Transformations & Coordinate Systems

EDAF80
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Slides by Jacob Munkberg 2012-13

2024

Labs

- Wednesday and Friday are crowded
    - If you can move to Thursday 15, that would help
- Start work on the lab before coming to the lab

Today

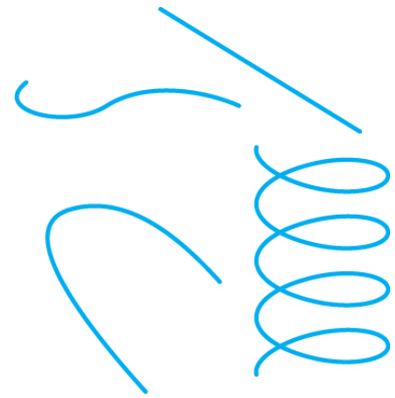
- Position objects in 3D
  - Transformations
  - Coordinate systems
- Reading
  - PBRT online : Section 2.7 Transformations

Last time

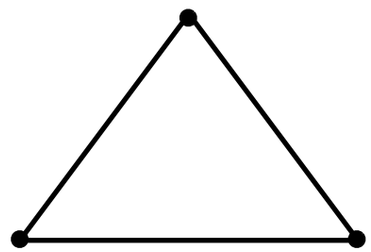
- Introduction to computer graphics
- Color, materials, real-time vs offline
- Linear algebra refresher

3D Primitives

- Curves - paths in space

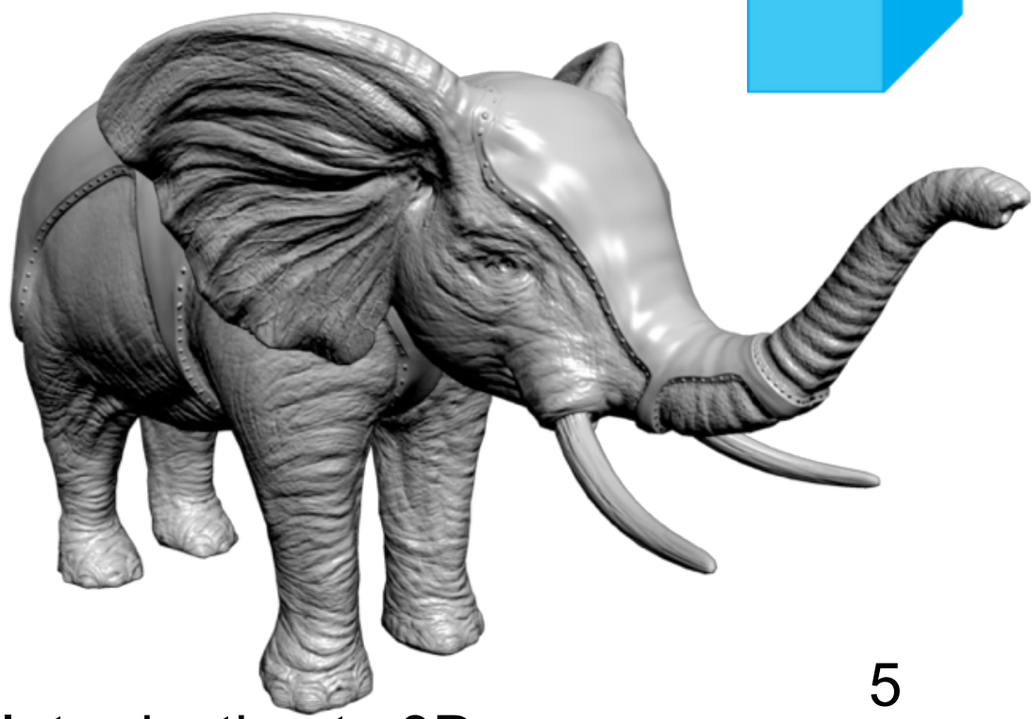


- Triangle
  - three connected points in 3D



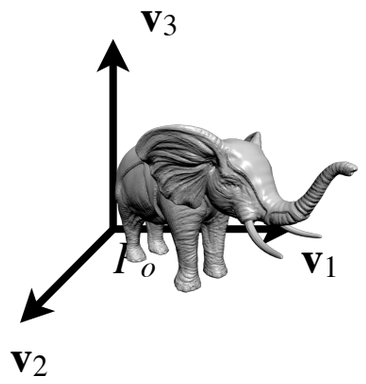
- Cube
  - 8 connected points in 3D (or 12 triangles)

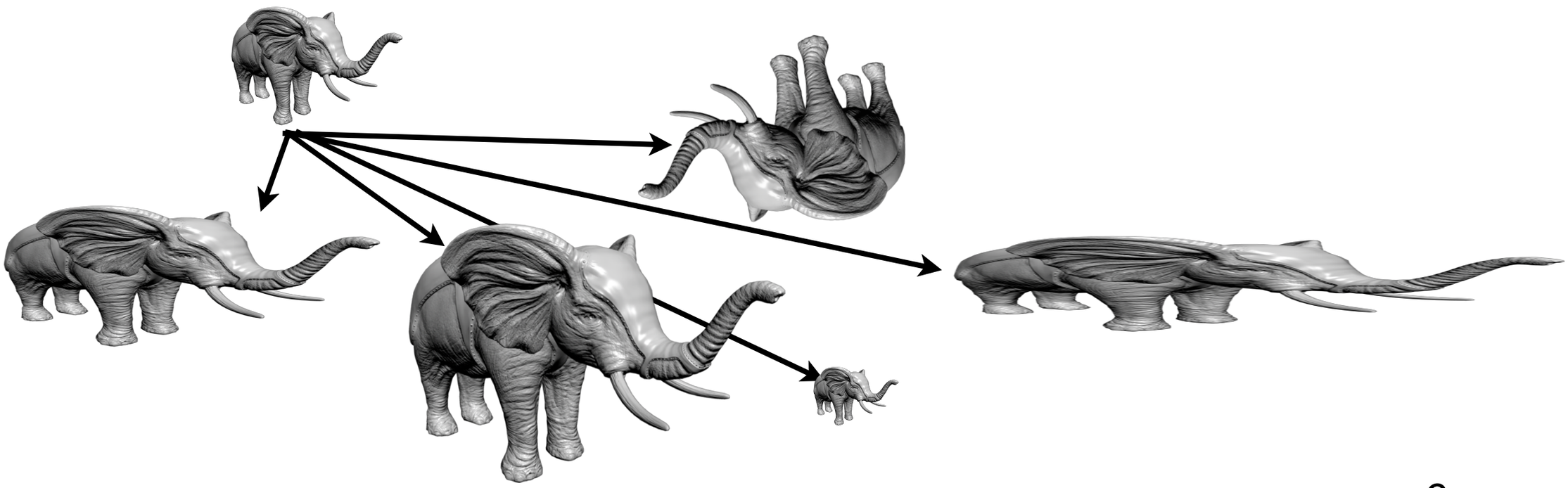
- Elephant
  - 22,840 triangles



Transformations

- Define object **once** in convenient local coordinate frame
- Apply transformation $\mathbf{M}$ to position each instance of the object



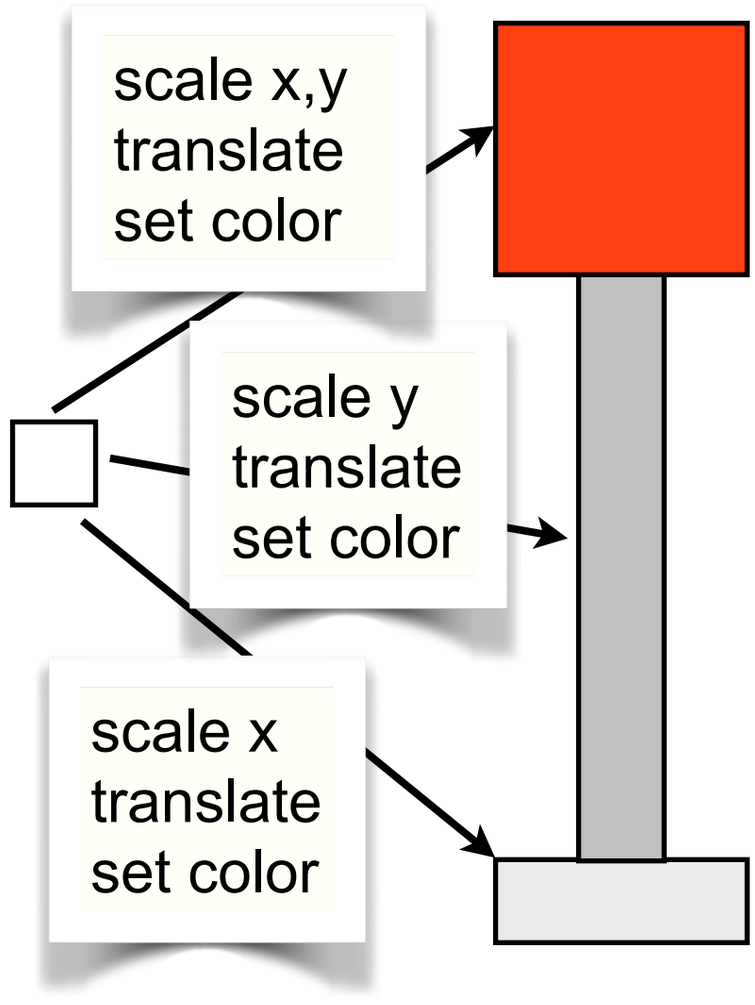


Transformations

- Maps points in one coordinate system to points in another coordinate system
- Useful for:
  - Position objects, lights and camera in a scene
  - Modeling
  - Change the shape of objects
  - Animations

