LUND UNIVERSITY

EDAP15: Program Analysis

FORMAL NOTATION

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Basic Formal Notation

- Tuples:
  - Notation: $\langle a \rangle$, $\langle a, b \rangle$ (pair), $\langle a, c, d \rangle$ (triple)
  - Fixed-length (unlike list)
  - Group items, analogous to (read-only) record/object

- Sets:
  - $\emptyset = \{\}$ (the empty set)
  - $\{1\}$ (*singleton* set containing precisely the number 1)
  - $\{2, 3\}$ (Set with two elements)
  - $\mathbb{Z}$ (The (infinite) set of integers)
  - $\mathbb{R}$ (The (infinite) set of real numbers)

Basic operations on sets

$x \in S$	Is x contained in S?	True: $1 \in \{1\}$ and $1 \in \mathbb{Z}$ False: $2 \in \{1\}$ or $\pi \in \mathbb{Z}$
$x \notin S$	Is x NOT containd in S?	
$A \cup B$	Set union	$\{1\} \cup \{2\} = \{1, 2\}$ $\{1, 3\} \cup \{2, 3\} = \{1, 2, 3\}$
$A \cap B$	Set intersection	$\{1\} \cap \{2\} = \emptyset$ $\{1, 3\} \cap \{2, 3\} = \{3\}$
$A \subseteq B$	Subset relationship	True: $\emptyset \subseteq \{1\}$ and $\mathbb{Z} \subseteq \mathbb{R}$ False: $\{2\} \subseteq \{1\}$
$A \times B$	Product set	$\{1, 2\} \times \{3, 4\}$ $= \{\langle 1, 3\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 2, 4\rangle\}$

Relations

A relation R of arity n is a set of tuples of the form

$$R = \{ \begin{array}{c} \langle v_1^1, v_2^1, \ldots, v_n^1 \rangle \\ \vdots \\ \langle v_1^k, v_2^k, \ldots, v_n^k \rangle \end{array} \}$$

- Notation:

$$R(x_1, \ldots, x_n) \iff \langle x_1, \ldots, x_n \rangle \in R$$

- Example: the less-than-or-equals relation over integers:

$$(\leq) \subseteq \mathbb{Z} \times \mathbb{Z}$$

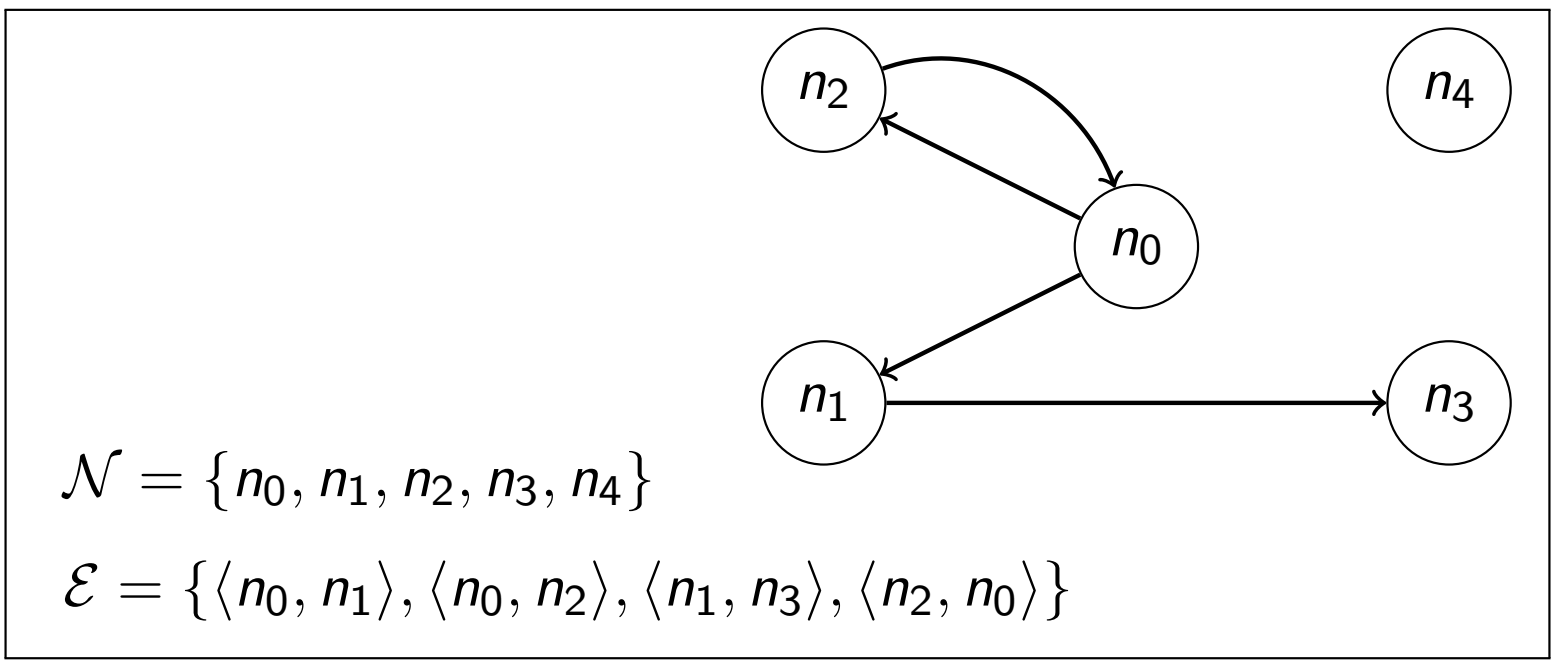
- Relations of arity 2 are called *binary* relations:
  - Notation:

$$R(x, y) \iff x \; R \; y$$

Graphs

A (directed) graph $\mathcal{G}$ is a tuple $\mathcal{G} = \langle \mathcal{N}, \mathcal{E} \rangle$, where:

- $\mathcal{N}$ is the set of *nodes* of $\mathcal{G}$
- $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of *edges* of $\mathcal{G}$
- Often: Add function $f : \mathcal{E} \to X$ to *label* edges



Summary

- **Tuples** group a fixed number of items
- **Sets** represent a (possibly infinite) number of distinct elements
  - We use them e.g. to represent possible analysis results
- **Relations** are sets of equal-sized tuples
  - We mostly use them implicitly
  - Similar to database tables, but:
    - No duplicate rows
    - No row order
- **(Directed) Graphs** represent *nodes* and *edges* between them
  - Optional *labels* on edges possible
  - We use them e.g. for program dependencies