Tessellation & Interpolation

EDAF80: Computer Graphics

Rikard Olajos

ASSIGNMENT 1

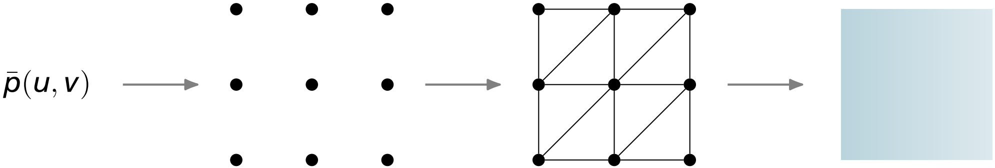
- Transformation hierarchy
- Scene graph traversal using a stack
- Any questions?

AGENDA

1. Tessellation
2. Interpolation
3. Assignment 2

Tessellation

TESSELLATION



1. Setup ***vertex array***
2. Setup ***index array*** (triangulate)

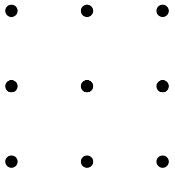
CREATE VERTEX ARRAY

- Create vertex array (e.g. 3×3 vertices)

```
auto vertices = std::vector<glm::vec3>(9);
```

- Assign vertex

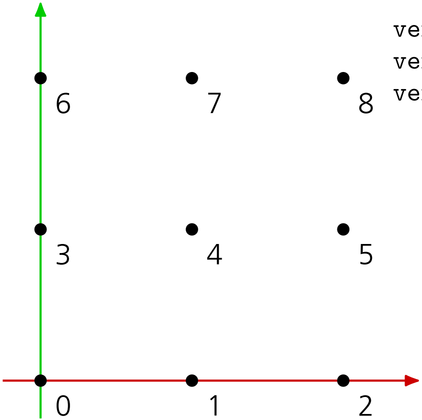
```
vertices[index] = glm::vec3(x, y, z);
```



VERTEX ARRAY LAYOUT


```
vertices[8].x = 1.0;
vertices[8].y = 1.0;
vertices[8].z = 0.0;
```

```
vertices[3].x = 0.0;
vertices[3].y = 0.5;
vertices[3].z = 0.0;
```



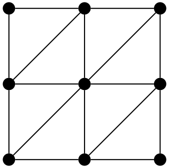
TRIANGULATION

- Create index array ($2 \times (3-1)(3-1)$ triangles)

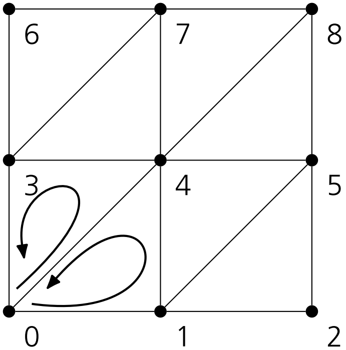
```
auto indices = std::vector<glm::uvec3>(8);
```

- Define triangle (indices for the three vertices)

```
indices[index] = glm::uvec3(v0, v1, v2);
```



INDEX ARRAY LAYOUT



- Indices in counter-clockwise order (CCW)
- Backface culling is off by default, turn it on to improve performance!

```
indices[0].x = 0;
indices[0].y = 1;
indices[0].z = 4;

indices[1].x = 0;
indices[1].y = 4;
indices[1].z = 3;
```

```
glEnable(GL_CULL_FACE);
glCullFace(GL_BACK); // GL_BACK (default), GL_FRONT, GL_FRONT_AND_BACK
glFrontFace(GL_CCW); // GL_CCW (default), GL_CW
```

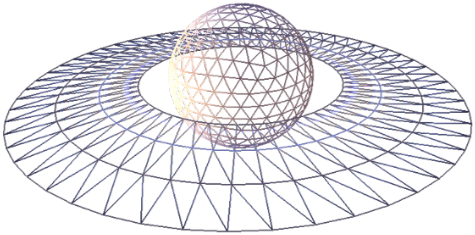
Seminar Exercise 2-1: Indexed Draw and Vertex Attributes

1. Change the index list to draw a triangle between vertices 3, 4 and 5.
2. Keep the green triangle and add back the original red triangle.
3. Try changing the color of the vertices by changing the vertex attributes.

FOLLOW-UP QUESTIONS

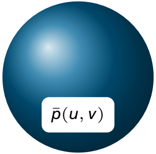
1. Can you control which triangle that ends up on top?
2. What happens when attributes within a triangle have different values?

