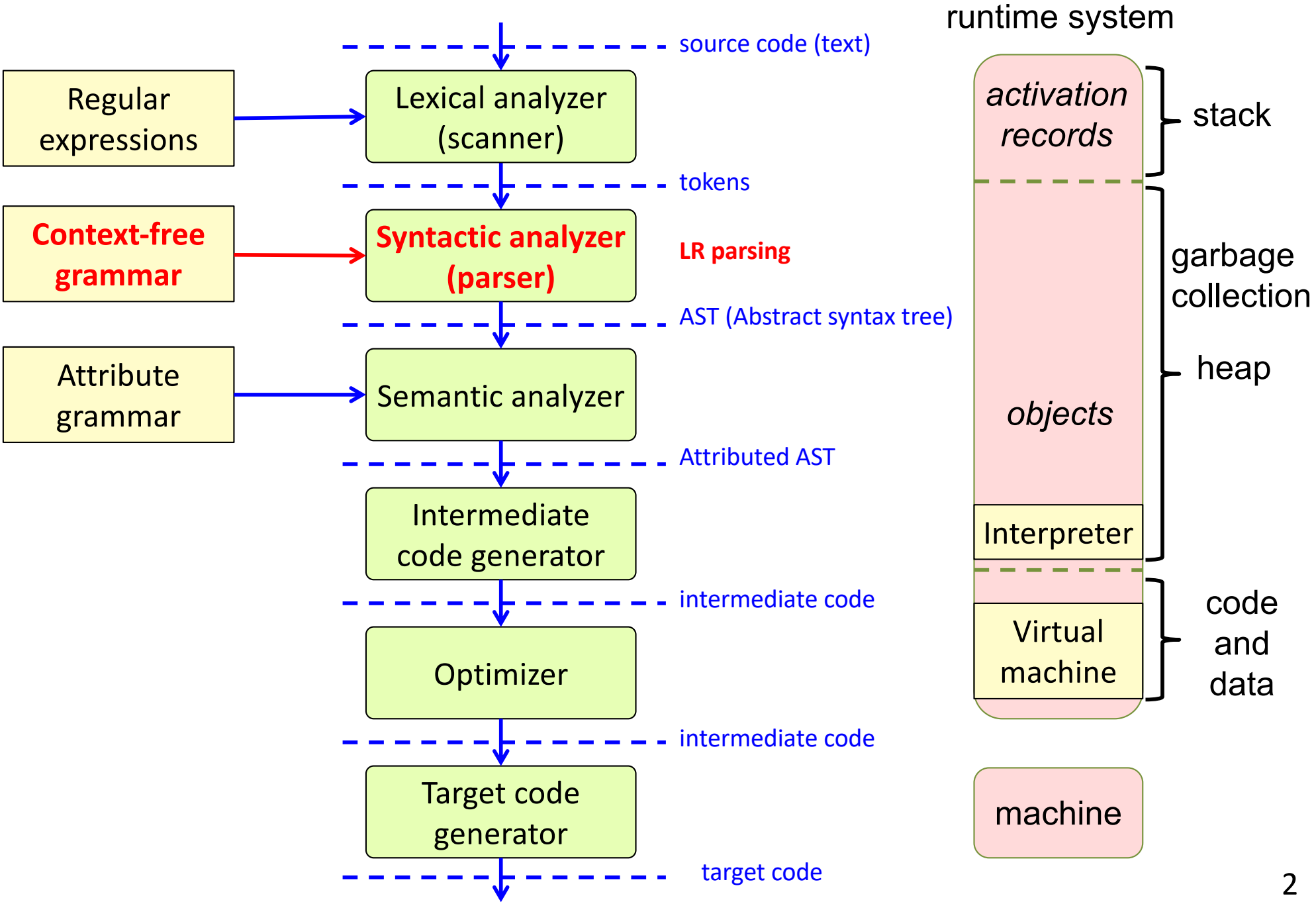
EDAN65: Compilers, Lecture 06 A

LR parsing

Görel Hedin

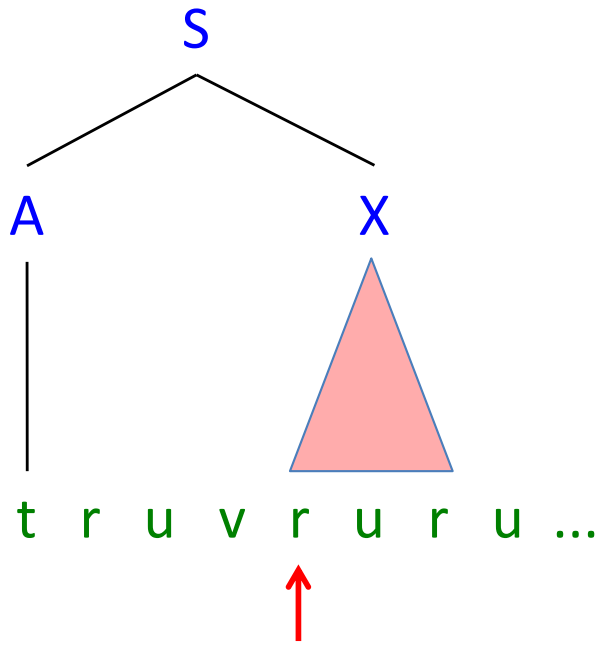
Revised: 2024-09-16

This lecture

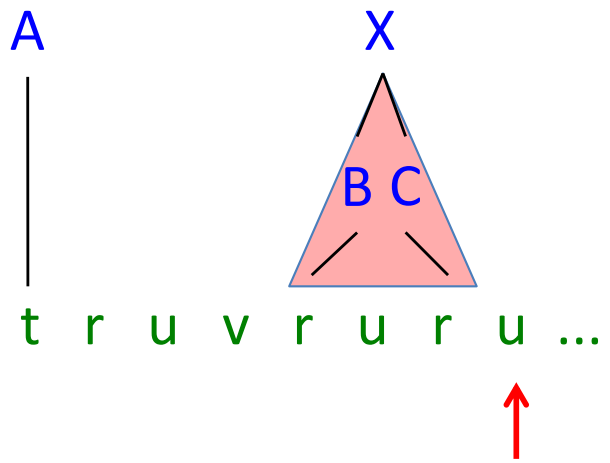


LR parsing

Recall main parsing ideas

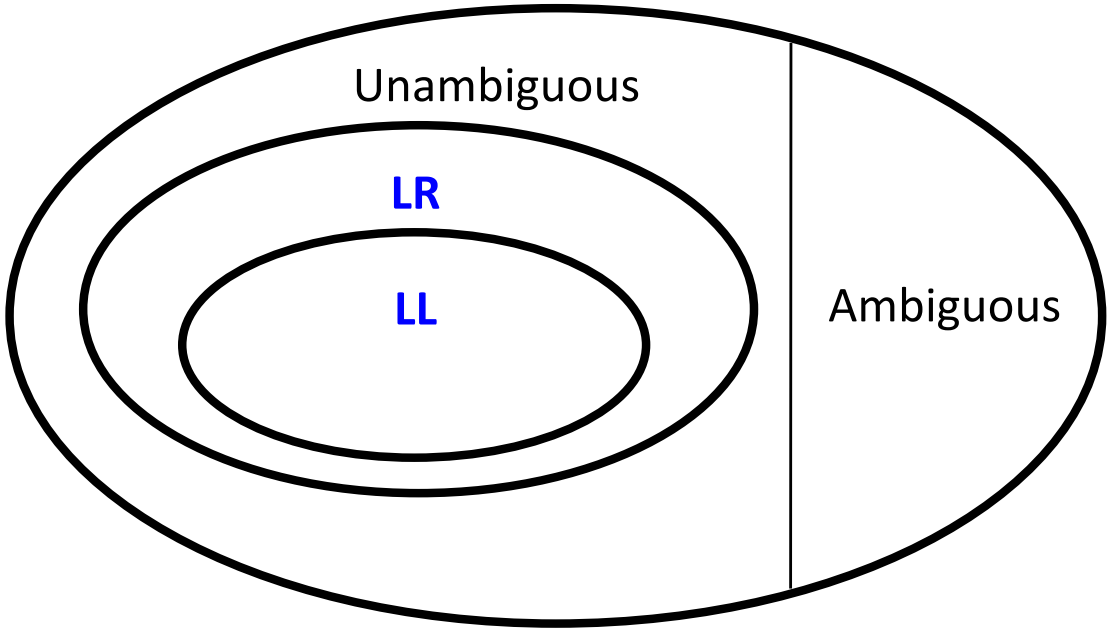


LL(1): decides to build X after seeing the first token of its subtree.
The tree is built top down.



LR(1): decides to build X after seeing the first token following its subtree.
The tree is built bottom up.

Recall different parsing algorithms



All context-free grammars

This lecture

LL:
- **L**eft-to-right scan
- **L**eftmost derivation

Builds tree top-down

Simple to understand

LR:
- **L**eft-to-right scan
- **R**ightmost derivation

Builds tree bottom-up

More powerful

Can handle left recursion and common prefix

Recall: LL(k) vs LR(k)

	LL(k)	LR(k)
Parses input	Left-to-right	
Derivation	Leftmost	Rightmost
Lookahead	k symbols	
Build the tree	top down	bottom up
Select rule	after seeing its first k tokens	after seeing all its tokens, and an additional k tokens
Left recursion	No	Yes
Unlimited common prefix	No	Yes
Resolve ambiguities through rule priority	Dangling else	Dangling else, associativity, priority
Error recovery	Trial-and-error	Good algorithms exist
Implement by hand?	Possible.	Too complicated. Use a generator.