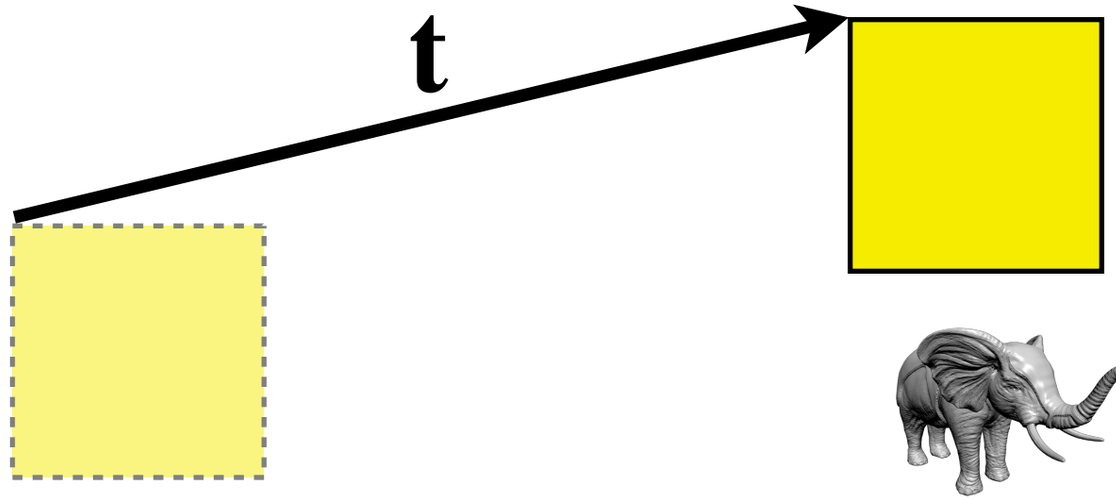
Example: Modeling

- Simple Modeling
  - Use basic primitives
  - Apply transforms to create more complex objects

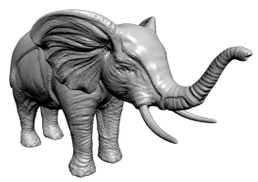


Translation

- Move along a vector
- Preserve distances
- Preserve angles



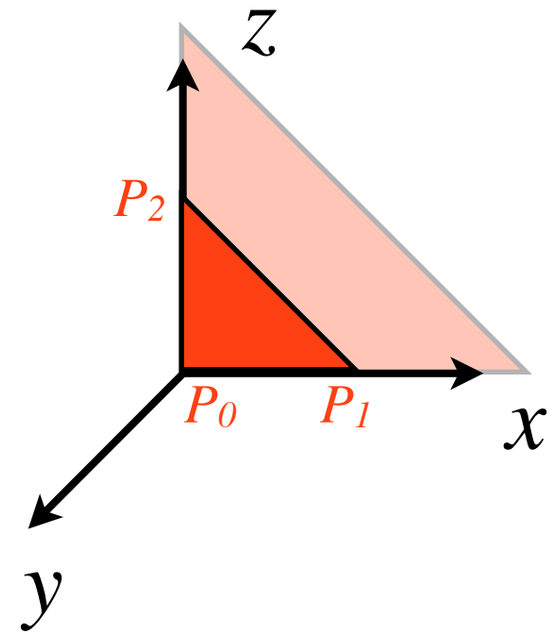
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ z + t_z \end{bmatrix}$$



$$P' = P + \mathbf{t}$$

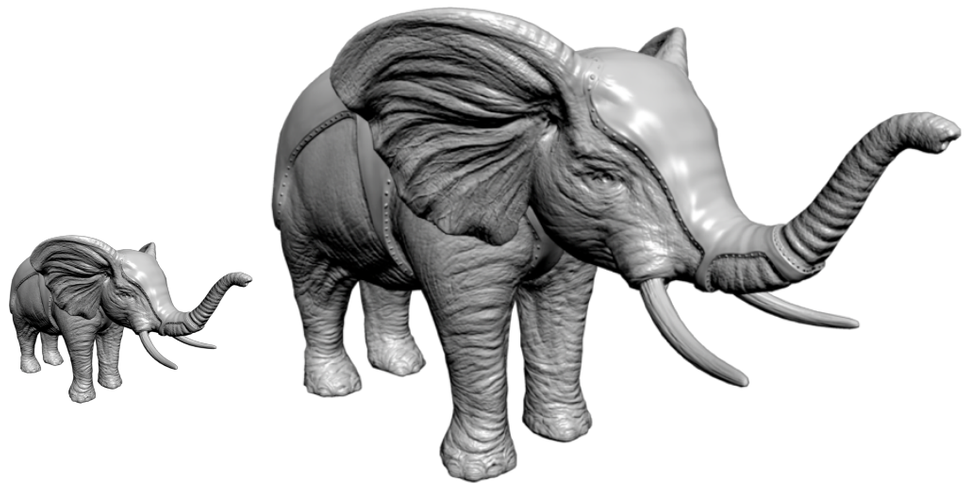
Uniform Scaling

- Scale each component with a scalar s
- Preserve angles
- **Doesn't** preserve distances



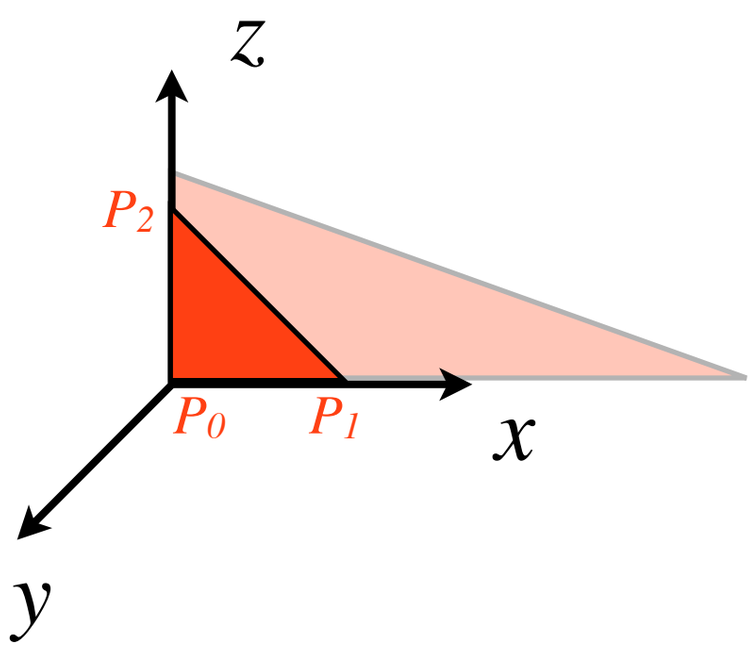
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} sx \\ sy \\ sz \end{bmatrix}$$

$$P_{\text{scaled}} = \mathbf{S}P$$



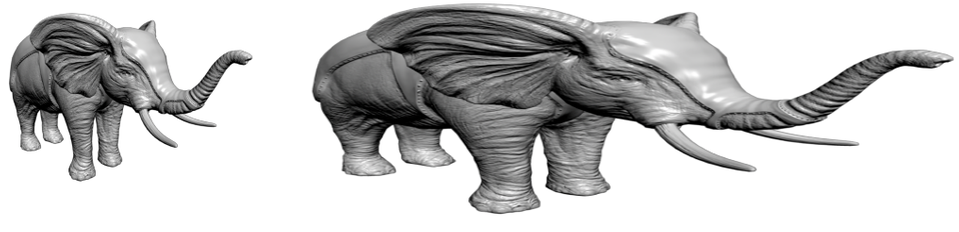
Non-uniform Scaling

- Scale each component with a different scalar s_i
- **Doesn't** preserve angles
- **Doesn't** preserve distances



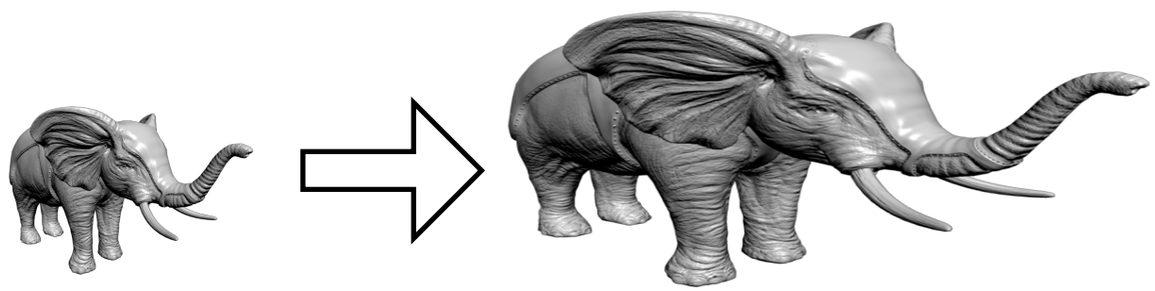
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \\ s_z z \end{bmatrix}$$

$$P_{\text{scaled}} = \mathbf{S}P$$

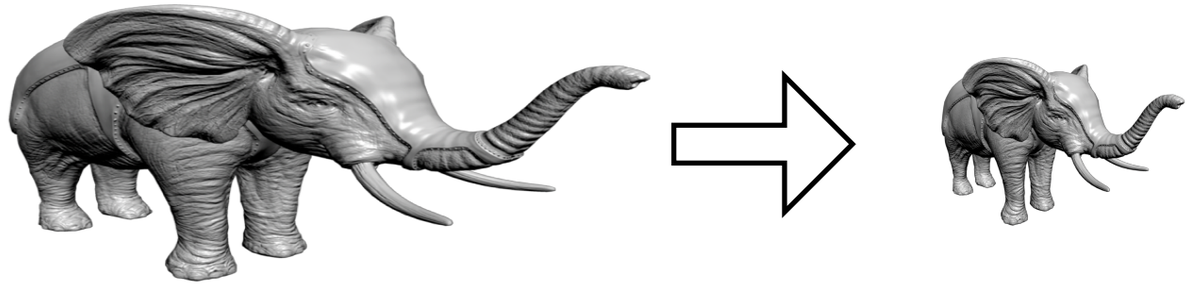


Inverse scaling transform

$$P_{\text{scaled}} = \mathbf{S}P$$



$$P = \mathbf{S}^{-1}P_{\text{scaled}}$$



$$\mathbf{S} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

$$\mathbf{S}^{-1} = \begin{bmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & \frac{1}{s_z} \end{bmatrix}$$