PARAMETRIC SURFACES



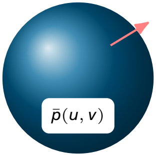
PARAMETRIC SURFACE & TANGENT SPACE

- Map surface from 2D: $\bar{p}(x, y, z) = \bar{p}(u, v)$
- $\mathbb{R}^2 \mapsto \mathbb{R}^3$



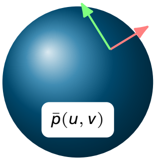
PARAMETRIC SURFACE & TANGENT SPACE

- Map surface from 2D: $\bar{p}(x, y, z) = \bar{p}(u, v)$
- $\mathbb{R}^2 \mapsto \mathbb{R}^3$
  - Tangent $t = \frac{\partial \bar{p}}{\partial u}$



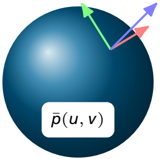
PARAMETRIC SURFACE & TANGENT SPACE

- Map surface from 2D: $\bar{p}(x, y, z) = \bar{p}(u, v)$
- $\mathbb{R}^2 \mapsto \mathbb{R}^3$
  - Tangent $t = \frac{\partial \bar{p}}{\partial u}$
  - Binormal $b = \frac{\partial \bar{p}}{\partial v}$



PARAMETRIC SURFACE & TANGENT SPACE

- Map surface from 2D: $\bar{p}(x, y, z) = \bar{p}(u, v)$
- $\mathbb{R}^2 \mapsto \mathbb{R}^3$
  - Tangent $t = \frac{\partial \bar{p}}{\partial u}$
  - Binormal $b = \frac{\partial \bar{p}}{\partial v}$
  - Normal $n = \frac{\partial \bar{p}}{\partial u} \times \frac{\partial \bar{p}}{\partial v}$

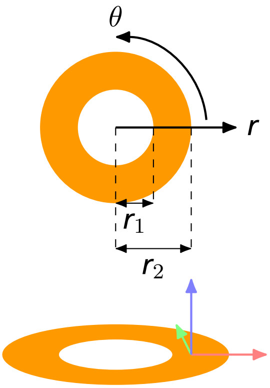


CIRCLE RING

$$\bar{p}(r,\theta) = \begin{Bmatrix} r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{Bmatrix} \text{ for } \begin{matrix} r_1 \leq r \leq r_2 \\ 0 \leq \theta < 2\pi \end{matrix}$$

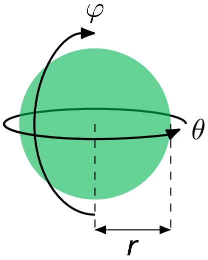
$$t = \frac{\partial \bar{p}}{\partial r} = \begin{Bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{Bmatrix}$$

$$b = \frac{\partial \bar{p}}{\partial \theta} = \begin{Bmatrix} -r\sin(\theta) \\ r\cos(\theta) \\ 0 \end{Bmatrix}$$

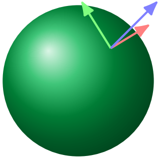


SPHERE

$$\bar{p}(\theta, \varphi) = \begin{Bmatrix} r\sin(\theta)\sin(\varphi) \\ -r\cos(\varphi) \\ r\cos(\theta)\sin(\varphi) \end{Bmatrix} \text{ for } \begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le \varphi \le \pi \end{matrix}$$



$$t = \frac{\partial \bar{p}}{\partial \theta} = \begin{Bmatrix} r\cos(\theta)\sin(\varphi) \\ 0 \\ -r\sin(\theta)\sin(\varphi) \end{Bmatrix}$$



$$b = \frac{\partial \bar{p}}{\partial \varphi} = \begin{Bmatrix} r\sin(\theta)\cos(\varphi) \\ r\sin(\varphi) \\ r\cos(\theta)\cos(\varphi) \end{Bmatrix}$$

SPHERE

$$\bar{p}(\theta, \varphi) = \begin{Bmatrix} r\sin(\theta)\sin(\varphi) \\ -r\cos(\varphi) \\ r\cos(\theta)\sin(\varphi) \end{Bmatrix} \text{ for } \begin{matrix} 0 \leq \theta \leq 2\pi \\ 0 \leq \varphi \leq \pi \end{matrix}$$

$$t = \frac{\partial \bar{p}}{\partial \theta} = \begin{Bmatrix} r\cos(\theta)\sin(\varphi) \\ 0 \\ -r\sin(\theta)\sin(\varphi) \end{Bmatrix}$$

$$b = \frac{\partial \bar{p}}{\partial \varphi} = \begin{Bmatrix} r\sin(\theta)\cos(\varphi) \\ r\sin(\varphi) \\ r\cos(\theta)\cos(\varphi) \end{Bmatrix}$$