LR parsing

Add the EOF token ($) and an extra start rule.

The parser uses a stack of symbols (terminals and nonterminals).

The parser looks at the current input token and decides to do one of the following actions:

shift – Push the input token onto the stack. Read the next token.
reduce –
  Match the top symbols on the stack with a production right-hand side.
  Pop those symbols and push the left-hand side nonterminal.
  At the same time, build this part of the tree.
accept – when the parser is about to shift $, the parse is complete.

The parser uses a finite state automaton (encoded in a table) to decide which action to take and which state to go to after each a shift action.

LR parsing example

Stack•Input

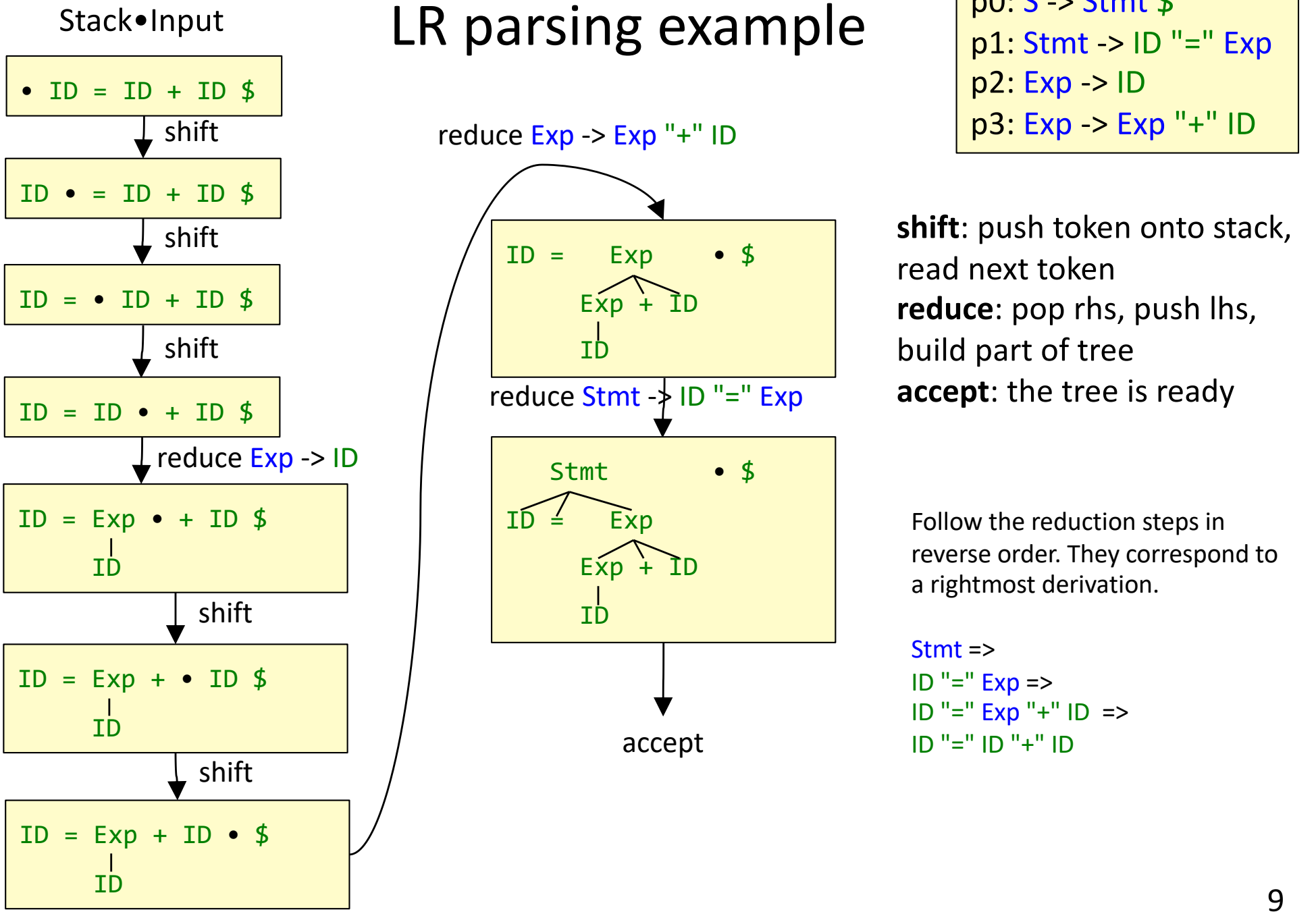
```
• ID = ID + ID $
```

Grammar:

```
p0: S -> Stmt $
p1: Stmt -> ID "=" Exp
p2: Exp -> ID
p3: Exp -> Exp "+" ID
```

shift: push token onto stack, read next token
reduce: pop rhs, push lhs, build part of tree
accept: the tree is ready

LR parsing example



Grammar:

p0: S -> Stmt $
p1: Stmt -> ID "=" Exp
p2: Exp -> ID
p3: Exp -> Exp "+" ID

shift: push token onto stack, read next token
reduce: pop rhs, push lhs, build part of tree
accept: the tree is ready

Follow the reduction steps in reverse order. They correspond to a rightmost derivation.

