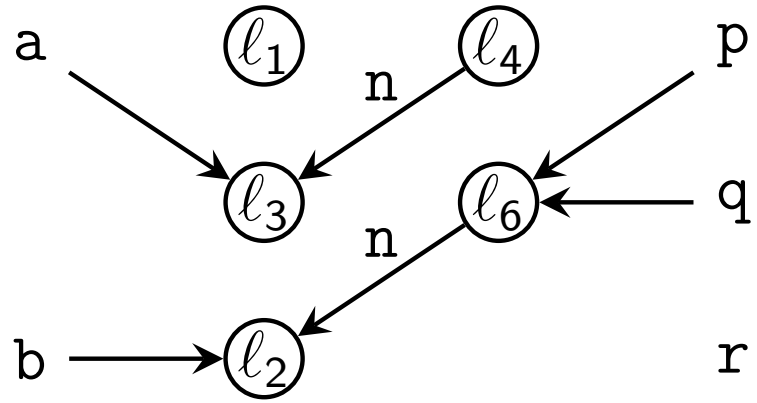
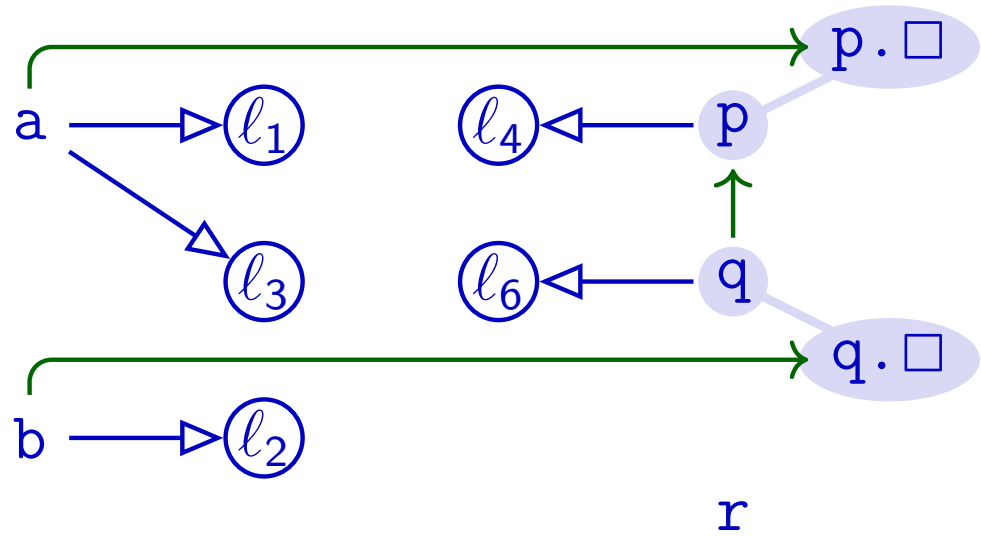
- **Actual**:



- **Andersen**:



Example

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q.n := b;
p := q;
var r := q.n;     //⇐
```

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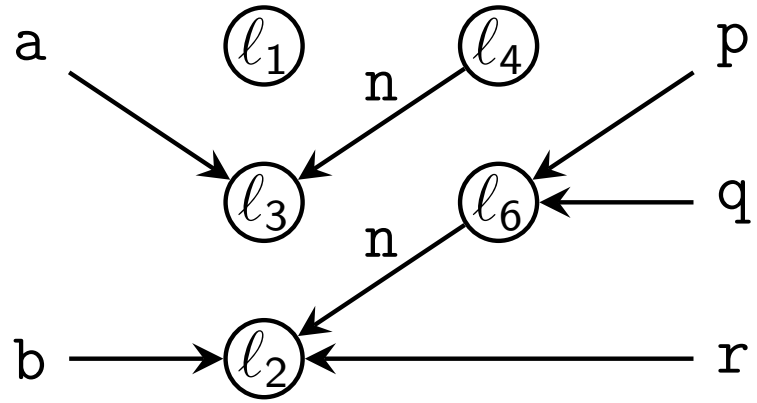

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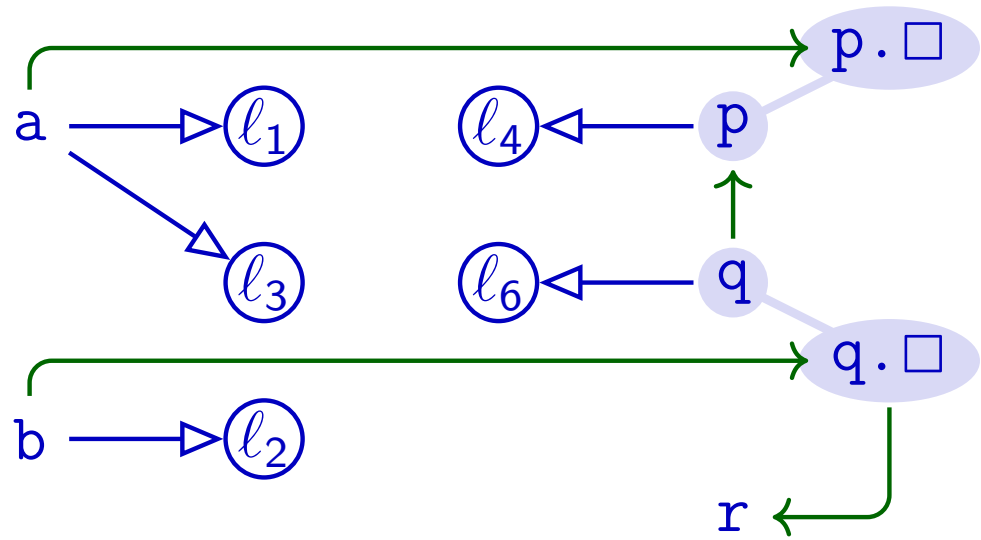

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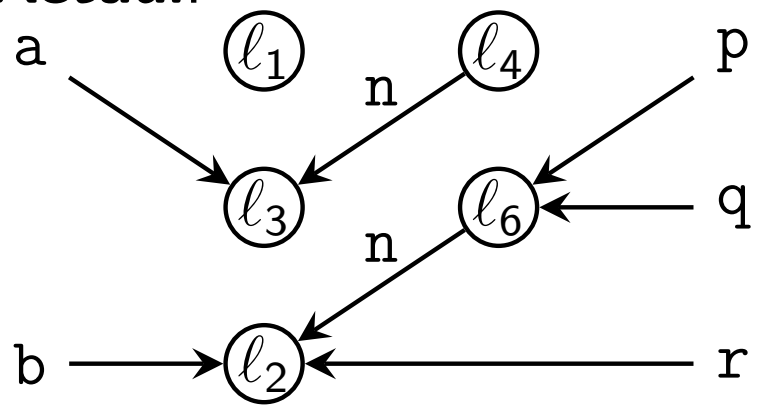
$\ell \leftarrow y$ and $x \leftarrow y \implies \ell \leftarrow x$
