

# EDDAN40

2nd June 2017

14:00 - 19:00

**WRITE ONLY ON ONE SIDE OF THE PAPER** - the exams will be scanned in and only the front/odd pages will be read.

**DO NOT WRITE WITH OTHER COLOUR THAN BLACK** - coloured text may disappear during scanning

**PUT YOUR ID AND PAGE NUMBER ON EACH PAGE YOU SUBMIT** - make sure that the amount of pages is equal to the amount you note on the front information page

**WRITE CLEARLY** - if we cannot read you we cannot grade you.

**PRELIMINARY AMOUNT OF POINTS : 6** (one per question)

```

{-A list of selected functions from the Haskell modules:
Prelude
Data.List
Data.Maybe
Data.Char -}
-----
-- standard type classes
class Show a where
  show :: a -> String

class Eq a where
  (==), (/=)      :: a -> a -> Bool

class (Eq a) => Ord a where
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min              :: a -> a -> a

class (Eq a, Show a) => Num a where
  (+), (-), (*)      :: a -> a -> a
  negate            :: a -> a
  abs, signum       :: a -> a
  fromInteger       :: Integer -> a

class (Num a, Ord a) => Real a where
  toRational        :: a -> Rational

class (Real a, Enum a) => Integral a where
  quot, rem         :: a -> a -> a
  div, mod          :: a -> a -> a
  toInteger         :: a -> Integer

class (Num a) => Fractional a where
  (/)              :: a -> a -> a
  fromRational     :: Rational -> a

class (Fractional a) => Floating a where
  exp, log, sqrt   :: a -> a
  sin, cos, tan    :: a -> a

class (Real a, Fractional a) => RealFrac a where
  truncate, round  :: (Integral b) => a -> b
  ceiling, floor   :: (Integral b) => a -> b
-----
-- numerical functions

even, odd :: (Integral a) => a -> Bool
even n    = n `rem` 2 == 0
odd       = not . even
-----
-- monadic functions

sequence :: Monad m => [m a] -> m [a]
sequence = foldr mcons (return [])

```

```

                                where mcons p q = do x <- p; xs <- q; return (x:xs)

sequence_ :: Monad m => [m a] -> m ()
sequence_ xs = do sequence xs; return ()
-----
-- functions on functions

id      :: a -> a
id x    = x

const   :: a -> b -> a
const x _ = x

(.)     :: (b -> c) -> (a -> b) -> a -> c
f . g   = \x -> f (g x)

flip   :: (a -> b -> c) -> b -> a -> c
flip f x y = f y x

($)    :: (a -> b) -> a -> b
f $ x  = f x
-----
-- functions on Bools

data Bool = False | True

(&&), (||) :: Bool -> Bool -> Bool
True && x = x
False && _ = False
True || _ = True
False || x = x
not      :: Bool -> Bool
not True = False
not False = True
-----
-- functions on Maybe

data Maybe a = Nothing | Just a

isJust   :: Maybe a -> Bool
isJust (Just a) = True
isJust Nothing = False

isNothing :: Maybe a -> Bool
isNothing = not . isJust

fromJust  :: Maybe a -> a
fromJust (Just a) = a

maybeToList :: Maybe a -> [a]
maybeToList Nothing = []
maybeToList (Just a) = [a]

```

```
listToMaybe      :: [a] -> Maybe a
listToMaybe []  = Nothing
listToMaybe (a:_) = Just a
```

-----  
-- a hidden goodie

```
instance Monad [] where
  return x = [x]
  xs >>= f = concat (map f xs)
```

-----  
-- functions on pairs

```
fst      :: (a, b) -> a
fst (x, y) = x
```

```
snd      :: (a, b) -> b
snd (x, y) = y
```

```
curry    :: ((a, b) -> c) -> a -> b -> c
curry f x y = f (x, y)
```

```
uncurry  :: (a -> b -> c) -> (a, b) -> c
uncurry f p = f (fst p) (snd p)
```