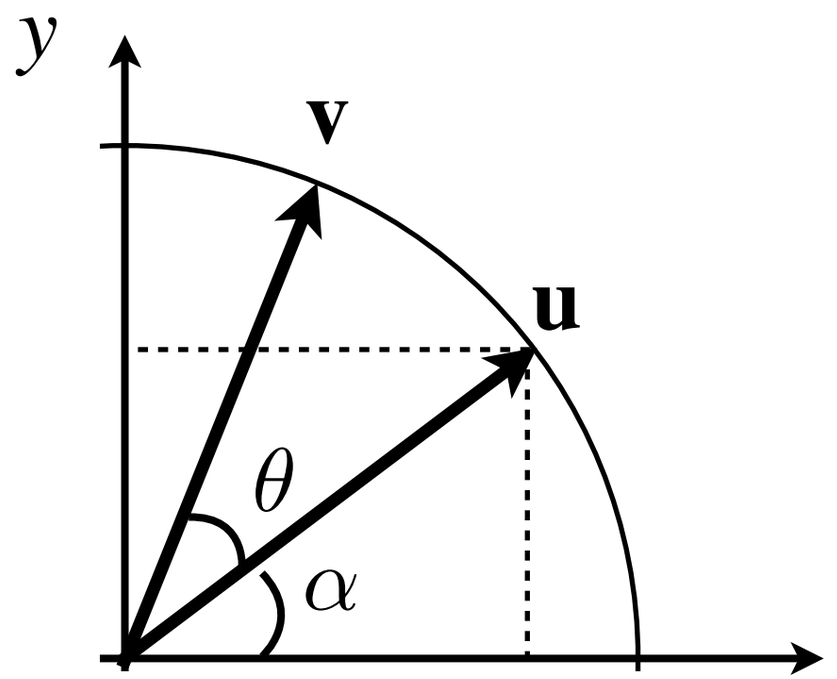
Rotation in the plane

- Rotate vector $\mathbf{u}$ to vector $\mathbf{v}$
- Express vectors in polar coordinates

$$\mathbf{u} = |\mathbf{u}|(\cos\alpha, \sin\alpha)$$
$$\mathbf{v} = |\mathbf{v}|(\cos(\alpha+\theta), \sin(\alpha+\theta))$$

- Use trigonometry

$$|\mathbf{u}| = |\mathbf{v}|$$
$$\cos(\alpha+\theta) = \cos\alpha\cos\theta - \sin\alpha\sin\theta$$
$$\sin(\alpha+\theta) = \cos\alpha\sin\theta + \sin\alpha\cos\theta$$



$$\mathbf{v} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \mathbf{u}$$

$$\mathbf{v} = \mathbf{R}(\theta)\mathbf{u}$$

Rotation in the plane

$$\mathbf{v} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \mathbf{u} \qquad \mathbf{v} = \mathbf{R}(\theta)\mathbf{u}$$

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Note that: $\mathbf{R}^{-1}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \mathbf{R}^T(\theta) = \mathbf{R}(-\theta)$

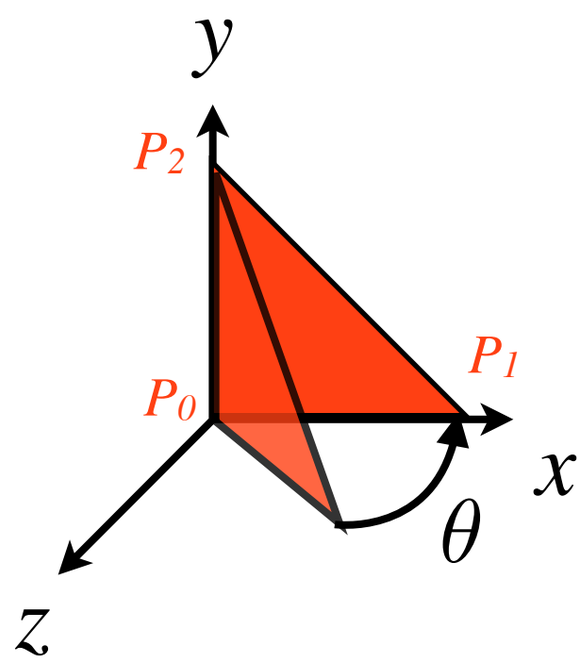
Rotation matrices are **orthogonal matrices**

Rotation

- 3D Example: Rotation around ***y***-axis

$$P_{r_y} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} P$$

$$\mathbf{R}_y(\theta)$$



- Rotations preserve angles and distances

Rotation Matrices

$$\mathbf{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation matrices are orthonormal