Stmt =>
ID "=" Exp =>
ID "=" Exp "+" ID =>
ID "=" ID "+" ID

LR(1) items

The parser uses a DFA (a deterministic finite automaton) to decide whether to shift or reduce.

The states in the DFA are sets of *LR items*.

LR(1) item:

X -> α • β ,t|s

LR(1) items

The parser uses a DFA (a deterministic finite automaton) to decide whether to shift or reduce.

The states in the DFA are sets of *LR items*.

LR(1) item:

X -> α • β ,t|s

An LR(1) item is a production extended with:
- A dot (•), corresponding to the position in the input sentence.
- One or more possible lookahead terminal symbols, t,s
  (we will use ? when the lookahead doesn't matter)

The LR(1) item corresponds to a state where:
- The topmost part of the stack is α.
- The first part of the remaining input is expected to match β(t|s)

Grammar:

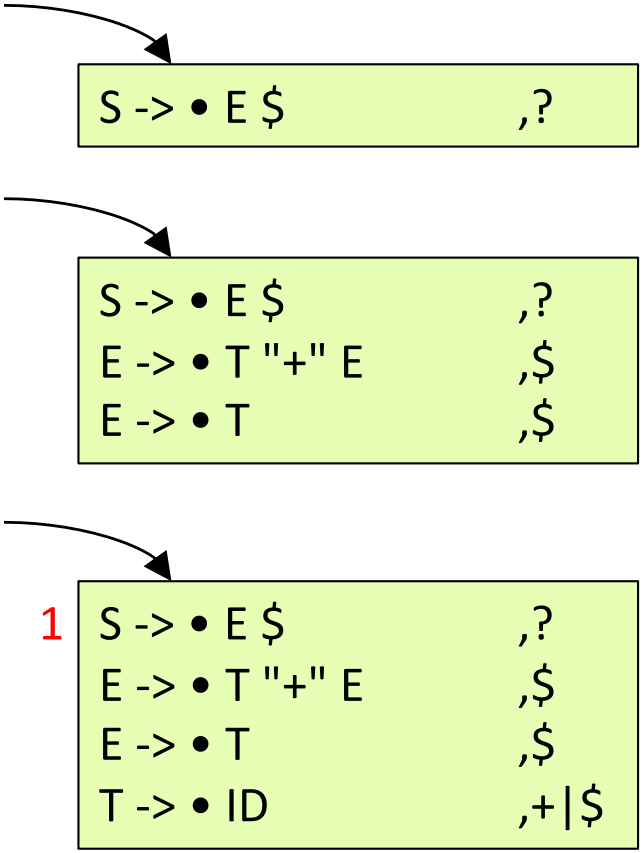
p0: S -> E $
p1: E -> T "+" E
p2: E -> T
p3: T -> ID

Constructing the LR state machine

Constructing state 1

Grammar:

```
p0: S -> E $
p1: E -> T "+" E
p2: E -> T
p3: T -> ID
```



First, take the start production and place the dot in the beginning...

Note that there is a nonterminal E right after the dot, and it is followed by a terminal $. Add the productions for E, with $ as the lookahead.

Note that there is a nonterminal T right after the dot, and which is followed by either "+" or $. Add the productions for T, with "+" and $ as the lookahead. (We write them on the same line as a shorthand.)

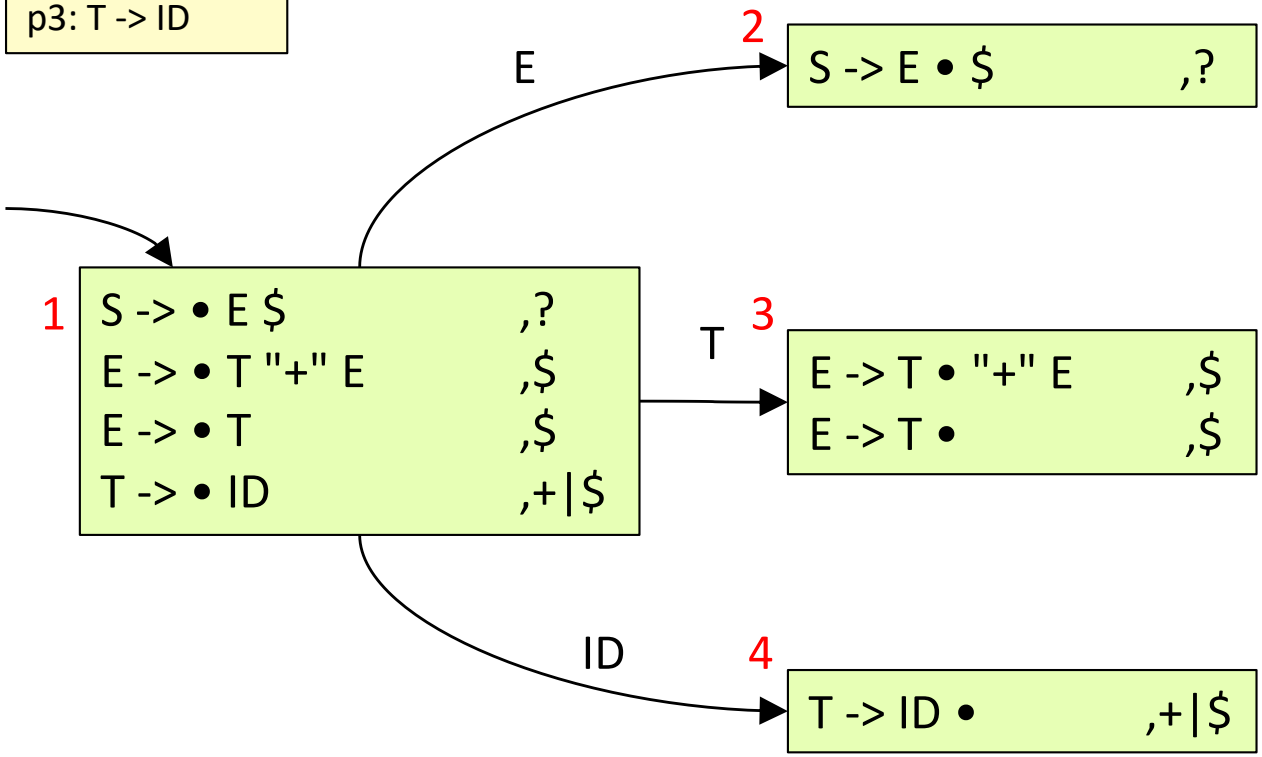
We have already added productions for all nonterminals that are right after the dot. Nothing more can be added.
We are finished constructing state 1.