-----  
-- functions on lists

```
map      :: (a -> b) -> [a] -> [b]
map f xs = [ f x | x <- xs ]
```

```
(++)     :: [a] -> [a] -> [a]
xs ++ ys = foldr (:) ys xs
```

```
filter   :: (a -> Bool) -> [a] -> [a]
filter p xs = [ x | x <- xs, p x ]
```

```
concat   :: [[a]] -> [a]
concat xss = foldr (++) [] xss
```

```
concatMap :: (a -> [b]) -> [a] -> [b]
concatMap f = concat . map f
```

```
head, last :: [a] -> a
head (x:_) = x
```

```
last [x] = x
last (_:xs) = last xs
```

```
tail, init :: [a] -> [a]
tail (_:xs) = xs
```

```
init [x] = []
init (x:xs) = x : init xs
```

```
null      :: [a] -> Bool
null []   = True
null (_:_) = False
```

```
length    :: [a] -> Int
length [] = 0
length (_:l) = 1 + length l
```

```
(!!)      :: [a] -> Int -> a
(x:_) !! 0 = x
(_:xs) !! n = xs !! (n-1)
```

```
foldr     :: (a -> b -> b) -> b -> [a] -> b
foldr f z [] = z
foldr f z (x:xs) = f x (foldr f z xs)
```

```
foldl    :: (a -> b -> a) -> a -> [b] -> a
foldl f z [] = z
foldl f z (x:xs) = foldl f (f z x) xs
```

```
iterate   :: (a -> a) -> a -> [a]
iterate f x = x : iterate f (f x)
```

```
repeat    :: a -> [a]
repeat x   = xs where xs = x:xs
```

```
replicate :: Int -> a -> [a]
replicate n x = take n (repeat x)
```

```
cycle     :: [a] -> [a]
cycle []  = error "Prelude.cycle: empty list"
cycle xs = xs' where xs' = xs++xs'
```

```
take, drop :: Int -> [a] -> [a]
take n _ | n <= 0 = []
take _ [] = []
take n (x:xs) = x : take (n-1) xs
```

```
drop n xs | n <= 0 = xs
drop _ [] = []
drop n (_:xs) = drop (n-1) xs
```

```
splitAt   :: Int -> [a] -> ([a],[a])
splitAt n xs = (take n xs, drop n xs)
```

```
takeWhile, dropWhile :: (a -> Bool) -> [a] -> [a]
takeWhile p [] = []
takeWhile p (x:xs) = x : takeWhile p xs
                    | p x
                    | otherwise = []
```

```
dropWhile p [] = []
dropWhile p xs@(x:xs') = dropWhile p xs'
                        | p x
                        | otherwise = xs
```

```

lines, words      :: String -> [String]
-- lines "apa\nbepa\ncepa\n" == ["apa","bepa","cepa"]
-- words "apa bepa\n cepa" == ["apa","bepa","cepa"]

unlines, unwords  :: [String] -> String
-- unlines ["apa","bepa","cepa"] == "apa\nbepa\ncepa"
-- unwords ["apa","bepa","cepa"] == "apa bepa cepa"

and, or           :: [Bool] -> Bool
and               = foldr (&&) True
or                = foldr (||) False

any, all          :: (a -> Bool) -> [a] -> Bool
any p             = or . map p
all p             = and . map p

elem, notElem    :: (Eq a) => a -> [a] -> Bool
elem x            = any (== x)
notElem x         = all (/= x)

lookup           :: (Eq a) => a -> [(a,b)] -> Maybe b
lookup key []    = Nothing
lookup key ((x,y):xys)
  | key == x     = Just y
  | otherwise    = lookup key xys

sum, product     :: (Num a) => [a] -> a
sum              = foldl (+) 0
product          = foldl (*) 1

maximum, minimum :: (Ord a) => [a] -> a
maximum []       = error "Prelude.maximum: empty list"
maximum xs       = foldl1 max xs

minimum []       = error "Prelude.minimum: empty list"
minimum xs       = foldl1 min xs

zip              :: [a] -> [b] -> [(a,b)]
zip              = zipWith (,)

zipWith          :: (a->b->c) -> [a]->[b]->[c]
zipWith z (a:as) (b:bs)
  = z a b : zipWith z as bs
zipWith _ _ _   = []

unzip            :: [(a,b)] -> ([a],[b])
unzip            = foldr (\(a,b) ~(as,bs) -> (a:as,b:bs)) ([],[])

nub              :: (Eq a) => [a] -> [a]
nub []           = []
nub (x:xs)       = x : nub [ y | y <- xs, x /= y ]

delete           :: Eq a => a -> [a] -> [a]
delete y []      = []

```

```

delete y (x:xs) = if x == y then xs else x : delete y xs

(\\)            :: Eq a => [a] -> [a]-> [a]
(\\)            = foldl (flip delete)

union           :: Eq a => [a] -> [a] -> [a]
union xs ys     = xs ++ ( ys \\ xs )

intersect       :: Eq a => [a] -> [a]-> [a]
intersect xs ys = [ x | x <- xs, x `elem` ys ]

intersperse     :: a -> [a] -> [a]
-- intersperse 0 [1,2,3,4] == [1,0,2,0,3,0,4]

transpose       :: [[a]] -> [[a]]
-- transpose [[1,2,3],[4,5,6]] == [[1,4],[2,5],[3,6]]

partition       :: (a -> Bool) -> [a] -> ([a],[a])
partition p xs  = (filter p xs, filter (not . p) xs)

group           :: Eq a => [a] -> [[a]]
-- group "aapaabbbbee" == ["aa","p","aa","bbb","eee"]

isPrefixOf, isSuffixOf :: Eq a => [a] -> [a] -> Bool
isPrefixOf [] _      = True
isPrefixOf _ []      = False
isPrefixOf (x:xs) (y:ys) = x == y && isPrefixOf xs ys

isSuffixOf x y       = reverse x `isPrefixOf` reverse y

sort              :: (Ord a) => [a] -> [a]
sort              = foldr insert []

insert            :: (Ord a) => a -> [a] -> [a]
insert x []        = [x]
insert x (y:xs)    = if x <= y then x:y:xs else y:insert x xs

-----
-- functions on Char

type String = [Char]

toUpper, toLower :: Char -> Char
-- toUpper 'a'    == 'A'
-- toLower 'Z'    == 'z'

digitToInt       :: Char -> Int
-- digitToInt '8' == 8

intToDigit       :: Int -> Char
-- intToDigit 3   == '3'

ord              :: Char -> Int
chr              :: Int -> Char