$$\mathbf{R}^{-1}(\theta) = \mathbf{R}^{\top}(\theta) = \mathbf{R}(-\theta)$$

$$\mathbf{R}\mathbf{R}^{\top} = 1$$

Classification of Transforms

Translation

Rotation

Rigid Body
preserves angles and distances

Uniform Scaling

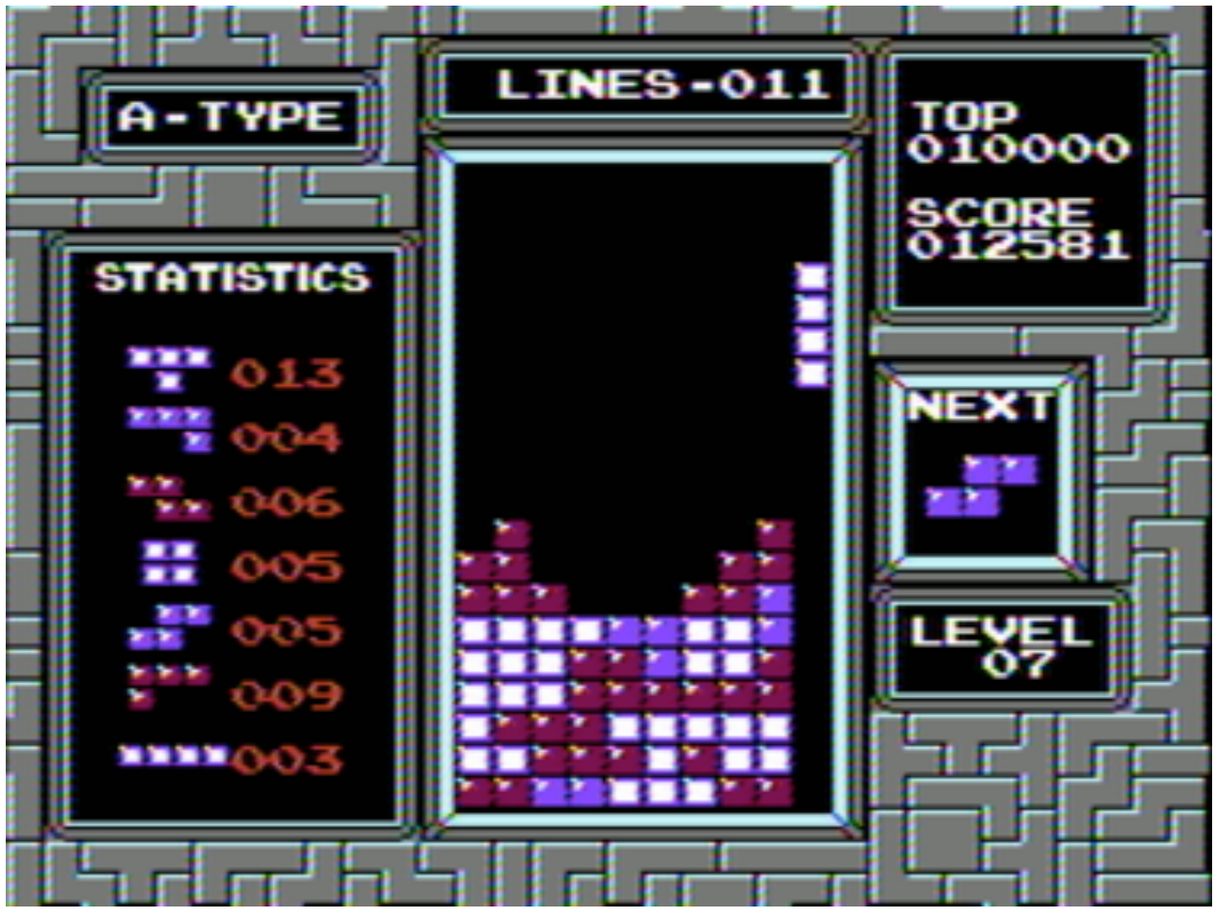
Similarity
preserves angles

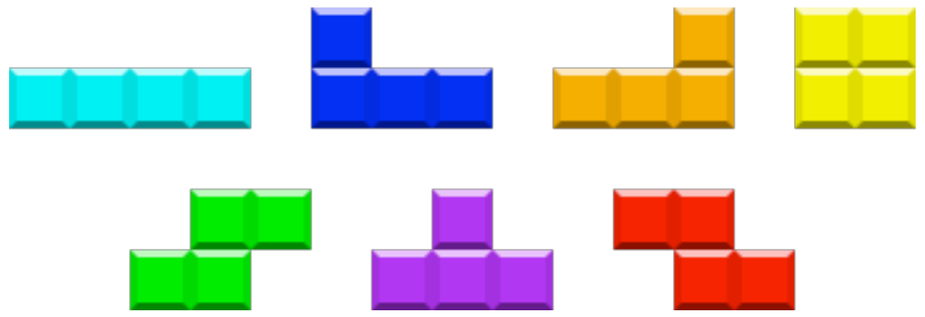
Non-Uniform Scaling

Affine
preserves parallel lines

Tetris

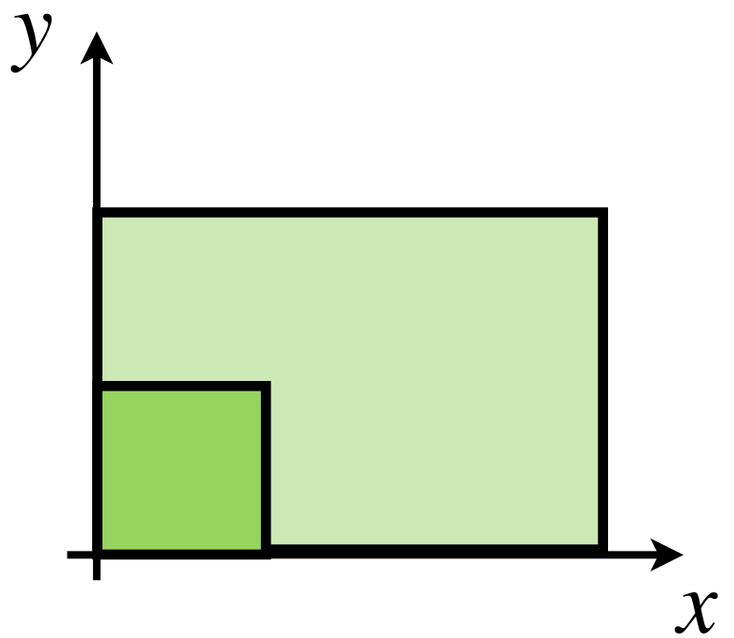
- Translation
- Rotation (increments of 90 degrees)



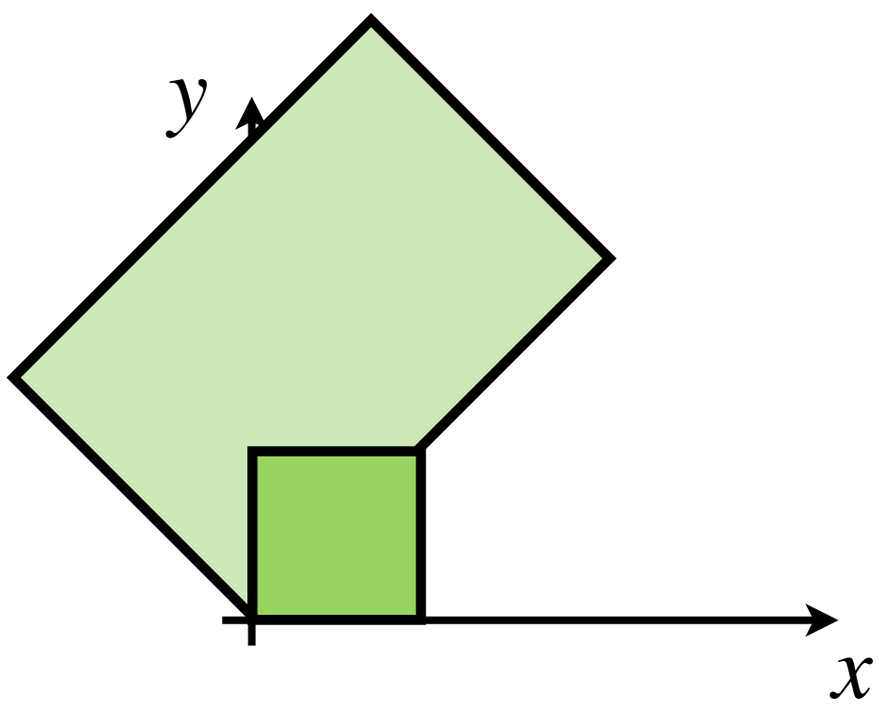


Composite Transform

- First: scale x by 3 and y by 2



... then rotate $\pi/4$ degrees around z-axis



Composite Transform

- First scale x by 3 and y by 2

$$P_s = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} P$$

... then rotate $\pi/4$ degrees around z-axis

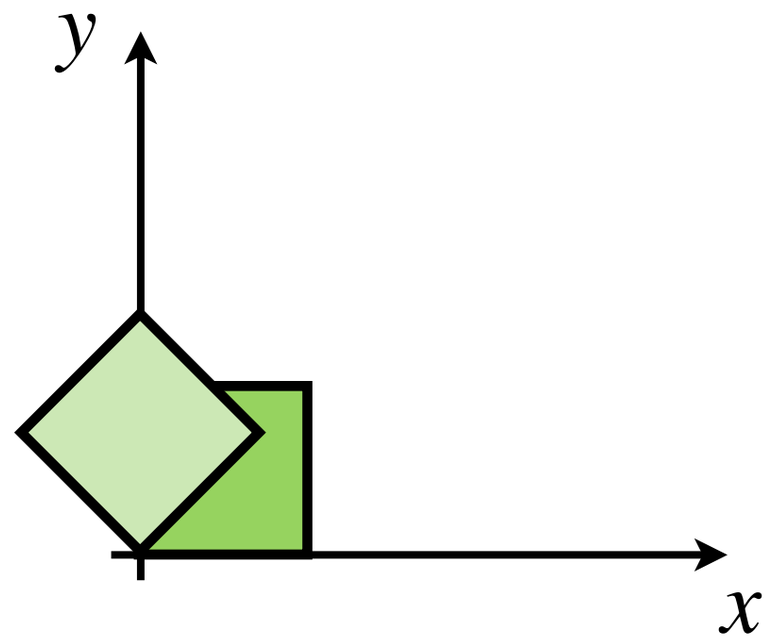
$$P_{s+r} = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} P$$

$$P_{s+r} = \begin{bmatrix} 3\cos\frac{\pi}{4} & -2\sin\frac{\pi}{4} & 0 \\ 3\sin\frac{\pi}{4} & 2\cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} P$$

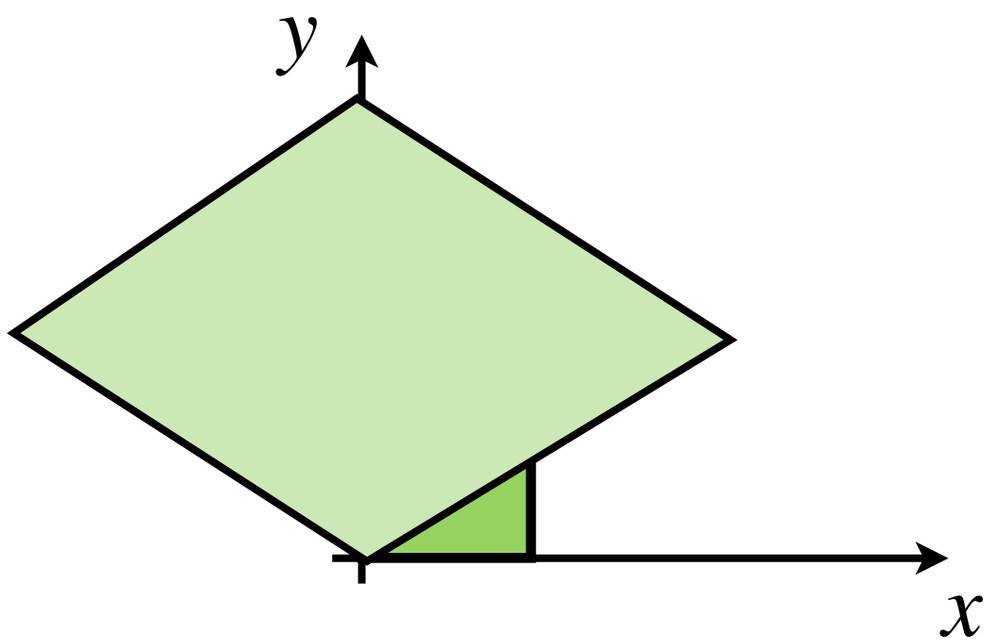
$$P_{s+r} = \mathbf{RS}\ P = \mathbf{M}P$$

Composite Transform II

- Now: **Swap order** of transforms
- First rotate $\pi/4$ degrees around z-axis



... then scale x by 3 and y by 2



Composite Transform II

- First rotate $\pi/4$ degrees around z-axis, then scale x by 3 and y by 2

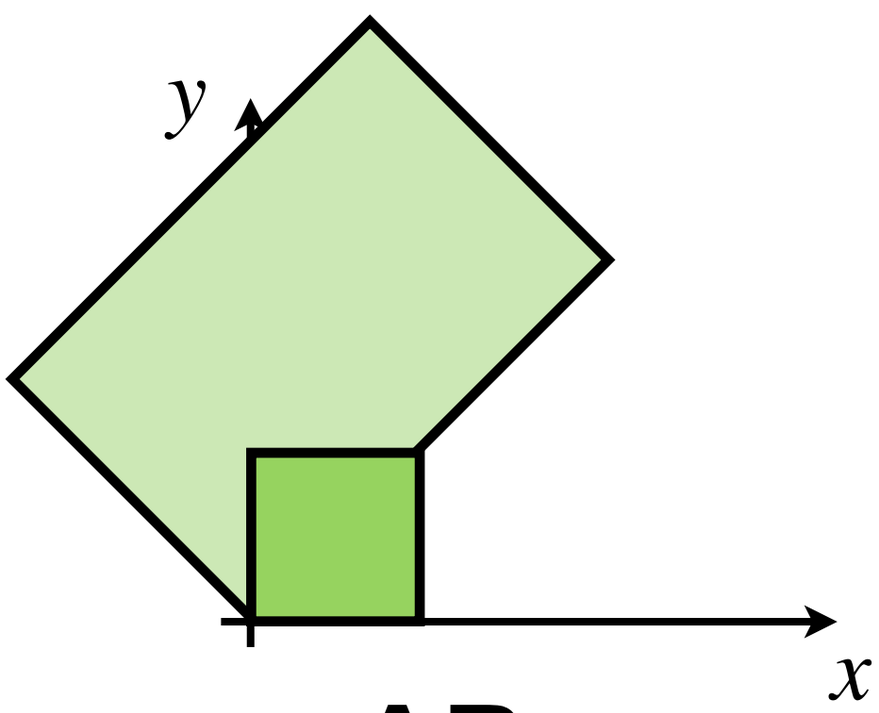
$$P_{r+s} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} P$$

$$P_{r+s} = \begin{bmatrix} 3\cos\frac{\pi}{4} & -3\sin\frac{\pi}{4} & 0 \\ 2\sin\frac{\pi}{4} & 2\cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} P$$