Adding new productions for nonterminals following the dot, until no more productions can be added, is called taking the closure of the LR item set.

Constructing the next states

Grammar:

```
p0: S -> E $
p1: E -> T "+" E
p2: E -> T
p3: T -> ID
```

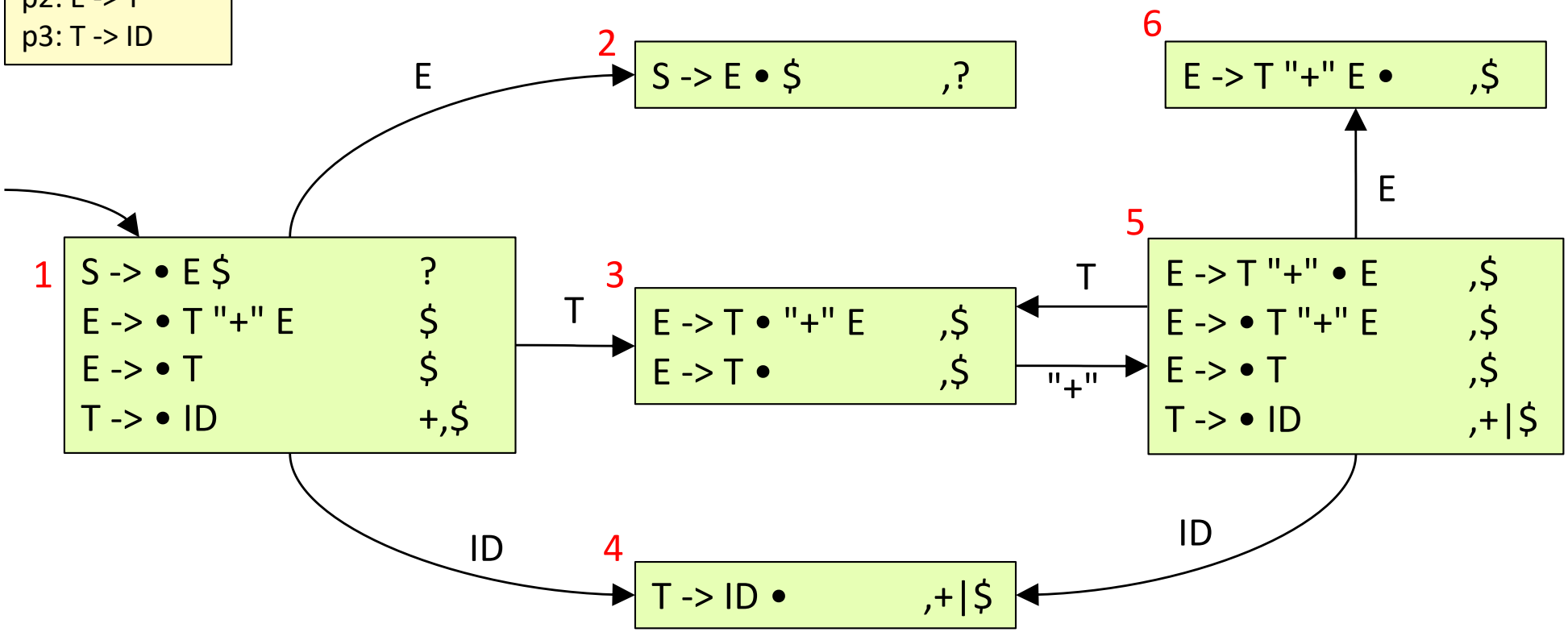


Note that the dot is followed by E, T, and ID in state 1. For each of these symbols, create a new set of LR items, by advancing the dot passed that symbol. Then complete the states by taking the closure.
(Nothing had to be added for these states.)

Completing the LR DFA

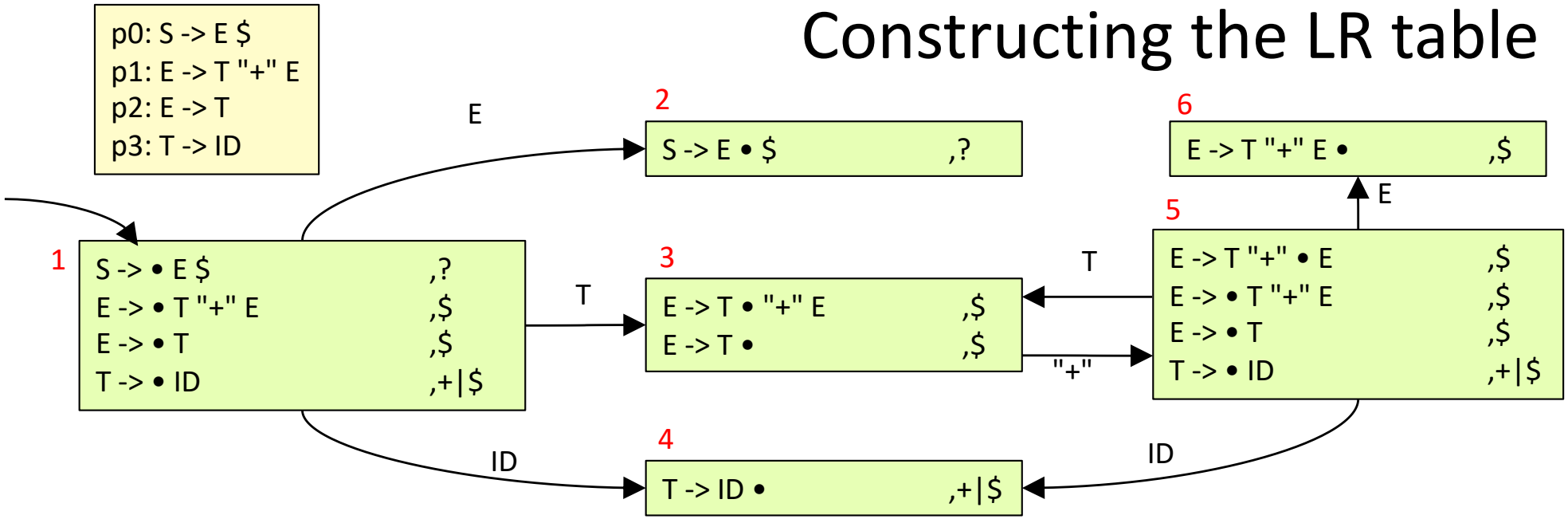
Grammar:

p0: S -> E $
p1: E -> T "+" E
p2: E -> T
p3: T -> ID



Complete the DFA by advancing the dot, creating new states, completing them by taking the closure. If there is already a state with the same items, we use that state instead.