$\ell \leftarrow y$ and $x \leftarrow y.\square \implies x \leftarrow \ell$
$\ell \leftarrow x$ and $x.\square \leftarrow y \implies \ell \leftarrow y$

- **Actual**:



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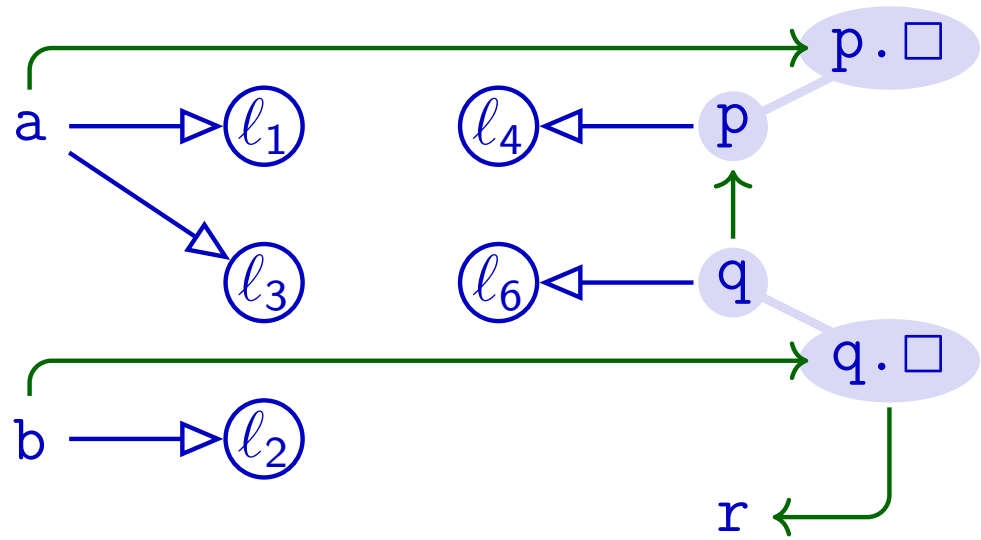

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Andersen's algorithm must propagate along inclusion graph

Example

$\ell \leftarrow y$ and $x \leftarrow y \implies \ell \leftarrow x$
$\ell \leftarrow y$ and $x \leftarrow y.\square \implies x \leftarrow \ell$