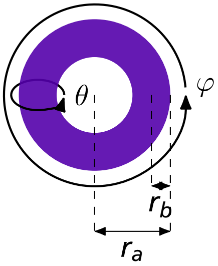
- t and b can be simplified since only direction is important
- t ***needs*** to be simplified, or it will be undefined for $\varphi = 0$

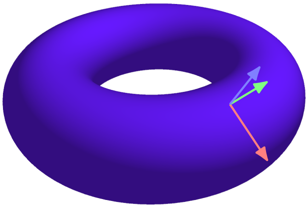
TORUS

$$\bar{p}(\theta, \varphi) = \begin{Bmatrix} (r_a + r_b \cos(\theta)) \cos(\varphi) \\ (r_a + r_b \cos(\theta)) \sin(\varphi) \\ -r_b \sin(\theta) \end{Bmatrix} \text{ for } \begin{matrix} 0 \leq \theta \leq 2\pi \\ 0 \leq \varphi \leq 2\pi \end{matrix}$$

$$t = \frac{\partial \bar{p}}{\partial \theta} = \begin{Bmatrix} -r_b \sin(\theta) \cos(\varphi) \\ -r_b \sin(\theta) \sin(\varphi) \\ -r_b \cos(\theta) \end{Bmatrix}$$

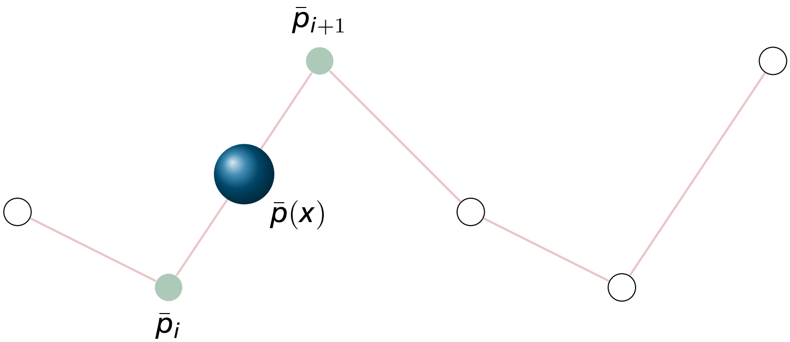
$$b = \frac{\partial \bar{p}}{\partial \varphi} = \begin{Bmatrix} -(r_a + r_b \cos(\theta)) \sin(\varphi) \\ (r_a + r_b \cos(\theta)) \cos(\varphi) \\ 0 \end{Bmatrix}$$





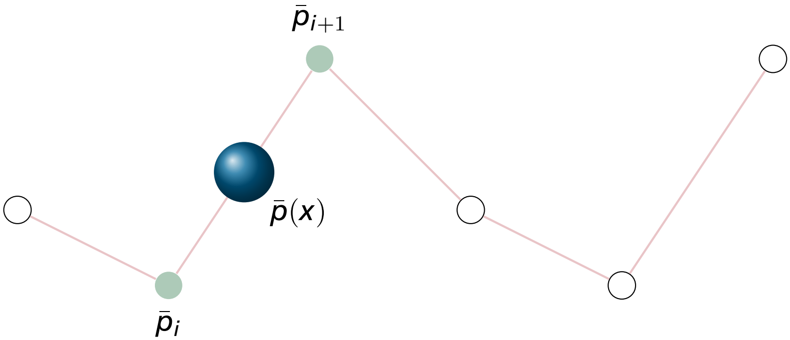
Interpolation

LINEAR INTERPOLATION (LERP)



$$\bar{p}(x) = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{p}_i \\ \bar{p}_{i+1} \end{bmatrix} \quad \text{for} \quad x \in [0, 1]$$

LINEAR INTERPOLATION (LERP)