Constructing the LR table



state	"+"	ID	$		E	T
1						
2						
3						
4						
5						
6						

Constructing the LR table

- For each **token edge t**, from state j to state k, add a **shift action** "s k" (shift and goto state k) to table[j,t]. (This corresponds to reading a token and pushing it onto the stack.)
- For each state j that contains an LR item **where the dot is at the end**, add a **reduce action** "r p" (reduce p) to table[j,t], where p is the production and t is the lookahead token.
  (This corresponds to popping the right-hand side of a production off the stack.)
- For each **nonterminal edge X**, from state j to state k, add a **goto action** "g k" (goto state k) to table[j,X].
  (This corresponds to pushing the left-hand side nonterminal onto the stack.)
- For a state j containing an LR item with **the dot to the left of $**, add an **accept action** "a" to table[j,$].
  (If we are about to shift $, the parse has succeeded.)

state	"+"	ID	$		E	T
1						
2						
3						
4						
5						
6						

Constructing the LR table

- For each **token edge t**, from state j to state k, add a **shift action** "s k" (shift and goto state k) to table[j,t]. (This corresponds to reading a token and pushing it onto the stack.)
- For each state j that contains an LR item **where the dot is at the end**, add a **reduce action** "r p" (reduce p) to table[j,t], where p is the production and t is the lookahead token.
  (This corresponds to popping the right-hand side of a production off the stack.)
- For each **nonterminal edge X**, from state j to state k, add a **goto action** "g k" (goto state k) to table[j,X].
  (This corresponds to pushing the left-hand side nonterminal onto the stack.)
- For a state j containing an LR item with **the dot to the left of $**, add an **accept action** "a" to table[j,$].
  (If we are about to shift $, the parse has succeeded.)

state	"+"	ID	$		E	T
1		s 4			g 2	g 3
2			a			
3	s 5		r p2			
4	r p3		r p3			
5		s 4			g 6	g 3
6			r p1			

Using the LR table for parsing

- Use a symbol stack and a state stack
- The current state is the state stack top.
- Push state 1 to the state stack
- Perform an action for each token:
- Case Shift s:
  - Push the token to the symbol stack
  - Push s to the state stack
  - The current state is now s.
- Case Reduce p:
  - Pop symbols for the rhs of p
  - Push the lhs symbol X of p
  - Pop the same number of states
  - Let s1 = the top of the state stack
  - Let s2 = table[s1,X]
  - Push s2 to the state stack
  - The current state is now s2.
- Case Accept: Report successful parse

state	"+"	ID	$		E	T
1		s 4			g 2	g 3
2			a			
3	s 5		r p2			
4	r p3		r p3			
5		s 4			g 6	g 3
6			r p1			

Example of LR parsing

Grammar:

p0: S -> E $
p1: E -> T "+" E
p2: E -> T
p3: T -> ID

Parse table:

state	"+"	ID	$		E	T
1		s 4			g 2	g 3
2			a			
3	s 5		r p2			
4	r p3		r p3			
5		s 4			g 6	g 3
6			r p1			

Parsing ID + ID $

State stack	Symbol stack	Input	action
1		ID + ID $	

Example of LR parsing

Grammar:

p0: S -> E $
p1: E -> T "+" E
p2: E -> T
p3: T -> ID

Parse table:

state	"+"	ID	$	E	T
1		s 4		g 2	g 3
2			a		
3	s 5		r p2		
4	r p3		r p3		
5		s 4		g 6	g 3
6			r p1		

Parsing ID + ID $

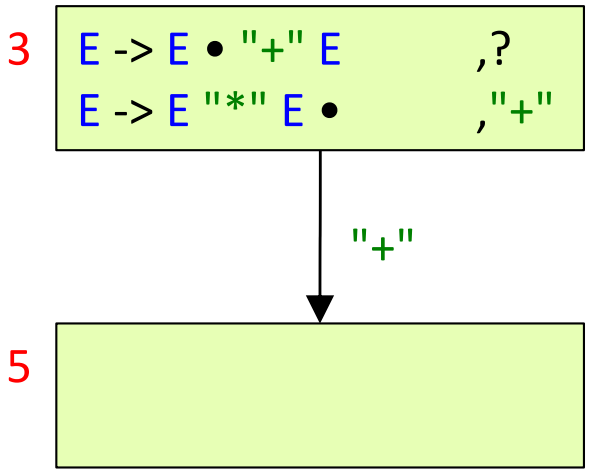
State stack	Symbol stack	Input	action
1		ID + ID $	shift 4
1 4	ID	+ ID $	reduce p3
1 3	T	+ ID $	shift 5
1 3 5	T +	ID $	shift 4
1 3 5 4	T + ID	$	reduce p3
1 3 5 3	T + T	$	reduce p2
1 3 5 6	T + E	$	reduce p1
1 2	E	$	accept

Conflict in an LR table

Grammar:

```
p0: S -> E $
p1: E -> E "+" E
p2: E -> E "*" E
p3: E -> ID
```

Parts of the DFA:



Parts of the parse table:

state	...	"+"	...		...	...
...						
3						
...						

Fill in the parse table.

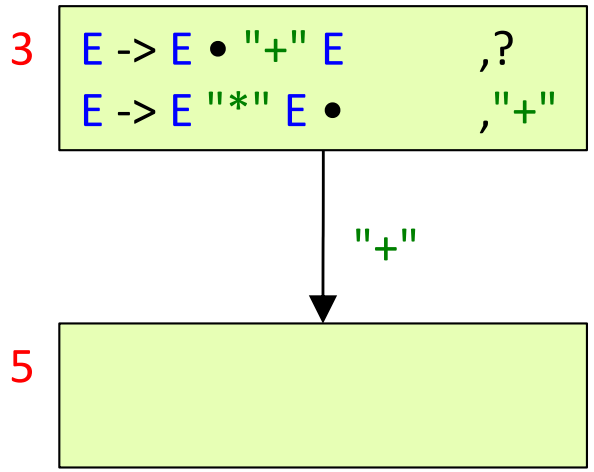
What is the problem?

Conflict in an LR table

Grammar:

```
p0: S -> E $
p1: E -> E "+" E
p2: E -> E "*" E
p3: E -> ID
```

Parts of the DFA:



Parts of the parse table:

state	...	"+"	...		...	...
...						
3		s 5, r p2				
...						

There is a shift-reduce conflict.
The grammar is ambiguous.
In this case, we can resolve the conflict by selecting one of the actions.

To understand which one, think about what the top of the stack looks like. Think about what will happen later if we take the shift rule or the reduce rule.

Analyzing LR conflicts ...

Example output from parser generator (NeoBeaver):

```
WARNING: resolved SHIFT/REDUCE conflict on [PLUS] by selecting SHIFT:
  REDUCE exp = exp PLUS exp
  SHIFT PLUS
Context:
 exp = exp PLUS exp . [PLUS]
 exp = exp . PLUS exp [PLUS]
```

Note! The parser generator automatically resolves the conflict by shifting.

Is this what we want???

"Context" lists the LR-items in the conflicting state.

Analyzing LR conflicts ...