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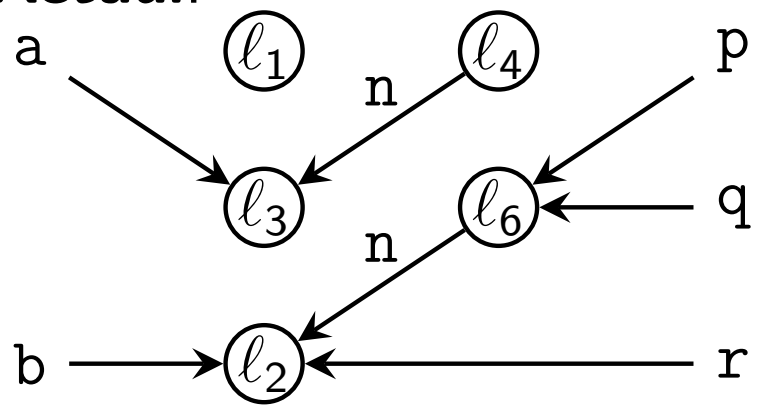

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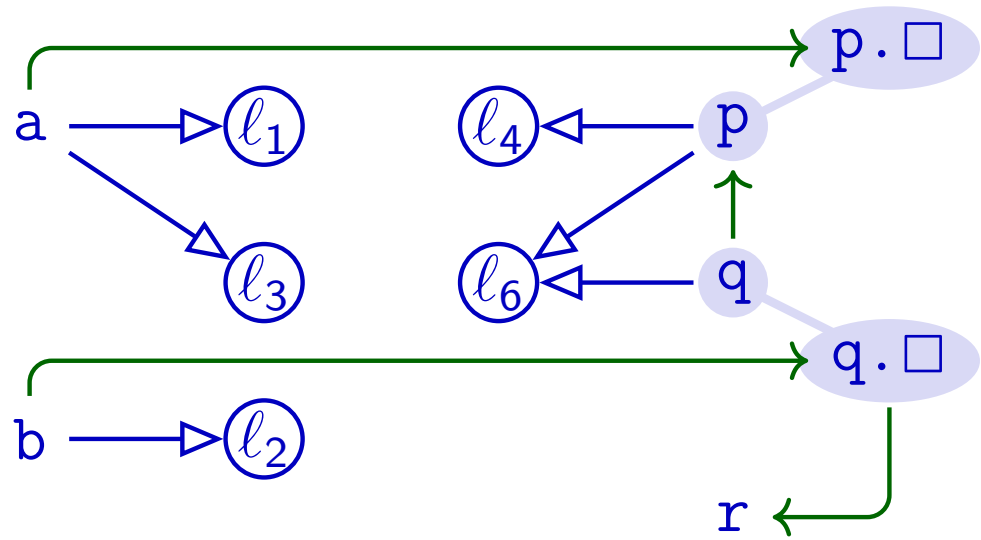

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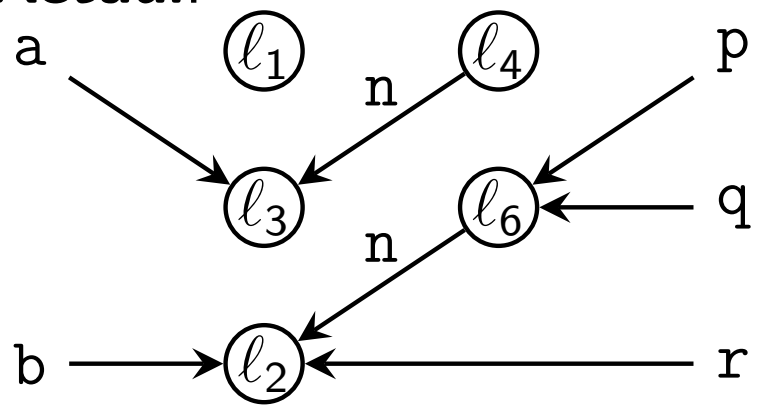

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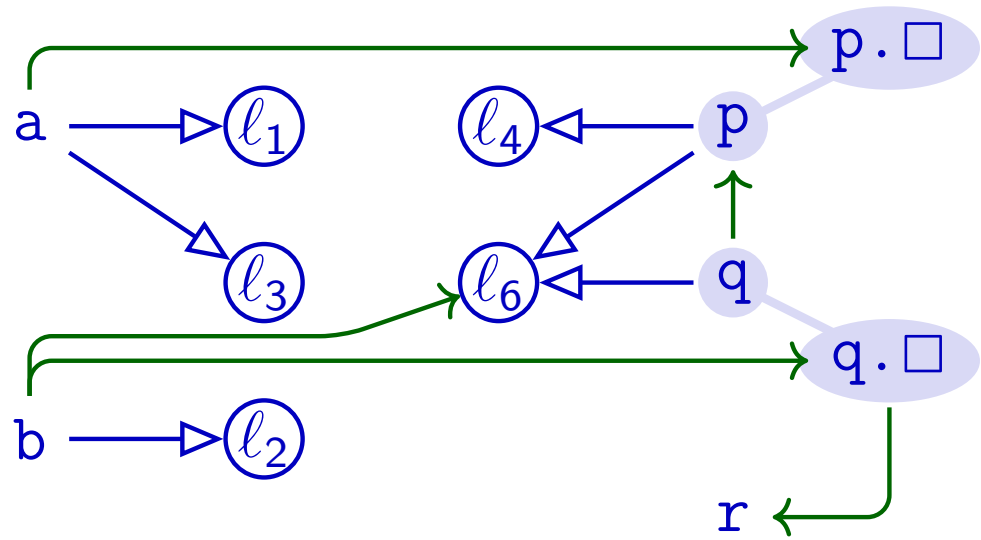

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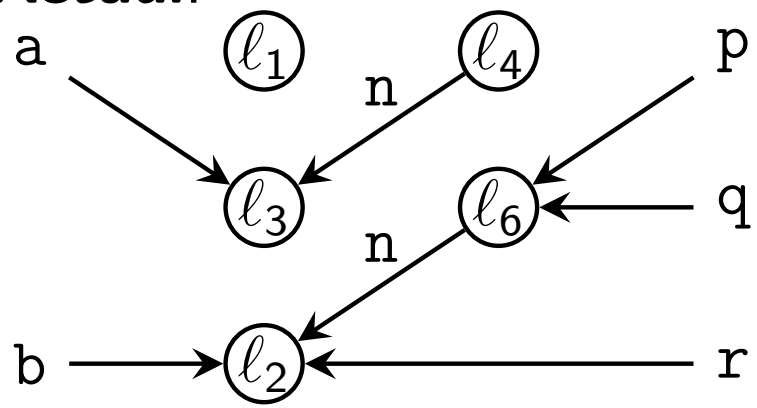

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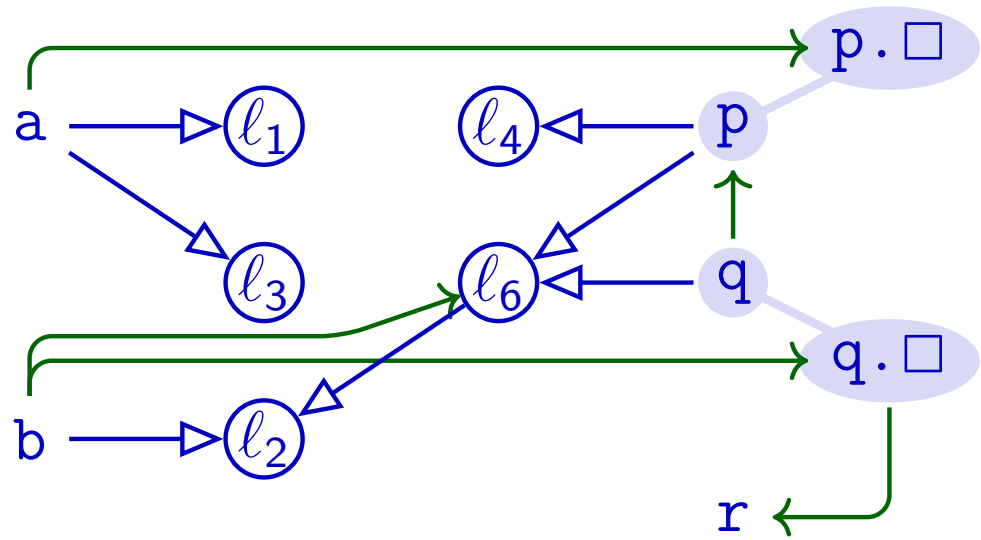