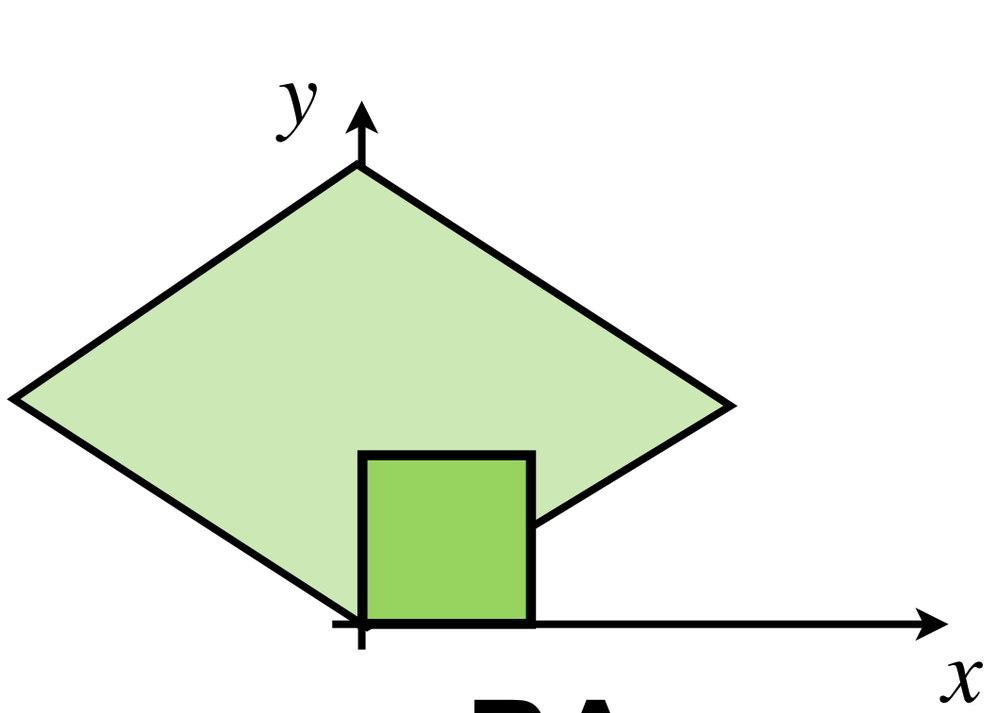
$$P_{r+s} = \mathbf{SR}P = \mathbf{N}P$$

Composite Transforms

- The concatenation of transform matrices is a new matrix with the same dimensions
- Order matters. In general: **AB ≠ BA**