Example output from parser generator (NeoBeaver):

```
WARNING: resolved SHIFT/REDUCE conflict on [PLUS] by selecting SHIFT:
  REDUCE exp = exp PLUS exp
  SHIFT PLUS
Context:
 exp = exp PLUS exp . [PLUS]
 exp = exp . PLUS exp [PLUS]
```

Note! The parser generator automatically resolves the conflict by shifting.

Is this what we want???

"Context" lists the LR-items in the conflicting state.

In assignment 2: Make sure you change the grammar to resolve all conflicts, even if they are only warnings.

Line up the dots in the state:

```
expr -> expr PLUS expr •
expr ->           expr • PLUS expr
```

The top of stack and input may look like:

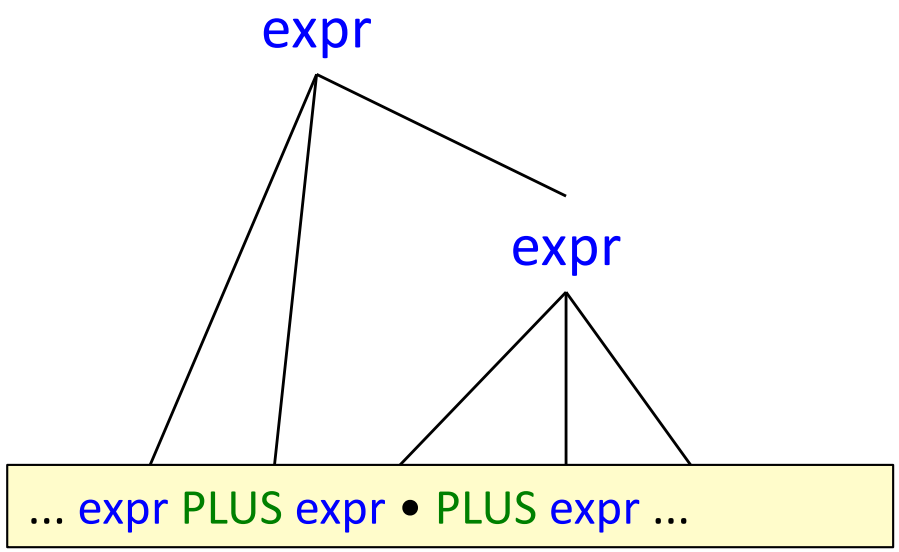
... expr PLUS expr • PLUS expr ...

top of stack | remaining input

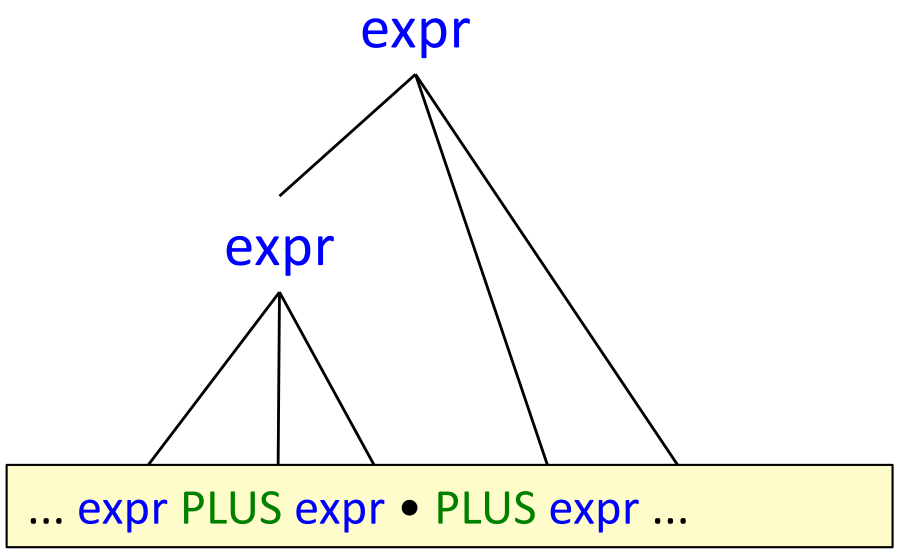
... Analyzing LR conflicts

Line up the dots in the state:

```
expr -> expr PLUS expr •              ,PLUS
expr ->           expr • PLUS expr    ,?
```



If we shift



If we reduce

Which rule should we choose?

Different kinds of conflicts

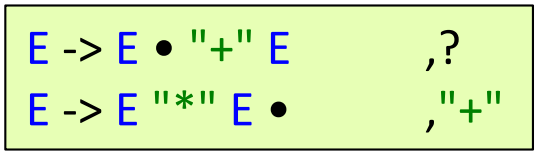
```
E -> E • "+" E        ,?
E -> E "*" E •        ,"+"
```

A shift-reduce conflict.

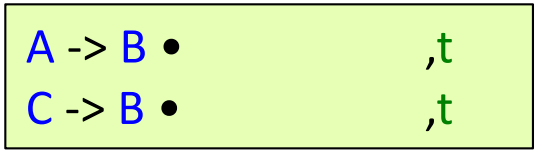
```
A -> B •              ,t
C -> B •              ,t
```

A reduce-reduce conflict.

Different kinds of conflicts



A shift-reduce conflict.



A reduce-reduce conflict.

Shift-reduce conflicts can sometimes be solved with *precedence rules*. In particular for binary expressions with priority and associativity.

For other cases, you need to carefully analyze the shift-reduce conflicts to see if precedence rules are applicable, or if you need to change the grammar.

For reduce-reduce conflicts, it is advisable to think through the problems, and change the grammar.

Typical precedence rules for an LR parser generator

```
E -> E "==" E
E -> E "**" E
E -> E "*" E
E -> E "/" E
E -> E "+" E
E -> E "-" E
E -> ID
E -> INT

// Precedence rules
%right "**"
%left "*", "/"
%left "+", "-"
%nonassoc "=="
```

Shift-reduce conflicts can be automatically resolved using *precedence rules*.

Operators in the same rule have the same priority (e.g., PLUS, MINUS).

Operators in an earlier rule have higher priority (e.g. TIMES has higher prio than PLUS.)

Typical precedence rules for an LR parser generator

```
E -> E "==" E
E -> E "**" E
E -> E "*" E
E -> E "/" E
E -> E "+" E
E -> E "-" E
E -> ID
E -> INT

// Precedence rules
%right "**"
%left "*", "/"
%left "+", "-"
%nonassoc "=="
```

Shift-reduce conflicts can be automatically resolved using *precedence rules*.

Operators in the same rule have the same priority (e.g., PLUS, MINUS).

Operators in an earlier rule have higher priority (e.g. TIMES has higher prio than PLUS.)

In assignment 2, you should NOT use precedence rules.
Instead, rewrite the grammar to use Term and Factor, etc.

How the precedence rules work

A rule is given the priority and associativity of its rightmost token.

For two conflicting rules with **different** priority, the rule with the highest priority is chosen:

```
E -> E • + E        ,?
E -> E * E •        ,+
```

```
E -> E • * E        ,?
E -> E + E •        ,*
```

How the precedence rules work

A rule is given the priority and associativity of its rightmost token.