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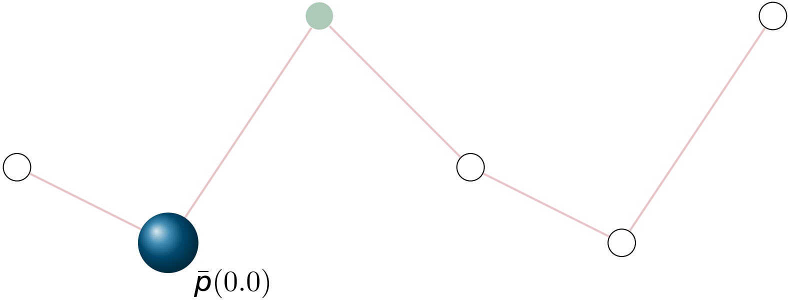
$$\bar{p}(x) = (1 - x)\bar{p}_i + x\bar{p}_{i+1}$$

LINEAR INTERPOLATION (LERP)



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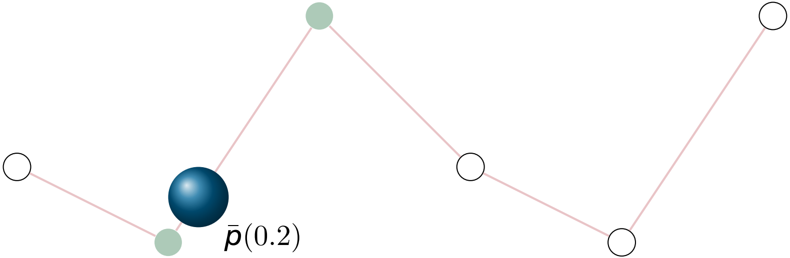

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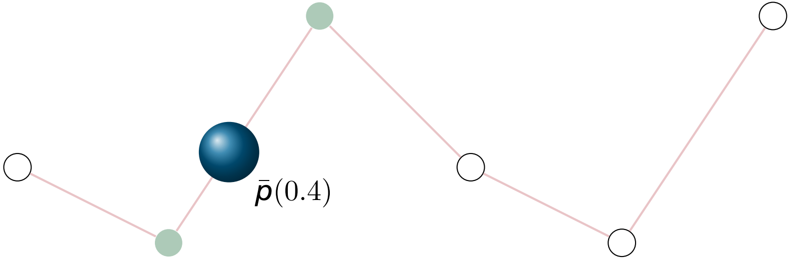

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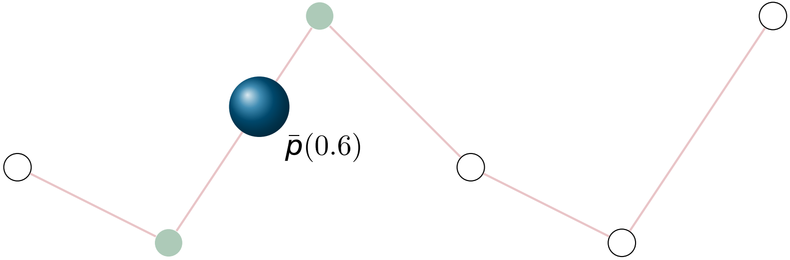

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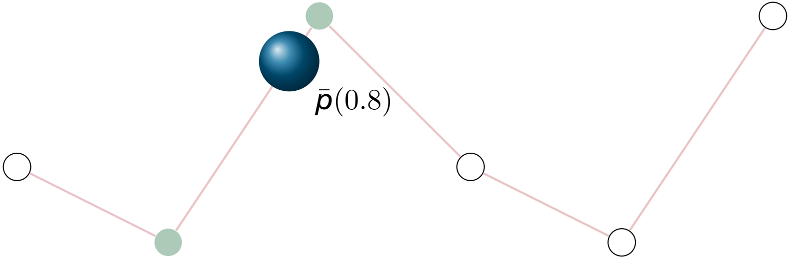

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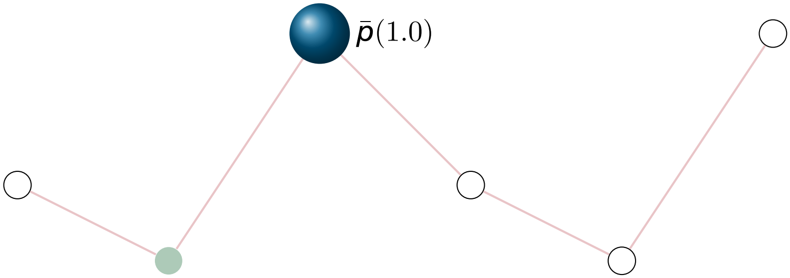