```

# Exam

1. Given the following typeclass definition:

```
class (Eq a, Show a) => Num a where
  (+), (-), (*)      :: a -> a -> a
  negate, abs, signum :: a -> a
  fromInteger        :: Integer -> a
```

and given the following definition of type `MyNatural`:

```
data MyNatural = Empty | () :-: MyNatural
  deriving (Eq, Show)
infixr 5 :-:
```

so that e.g.:

```
twoM = () :-: () :-: Empty
threeM = () :-: () :-: () :-: Empty
-- (or: threeM = () :-: twoM)
```

consider the following functions:

```
f1 Empty y = y
f1 ( () :-: x ) y = () :-: (f1 x y)
f2 Empty y = Empty
f2 ( () :-: x ) y = f1 y (f2 x y)
f3 x Empty = x
f3 Empty x = error "foo"
f3 ( () :-: x ) ( () :-: y ) = f3 x y
```

and make the following definition complete:

```
instance Num MyNatural where
  ...
```

Define appropriate auxiliary functions, if necessary.

Please note that the following equation must be obeyed in order to make `abs` and `signum` correctly defined:

```
(abs x) * (signum x) == x
```

`signum` is either 1 (positive argument), 0 (zero) or -1 (negative argument) in general case. For natural numbers that we try to define here, it may obviously be only zero or one. The same note applies to `negate` function: it should yield error on non-zero values, like in `f3` above.

2. Consider the following two versions of similarity score computations. The difference is in the expression defining value for `simEntry i j`.

- (a) Which of the versions is much faster than the other?
- (b) Why?

Answering (a) but not (b) does not give much credit. Wrong answer is worth less than “I don’t know”.

VERSION 1:

```

similScore :: String -> String -> Int
similScore xs ys = simScore (length xs) (length ys)
  where
    simScore i j = simTable!!i!!j
    simTable = [[ simEntry i j | j<-[0..]] | i<-[0..] ]

    simEntry :: Int -> Int -> Int
    simEntry 0 0 = 0
    simEntry i 0 = (i * scoreSpace)
    simEntry 0 j = (scoreSpace * j)
    simEntry i j = maximum [((simScore (i-1) (j-1)) + (score x y)),
                             ((simScore (i-1) j) + (score x '-')),
                             ((simScore i (j-1)) + (score '- ' y))]
      where
        x = xs!!(i-1)
        y = ys!!(j-1)

```

VERSION 2:

```

similScore :: String -> String -> Int
similScore xs ys = simScore (length xs) (length ys)
  where
    simScore i j = simTable!!i!!j
    simTable = [[ simEntry i j | j<-[0..]] | i<-[0..] ]

    simEntry :: Int -> Int -> Int
    simEntry 0 0 = 0
    simEntry i 0 = (i * scoreSpace)
    simEntry 0 j = (scoreSpace * j)
    simEntry i j = maximum [((simEntry (i-1) (j-1)) + (score x y)),
                             ((simEntry (i-1) j) + (score x '-')),
                             ((simEntry i (j-1)) + (score '- ' y))]
      where
        x = xs!!(i-1)
        y = ys!!(j-1)

```

3. The function `unfoldr` may be defined as follows:

```
unfoldr :: (b -> Maybe (a,b)) -> b -> [a]
unfoldr f b = case f b of
    Nothing -> []
    Just (a,b) -> a : unfoldr f b
```

With a suitable function `g` it is possible to implement the prelude function

```
iterate :: (a -> a) -> a -> [a]
```

as:

```
iterate = unfoldr . g
```

- (a) Determine the type of function `g`.
- (b) Define the function `g`.

4. The `Functor` class is defined as follows:

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

It is mandatory that all instances of `Functor` should obey:

```
fmap id      = id
fmap (p . q) = (fmap p) . (fmap q)
```

Assume the following definition of lists as a functor instance:

```
instance Functor [] where
    fmap g []      = []
    fmap g (x:xs) = fmap g xs ++ [g x]
```

Is this a correct definition of a functor instance? Why or why not?

5. Explain the concept of a *spark* in Haskell. How does it relate to the following three functions

```
seq, pseq, par :: a -> b -> b
```

Explain what they do.

6. Type derivation

(a) Find the type of `(.) . (.)`

(b) Given that

```
map2 :: (a -> b, c -> d) -> (a, c) -> (b, d)
```

find the destination type `e` of the following function:

```
rulesCompile :: [(String, [String])] -> e
```

```
rulesCompile = (map . map2) (words . map toLower, map words)
```

(c) Given that

```
transformationApply :: Eq a => a -> ([a] -> [a]) -> [a] -> ([a], [a])  
                                                             -> Maybe [a]
```

```
orElse :: Maybe a -> Maybe a -> Maybe a
```

find the type of

```
foldr1 orElse (map (transformationApply wildcard f x) pats)
```

Good Luck!