For two conflicting rules with **different** priority, the rule with the highest priority is chosen:

```
E -> E • + E        ,?
E -> E * E •        ,+
```

Reduce is chosen

```
E -> E • * E        ,?
E -> E + E •        ,*
```

Shift is chosen

Two conflicting rules with the **same** priority have the same associativity.
Left-associativity favors reduce.
Right-associativity favors shift.
Non-associativity removes both rules from the table (input following that pattern will cause a parse error).

```
E -> E + E •        ,+
E -> E • + E        ,?
```

```
E -> E ** E •       ,**
E -> E • ** E       ,?
```

```
E -> E == E •       ,==
E -> E • == E       ,?
```

How the precedence rules work

A rule is given the priority and associativity of its rightmost token.

For two conflicting rules with **different** priority, the rule with the highest priority is chosen:

```
E -> E • + E        ,?
E -> E * E •        ,+
```

Reduce is chosen

```
E -> E • * E        ,?
E -> E + E •        ,*
```

Shift is chosen

Two conflicting rules with the **same** priority have the same associativity.
Left-associativity favors reduce.
Right-associativity favors shift.
Non-associativity removes both rules from the table (input following that pattern will cause a parse error).

```
E -> E + E •        ,+
E -> E • + E        ,?
```

Reduce is chosen

```
E -> E ** E •       ,**
E -> E • ** E       ,?
```

Shift is chosen

```
E -> E == E •       ,==
E -> E • == E       ,?
```

No rule is chosen

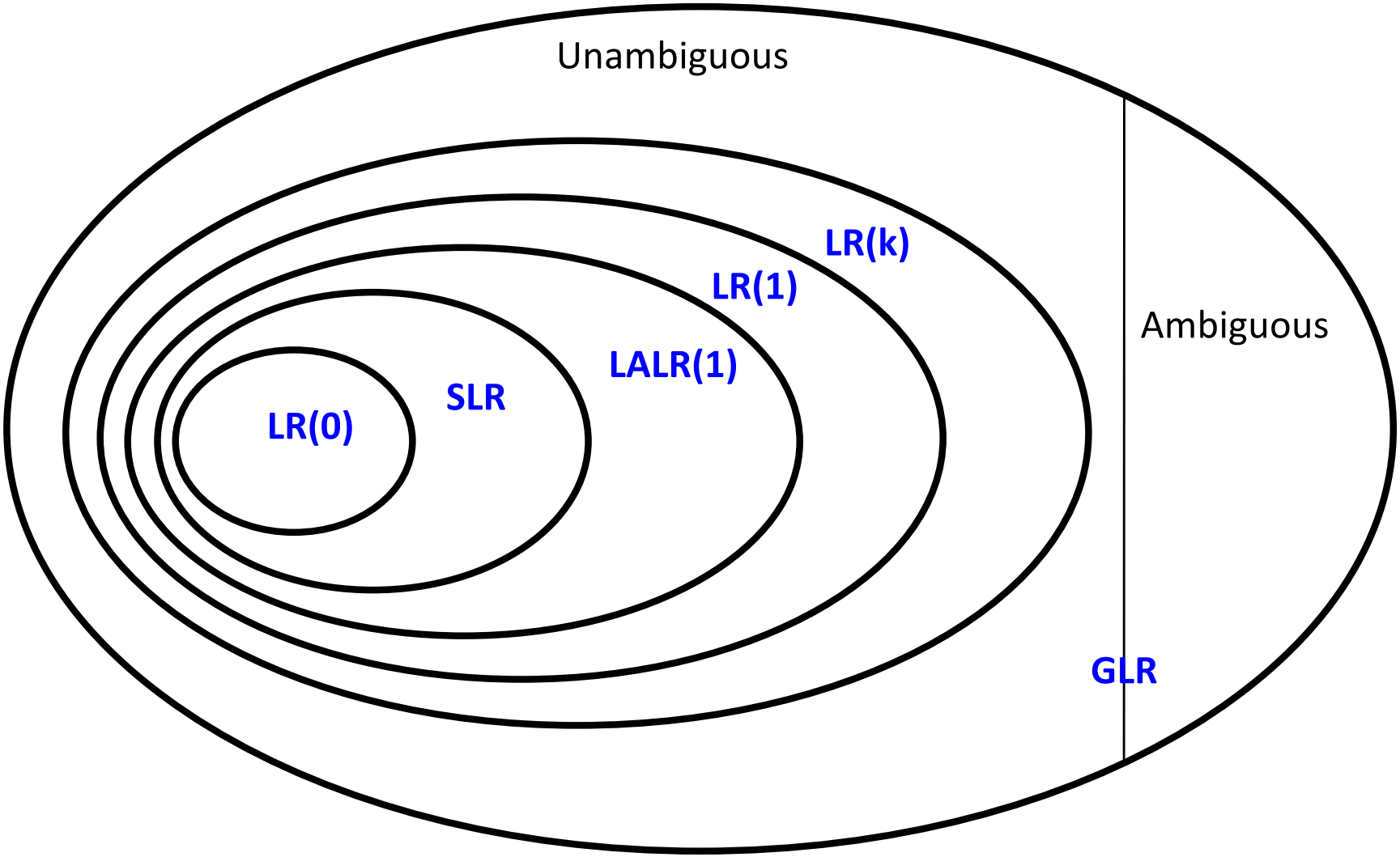
Different variants of LR(k) parsers

Type	Characteristics
LR(0)	
SLR Simple LR	
LALR(1) used in practice	
LR(1)	
LR(k)	
GLR	