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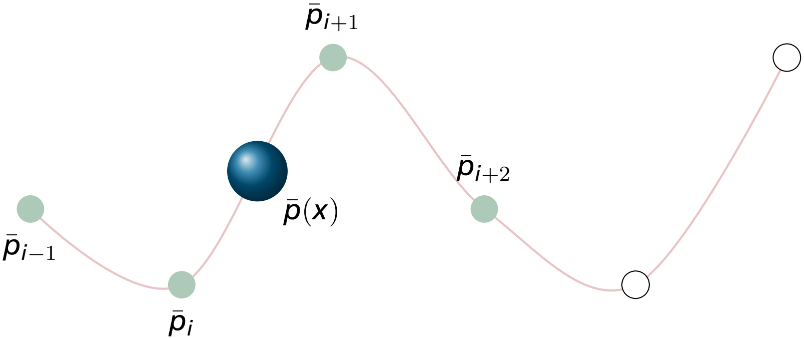
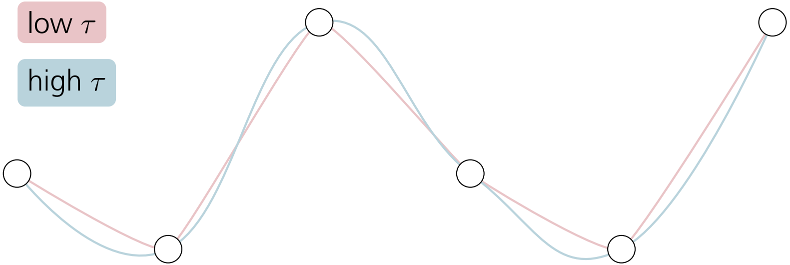
$$\bar{p}(x) = (1 - x)\bar{p}_i + x\bar{p}_{i+1}$$

CUBIC INTERPOLATION (CATMULL-ROM)



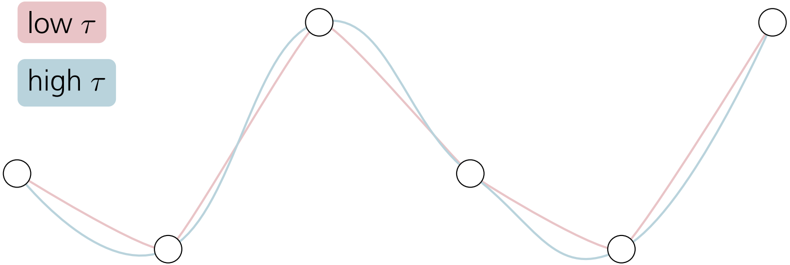
$$\bar{q}(x) = \begin{bmatrix}1 & x & x^2 & x^3\end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\tau & 0 & \tau & 0 \\ 2\tau & \tau - 3 & 3 - 2\tau & -\tau \\ -\tau & 2 - \tau & \tau - 2 & \tau \end{bmatrix} \begin{bmatrix} \bar{p}_{i-1} \\ \bar{p}_i \\ \bar{p}_{i+1} \\ \bar{p}_{i+2} \end{bmatrix} \quad \textit{for} \quad x \in [0, 1]$$

TENSION



$$\bar{q}(x) = \begin{bmatrix}1 & x & x^2 & x^3\end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\tau & 0 & \tau & 0 \\ 2\tau & \tau - 3 & 3 - 2\tau & -\tau \\ -\tau & 2 - \tau & \tau - 2 & \tau \end{bmatrix} \begin{bmatrix} \bar{p}_{i-1} \\ \bar{p}_i \\ \bar{p}_{i+1} \\ \bar{p}_{i+2} \end{bmatrix} \quad for \;\; x \in [0, 1]$$

TENSION



$$\bar{q}(x) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\tau & 0 & \tau & 0 \\ 2\tau & \tau - 3 & 3 - 2\tau & -\tau \\ -\tau & 2 - \tau & \tau - 2 & \tau \end{bmatrix} \begin{bmatrix} \bar{p}_{i-1} \\ \bar{p}_i \\ \bar{p}_{i+1} \\ \bar{p}_{i+2} \end{bmatrix} \quad for \;\; x \in [0, 1]$$

- $\tau =$ tension, how "stiff" the curve is at the control points
- Keep within $[0, 1]$
- Good initial value: 0.5

Seminar Exercise 2-2: Interpolation

1. Make the top vertex of the triangle (`gl_VertexID` 2) move between point `p0` at $(-1.0, 1.0, 0.0)$ and `p1` at $(1.0, -0.3, 0.0)$.
   - Use `mix()` to do linear interpolation in GLSL.
   - Use the `time` uniform to animate, but remember to keep the t value of the interpolation between 0 and 1. Try using `mod()` or `sin()`.
2. Animate the color of the triangle by changing the color output in the fragment shader. Have it interpolate between two colors `c0` and `c1`.
3. Spend a few minutes and trying different combinations of animations: different vertex ID, other colors, maybe throw in an extra point to interpolate with and use nested mix functions.