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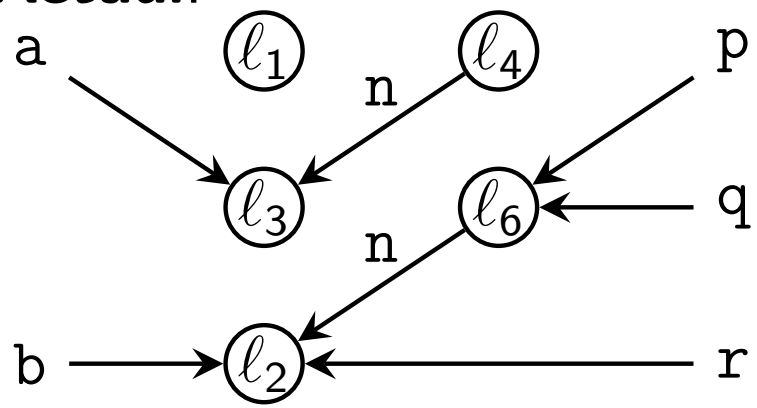

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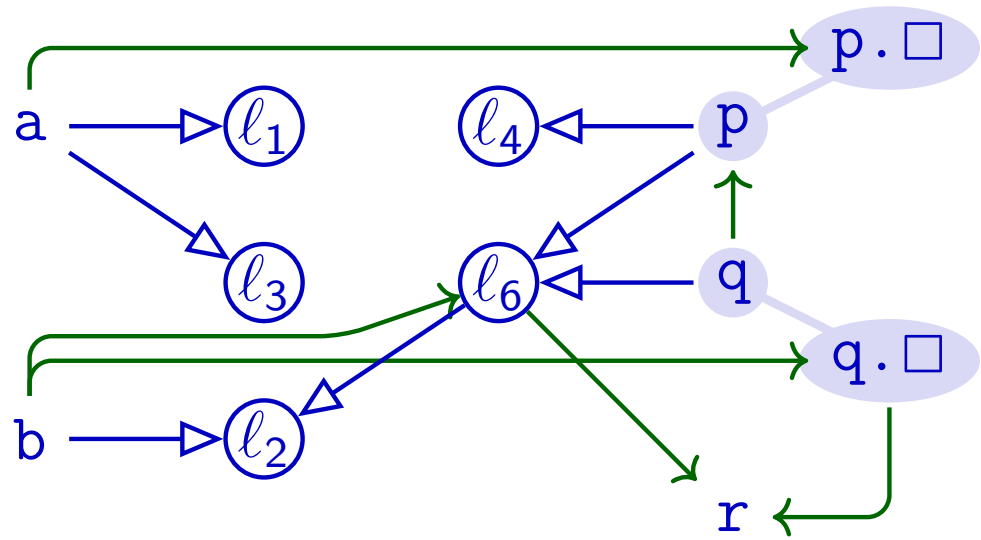

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Andersen's algorithm must propagate along inclusion graph

Example

$\ell \leftarrow y$ and $x \leftarrow y \implies \ell \leftarrow x$
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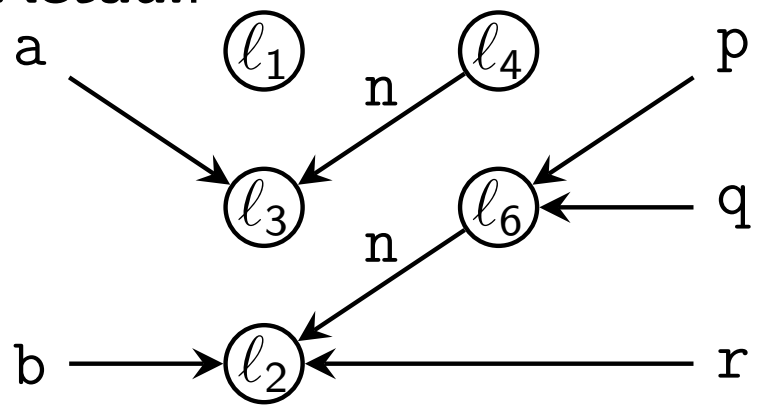
- **Actual**:



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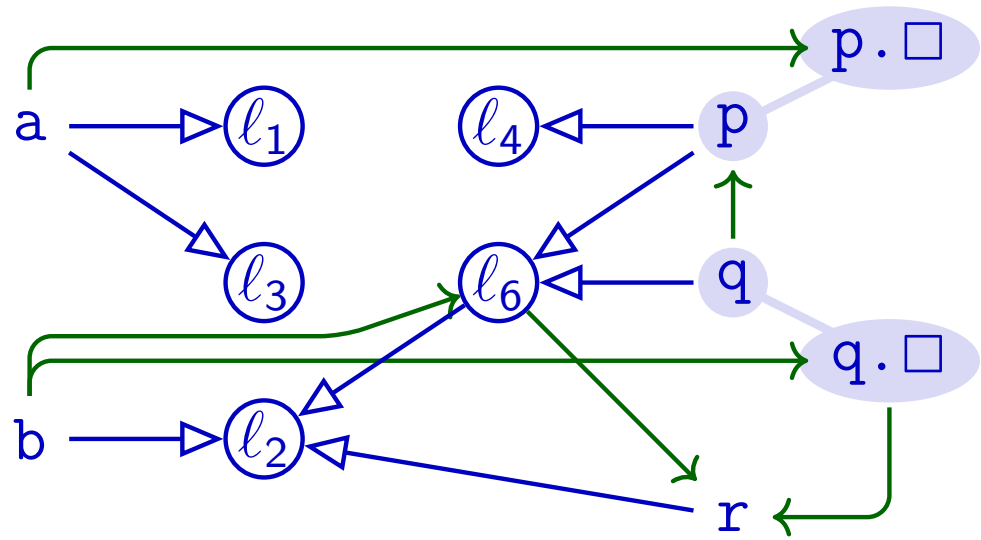