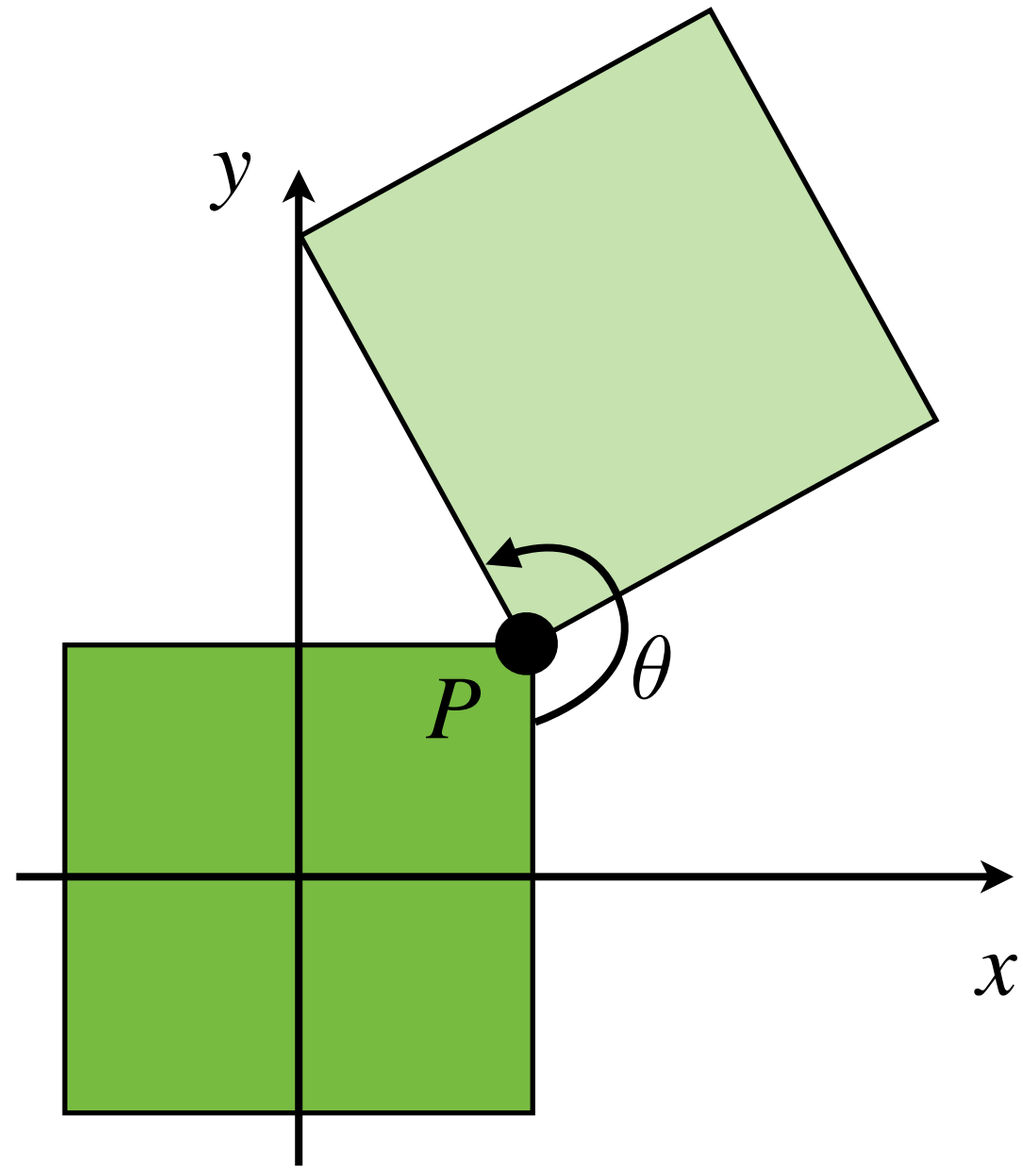
AB



BA

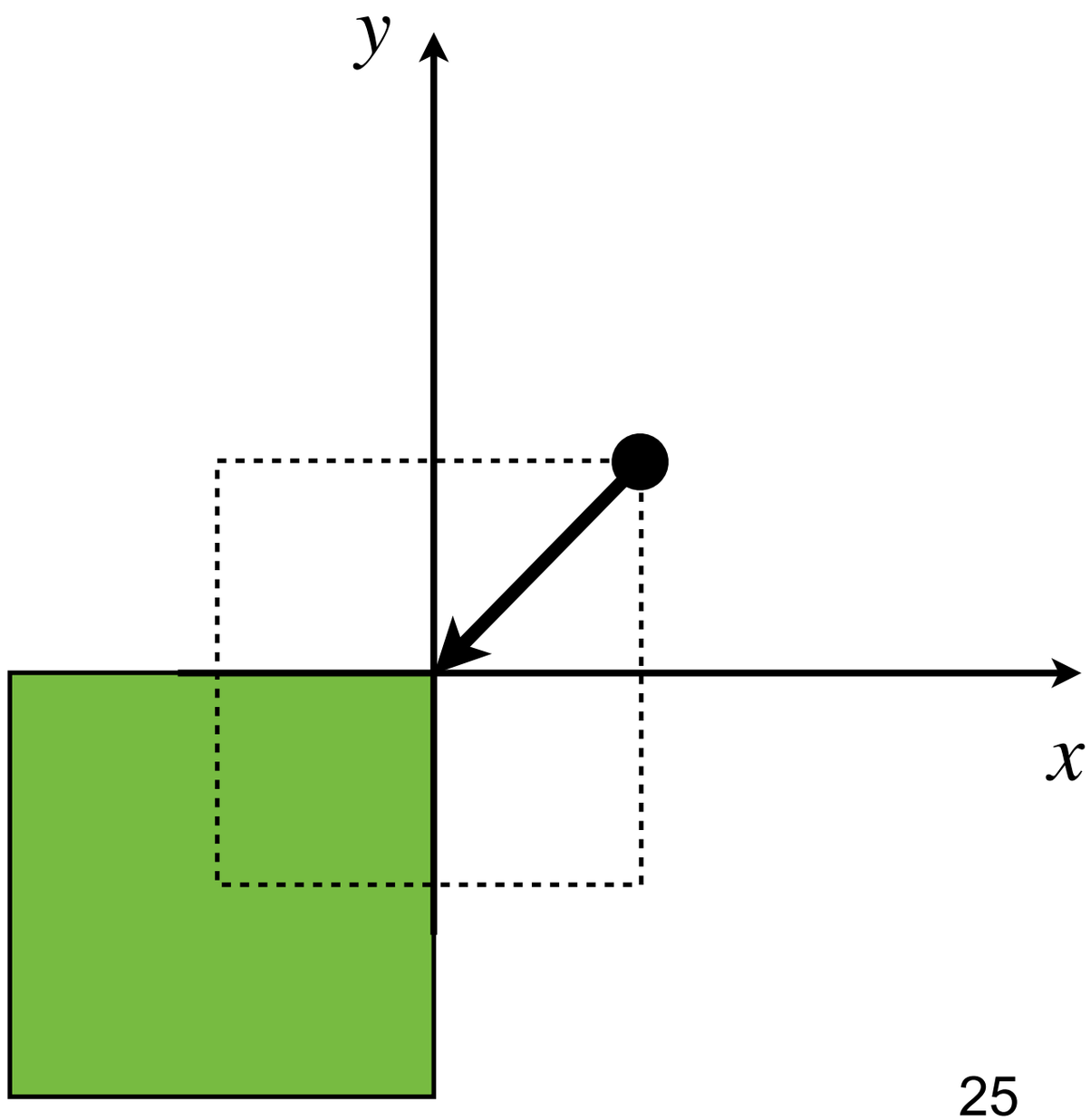
Composite Transforms III

- Example:
  - Z-rotation around point P



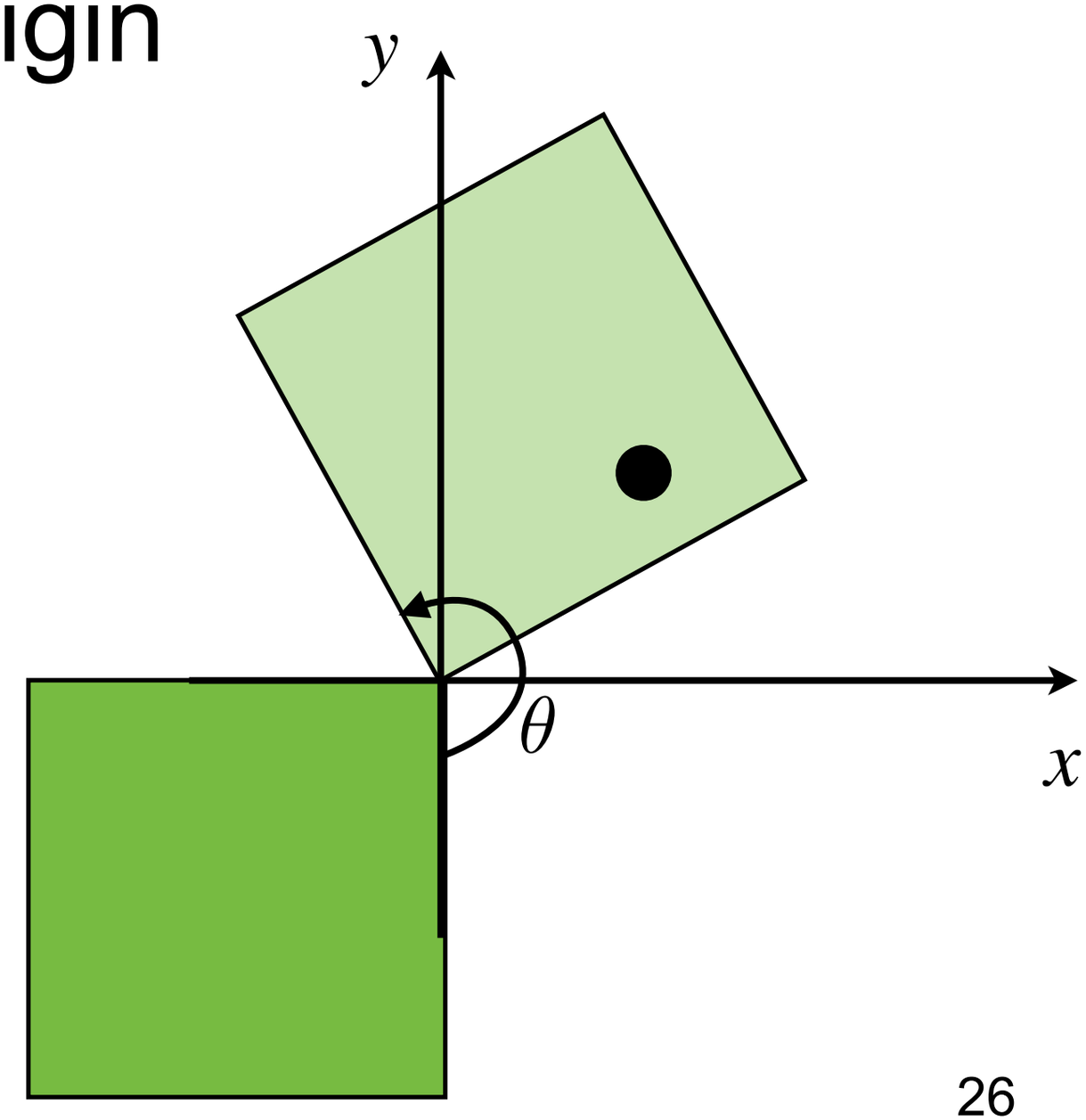
Combined Transforms III

1. Translate to origin: $\mathbf{T}(-P)$



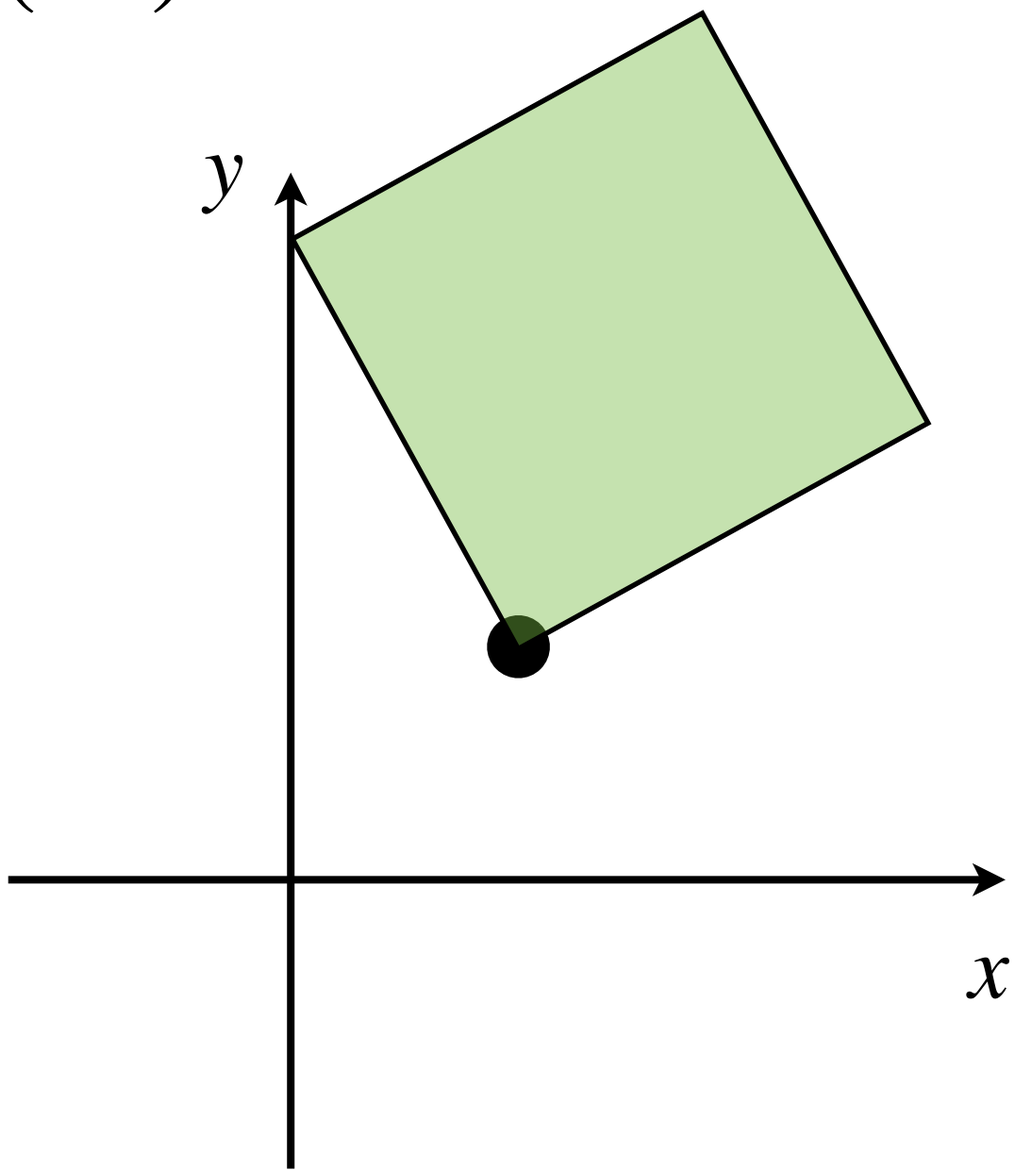
Combined Transforms III

1. Translate to origin: $\mathbf{T}(-P)$
2. Rotate around origin $\mathbf{R}_z(\theta)$



Combined Transforms III

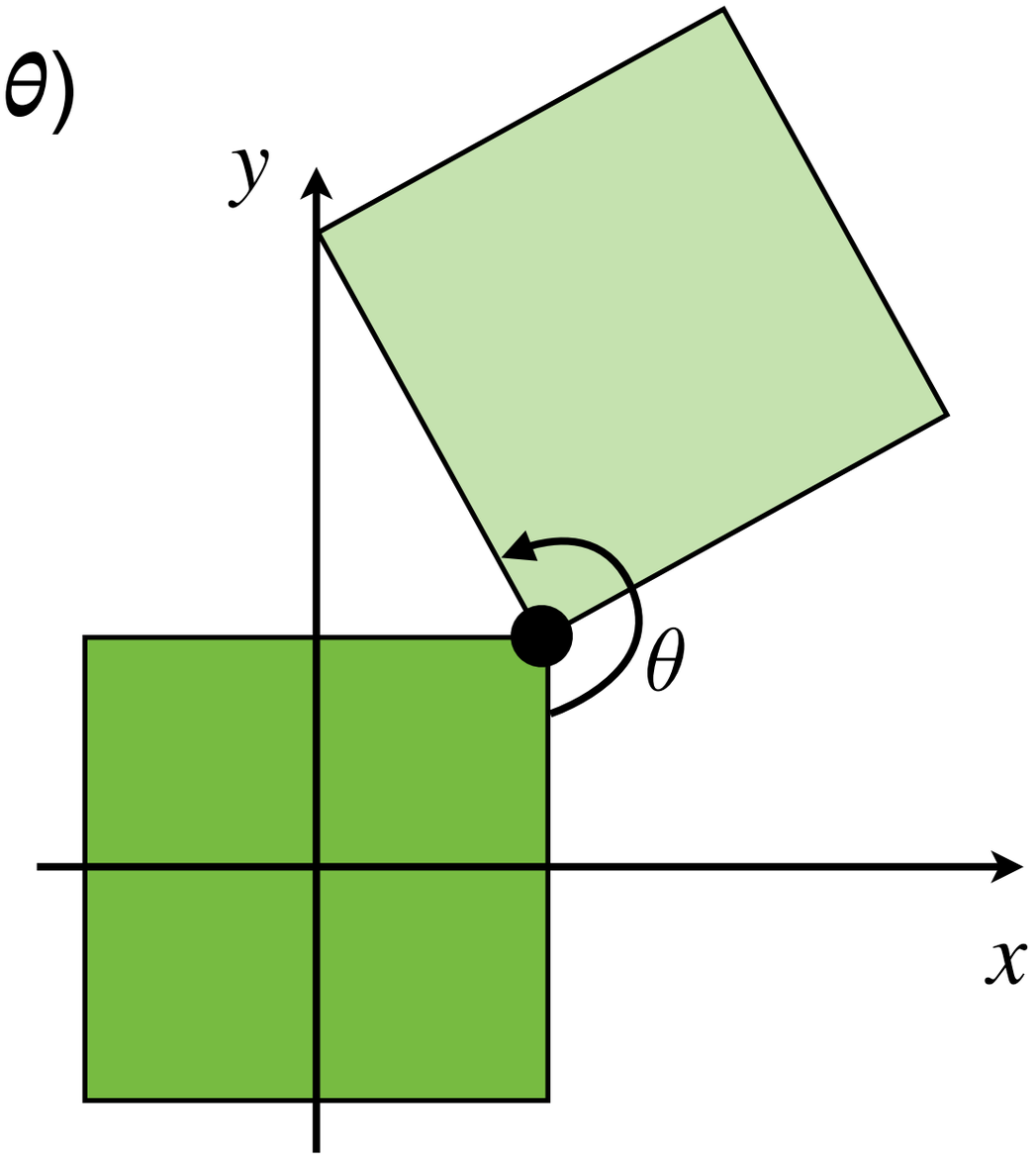
1. Translate to origin: $\mathbf{T}(-P)$
2. Rotate around origin $\mathbf{R}_z(\theta)$
3. Translate back: $\mathbf{T}(P)$