Assignment 2

ASSIGNMENT 2

- Tessellate objects from parametric equations
- Linear and cubic interpolation
- Files you have to modify
  - `src/EDAF80/assignment2.cpp`
  - `src/EDAF80/parametric_shapes.cpp`
  - `src/EDAF80/interpolation.cpp`

TESSELLATION

- Implement function bodies in `src/EDAF80/parametric_shapes.cpp`

```
bonobo::mesh_data parametric_shapes::createQuad(...);
bonobo::mesh_data parametric_shapes::createSphere(...);
bonobo::mesh_data parametric_shapes::createTorus(...); // Optional
```

TESSELLATION

- Implement function bodies in `src/EDAF80/parametric_shapes.cpp`

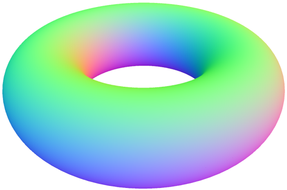
```
bonobo::mesh_data parametric_shapes::createQuad(...);
bonobo::mesh_data parametric_shapes::createSphere(...);
bonobo::mesh_data parametric_shapes::createTorus(...); // Optional
```

- Look at `createCircleRing(...)` in the same file for guidance
- Make sure parameter definitions and ranges are correct
  - Circle ring: $0 \leq \theta < 2\pi, r_1 \leq r \leq r_2$
  - Sphere: $0 \leq \theta \leq 2\pi, 0 \leq \varphi \leq \pi$

DEBUGGING NORMALS

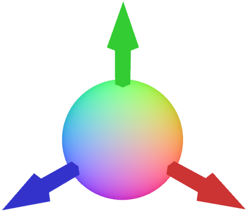
- Colourize, use the "Normals" shader to represent normals as RGB values

- Inspect illumination, is illumination consistent with the location of the light source?



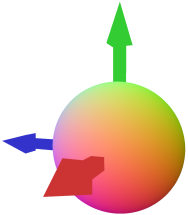
COLOURIZING NORMALS

- Map from $[-1, 1]$ to $[0, 1]$
  - $(\boldsymbol{N} \cdot 0.5) + 0.5$
- Example: Z axis $(0, 0, 1)$ becomes $(0.5, 0.5, 1)$
- Values are normalized: $(1, 1, 1) \mapsto (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$



$(1, 1, 1) \mapsto (1, 1, 1)$

$(-1, -1, -1) \mapsto (0, 0, 0)$



$(1, 0, 0) \mapsto (1, 0.5, 0.5)$

TOOLS IN GUI

- Change cull mode: Disabled, Back faces, Front faces
- Change polygon mode: Fill, Line, Point
- Change shaders: Fallback, Diffuse, Normal, Tangent, Bitangent, Texture coords

INTERPOLATION

- Implement linear and cubic interpolation
- Implement function bodies in `src/EDAF80/interpolation.cpp`

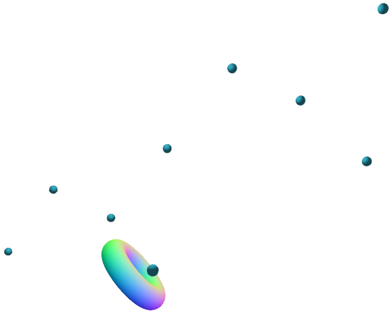
```
glm::vec3 interpolation::evalLERP(...);
glm::vec3 interpolation::evalCatmullRom(...);
```

- Test with just 2 (LERP) or 4 (cubic) points first
- Animate an object along the path using both function and the predefined control points
- `use_linear` and `catmull_rom_tension` variables are bound to the GUI and should be used

IMPLEMENTATION SKETCH

```
// Init:
std::array<glm::vec3, N> control_points = { ... };
float elapsed_time_s = 0.0f;

// Main loop:
    elapsed_time_s += delta_time_s;

    int i = floor(elapsed_time_s);
    float x = elapsed_time_s - i;

    // Pick indices for interpolation: i-1, i, i+1, i+2

    // Make sure indices wrap: 0, 1, ..., N-1, 0, 1, ...

    // Call interpolation function with points from control_points
```

DEMO



WEBGL2 STATE DIAGRAM

WebGL2 - State Diagram