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Andersen's algorithm must propagate along inclusion graph

Example

$\ell \leftarrow y$ and $x \leftarrow y \implies \ell \leftarrow x$
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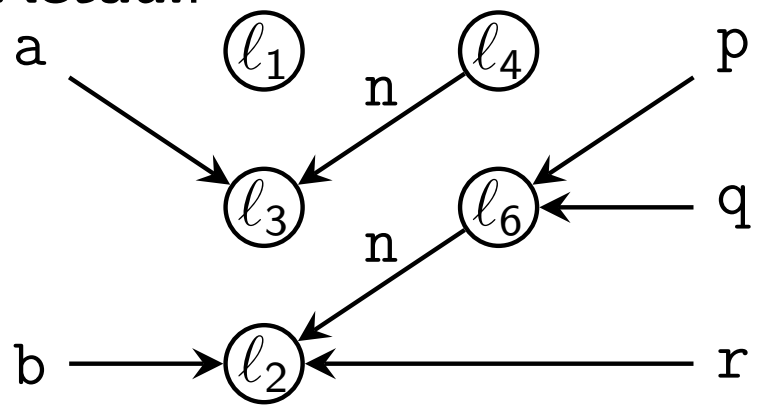

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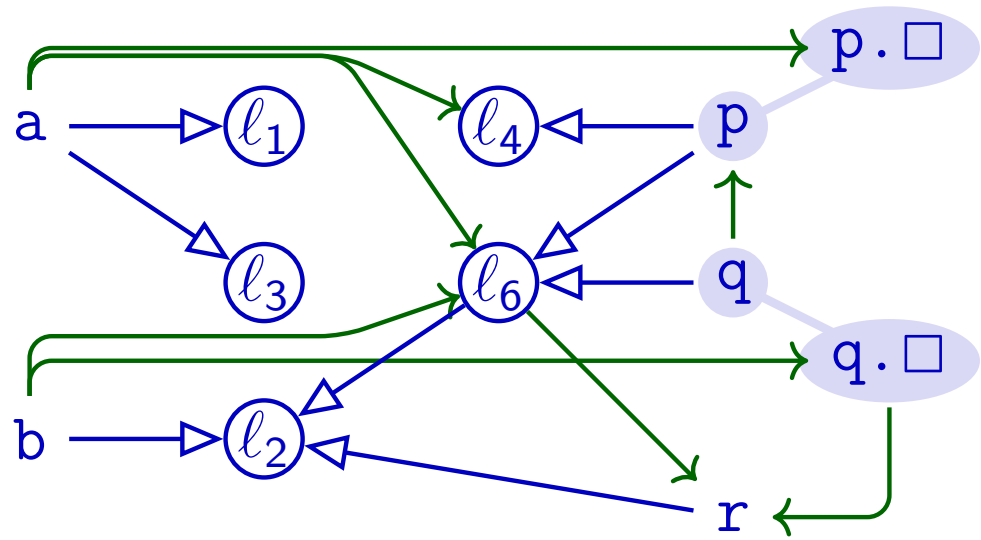

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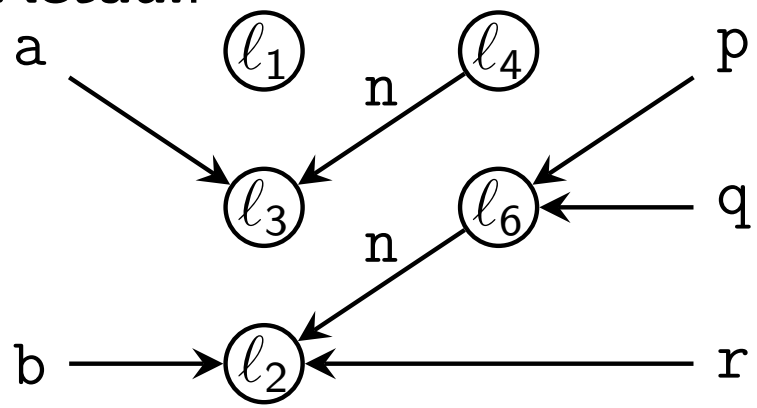
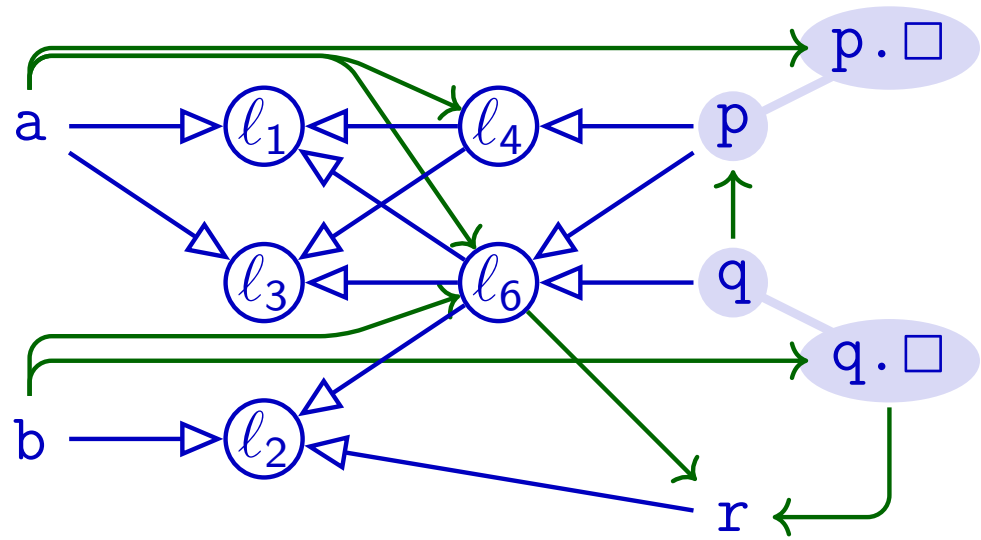