Combined Transforms III

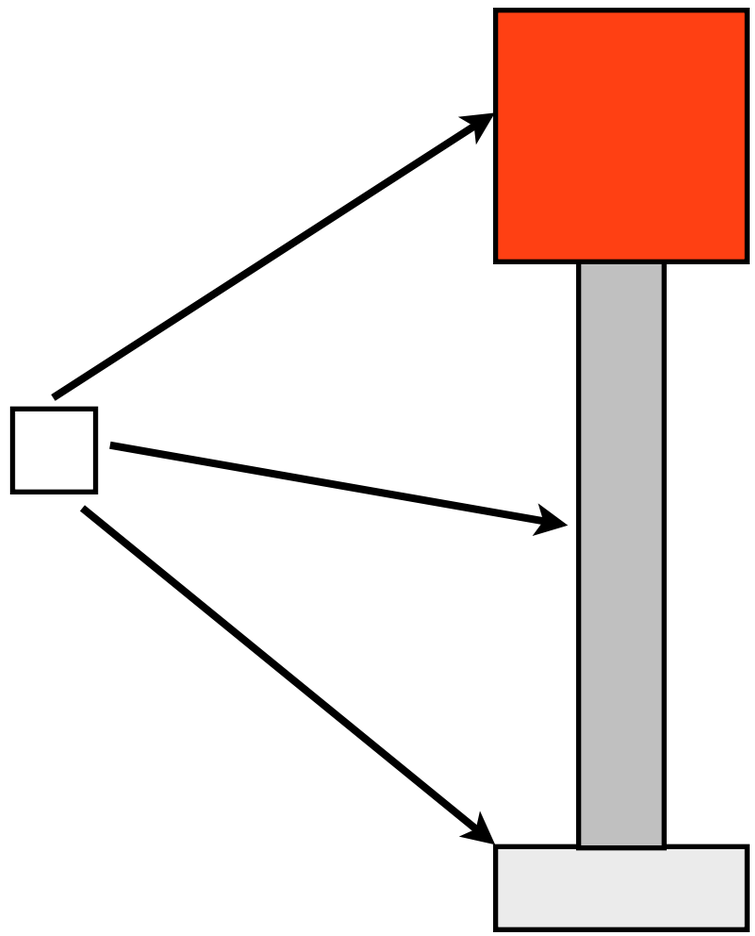
1. Translate to origin: $\mathbf{T}(-P)$
2. Rotate around origin: $\mathbf{R}_z(\theta)$
3. Translate back: $\mathbf{T}(P)$

- $\mathbf{M} = \mathbf{T}(P)\,\mathbf{R}_z(\theta)\,\mathbf{T}(-P)$



Modeling

- Simple Modeling
  - Use basic primitives
  - Apply transforms to create more complex objects

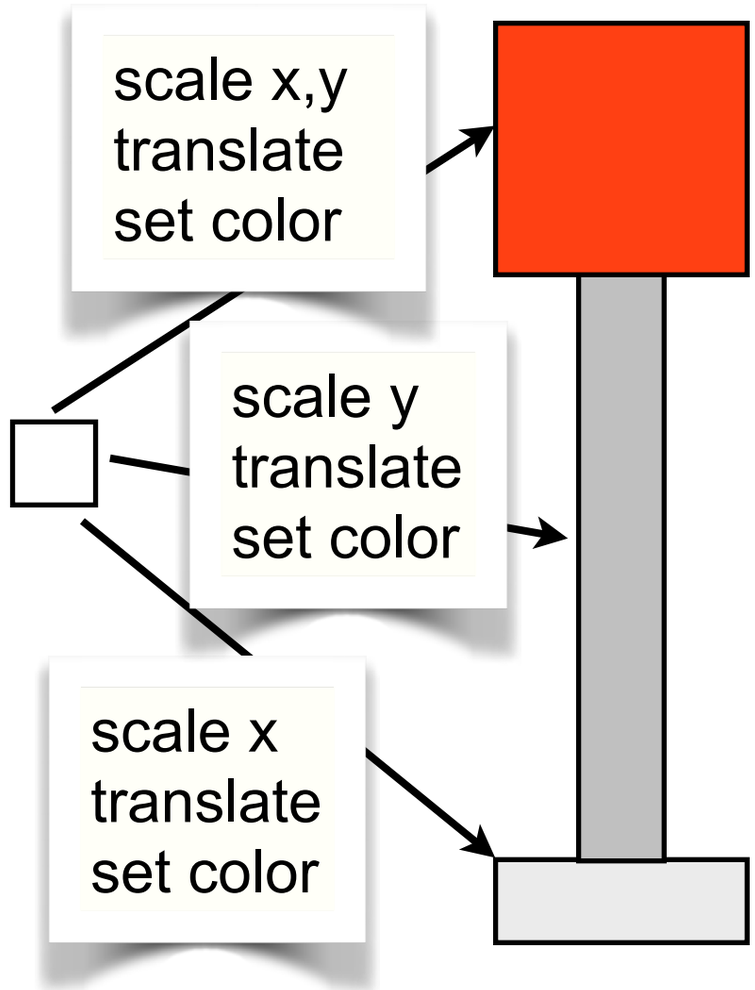


Use Blender or
Free student licenses of
Maya, 3D Studio Max, ...
http://students.autodesk.com/

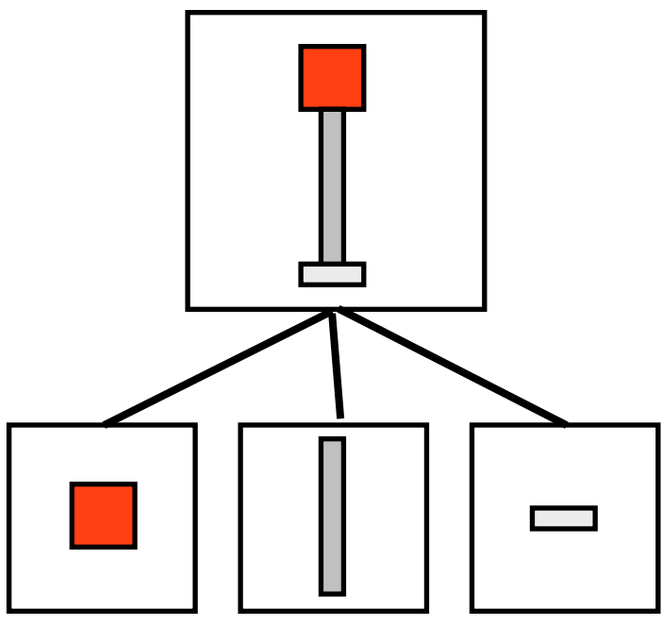
Blender Modeling Introduction : https://www.youtube.com/watch?v=7bHleRTEmZ0