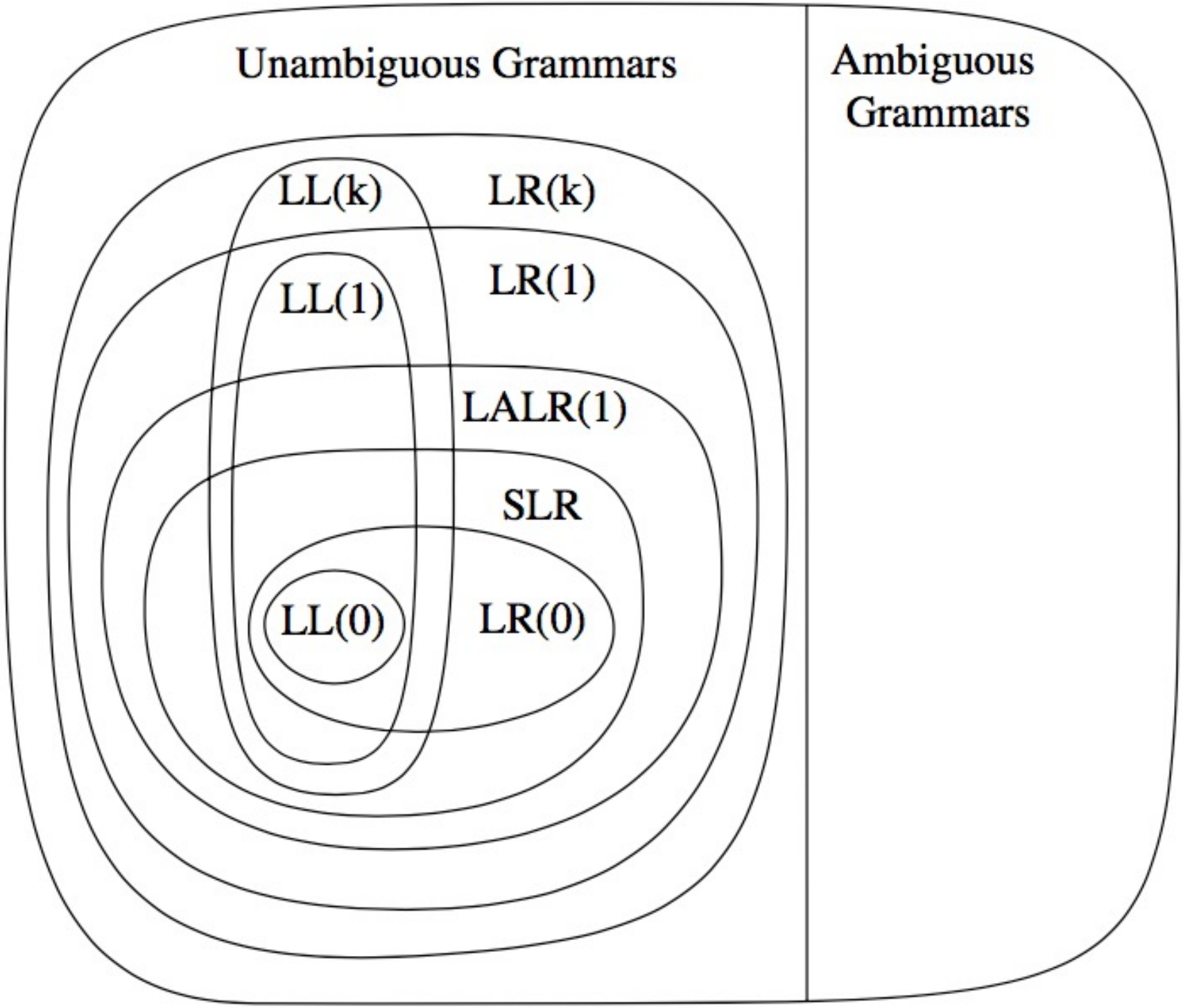
Different variants of LR(k) parsers

Type	Characteristics
LR(0)	LR items without lookahead. Not very useful in practice.
SLR Simple LR	Look at the FOLLOW set to decide where to put reduce actions. Can parse some useful grammars.
LALR(1) used in practice	Merges states that have the same LR items, but different lookaheads (LA). Leads to much smaller tables than LR(1). Used by most well known tools: Yacc, CUP, Beaver, SableCC, ... Sufficient for most practical parsing problems.
LR(1)	Slightly more powerful than LALR(1). Not used in practice – the tables become very large.
LR(k)	Much too large tables for k>1
GLR	Can handle arbitrary CFG, also ambiguous. Uses LR(1) table. Produces parse forest. Slow: $O(n^3)$ (n – length of input)

Different variants of LR parsers



All context-free grammars


Unambiguous Grammars
Ambiguous Grammars
LL(k)
LR(k)
LL(1)
LR(1)
LALR(1)
SLR
LL(0)
LR(0)

Universal parsing algorithms

GLR – Generalized LR

Can parse *any* context free grammar.

Including *ambiguous* grammars!

Returns a parse *forest* (all possible parse trees).
Additional mechanism needed to select which of the trees to use.

Can parse grammars with shift-reduce and reduce-reduce conflicts (spawns parallel parsers).

Has $O(n^3)$ worst-case time complexity, in the length of the input (n).

Is often much better than that in practice. But still slower than LALR.

Used in several research systems.

There is a universal LL algorithm as well: GLL (from 2010). Also $O(n^3)$.

Some well-known parser generators

Name	Type, host language, license
JavaCC	LL(1), Java, supports longer local lookahead, BSD
ANTLR	Adaptive LL(*), Java (also earlier versions for C, C#, ...), BSD
yacc	LALR(1), C, "yet another compiler compiler" Developed for AT&T Unix in 1970. GNU GPL
bison	LALR(1), GLR, C++, GNU GPL
CUP	LALR(1), Java, BSD-like
beaver/neobeaver	LALR(1), Java, BSD
SDF/SGLR	Scannerless GLR, C, Java, BSD

For more examples, see
https://en.wikipedia.org/wiki/Comparison_of_parser_generators

Parsing Expression Grammars (PEGs)

Look like CFGs, but productions are *ordered*
Always *unambiguous* (gives priority to first alternative)

CFG (unordered productions)
Ambiguous!

```
ifstmt -> "if" "(" expr ")" stmt "else" stmt
        | "if" "(" expr ")" stmt
```

PEG (ordered productions)
Unambiguous!

```
ifstmt -> "if" "(" expr ")" stmt "else" stmt
        | "if" "(" expr ")" stmt
```

Will match if-statements in the desired way!

Parsing Expression Grammars (PEGs)

Look like CFGs, but productions are *ordered*
Always *unambiguous* (gives priority to first alternative)

CFG (unordered productions)
Ambiguous!

```
ifstmt -> "if" "(" expr ")" stmt "else" stmt
        | "if" "(" expr ")" stmt
```

Equivalent CFG
Also ambiguous!

```
ifstmt -> "if" "(" expr ")" stmt
        | "if" "(" expr ")" stmt "else" stmt
```

PEG (ordered productions)
Unambiguous!

```
ifstmt -> "if" "(" expr ")" stmt "else" stmt
        | "if" "(" expr ")" stmt
```

Will match if-statements in the desired way!

Not equivalent PEG
Also unambiguous!

```
ifstmt -> "if" "(" expr ")" stmt
        | "if" "(" expr ")" stmt "else" stmt
```

Will not match if-then-else statements

Implementation of PEGs

Implementation similar to recursive-descent
Unlimited lookahead
Backtracks (tries the next alternative) if an alternative is not successful
Typically does not allow left recursion (but supports EBNF)

Straightforward implementation is exponential
"Packrat" implementation is linear by using memoization (caching of function call results), but uses more memory, proportional to input

Often implemented using *parser combinators* – higher-order functions that combine subparsers.
Parser combinator libraries are alternatives to using parser generators.

Summary questions: LR parsing

- How does LR differ from LL parsers?
- What does it mean to shift?
- What does it mean to reduce?
- Explain how LR parsing works on an example.
- What is an LR item?
- What does an LR state consist of?
- What does it mean to take the closure of a set of LR items?
- What do the edges in an LR DFA represent?
- How can an LR table be constructed from an LR DFA?
- How is the LR table used for parsing?
- What is meant by a shift-reduce conflict and a reduce-reduce conflict?
- How can such a conflict be analyzed?
- How can precedence rules be used in an LR parser?
- What is LR(0) and SLR parsing?
- What is the difference between LALR(1) and LR(1)?
- Explain why the LALR(1) algorithm is most commonly used in parser generators.
- What is a GLR parser?