$\ell \leftarrow x$ and $x.\square \leftarrow y \implies \ell \leftarrow y$

- **Actual**:



- **Andersen**:



Teal

```
var a := new_ℓ1();
var b := new_ℓ2();
a := new_ℓ3();
var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
q.n := b;
p := q;
var r := q.n;
```

Andersen's algorithm must propagate along inclusion graph

Implementation

- Graph structure
- Two types of edges
- Connection between x and $x.\square$
- Worklist:
  - Track all *new* edges (at start: *all* extracted edges)
  - Process one edge at a time:
    - Remove from worklist, add to "completed edges"
    - Check our thee rules: does current edge + completed edges allow producing new edge that is neither in worklist nor completed?
    - If so: add all such edges to worklist (may be several!)

$$\ell \leftarrow y \text{ and } x \leftarrow y \implies \ell \leftarrow x$$
$$v \leftarrow y \text{ and } x \leftarrow y.\square \implies x \leftarrow v$$
$$v \leftarrow x \text{ and } x.\square \leftarrow y \implies v \leftarrow y$$

Complexity

- Complexity of graph closure: $O(n^3)$
- Traditional assumption about Andersen's analysis
- Close to $O(n^2)$ if:
  1. Few statements dereference each variable
  2. Control flow graphs not too complex

  Both conditions are common in practical programs

  Manu Sridharan, Stephen J. Fink, "The Complexity of Andersen's Analysis in Practice", in SAS 2009

Summary

- Andersen's analysis:
  - Subset-based
  - Builds inclusion graph for propagating memory locations along subset constraints
  - $O(n^3)$ worst-case behaviour
  - Closer to $O(n^2)$ in practice
  - More precise than Steensgaard's analysis
  - Less scalable than Steensgaard's analysis