Modeling

- Simple Modeling
  - Use basic primitives
  - Apply transforms to create more complex objects

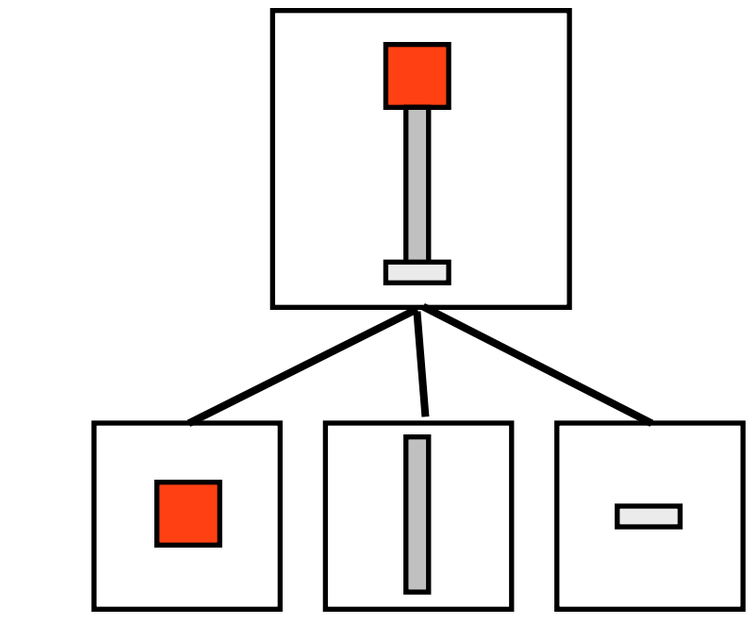


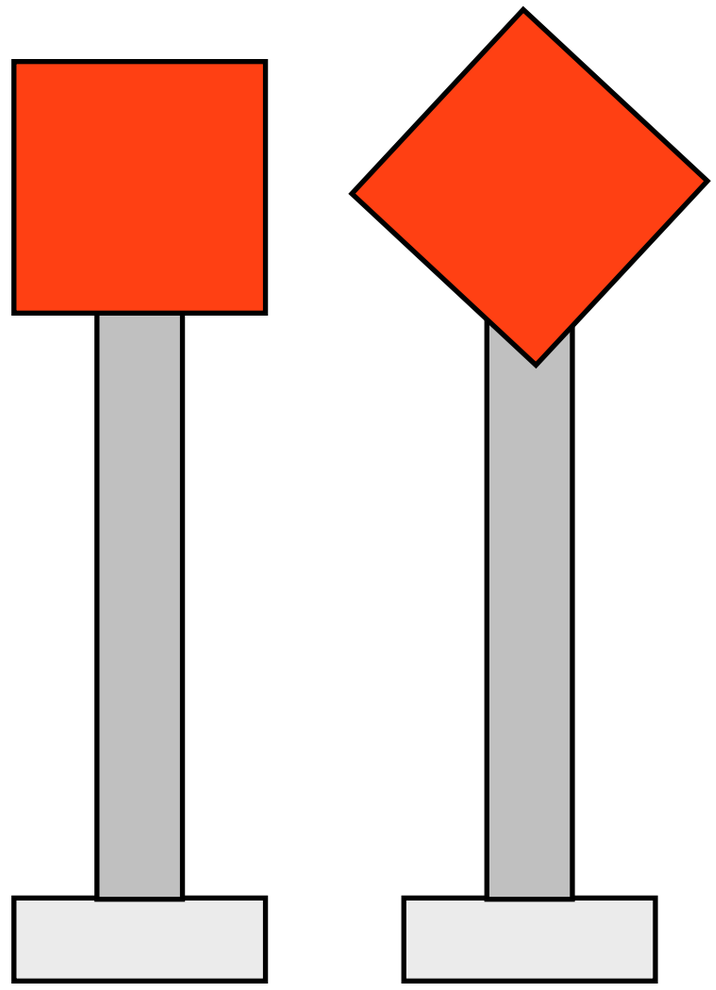
- Scene graph:



Hierarchical Transforms

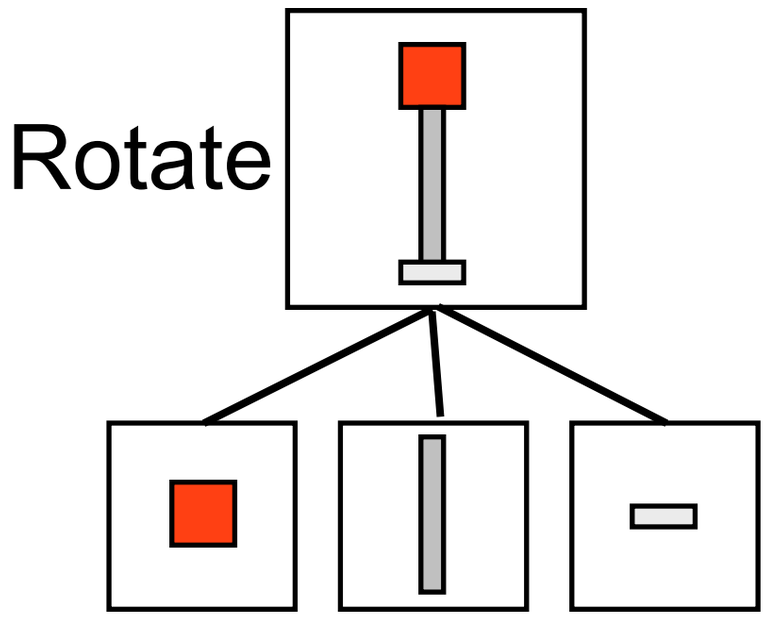
- Create scene graph
  - Component of a scene
  - Each **node** can have a unique transform

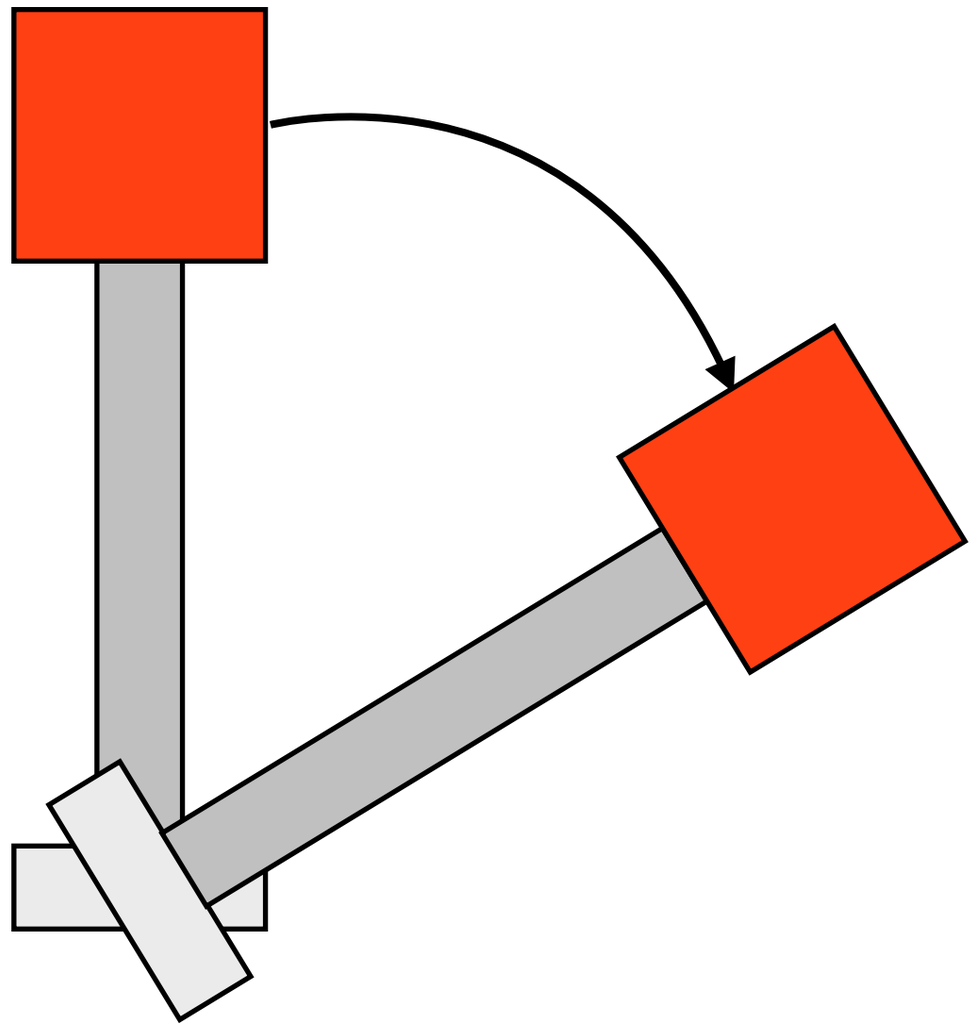




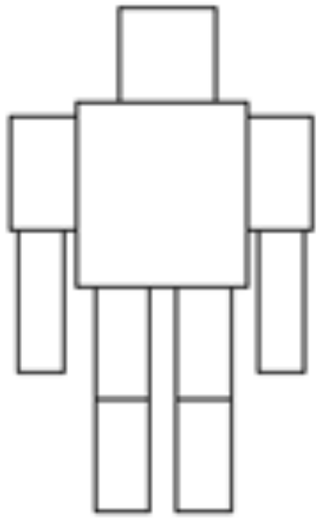
Hierarchical Transforms

- Create new nodes
  - Apply transformation to nodes instead of individual parts



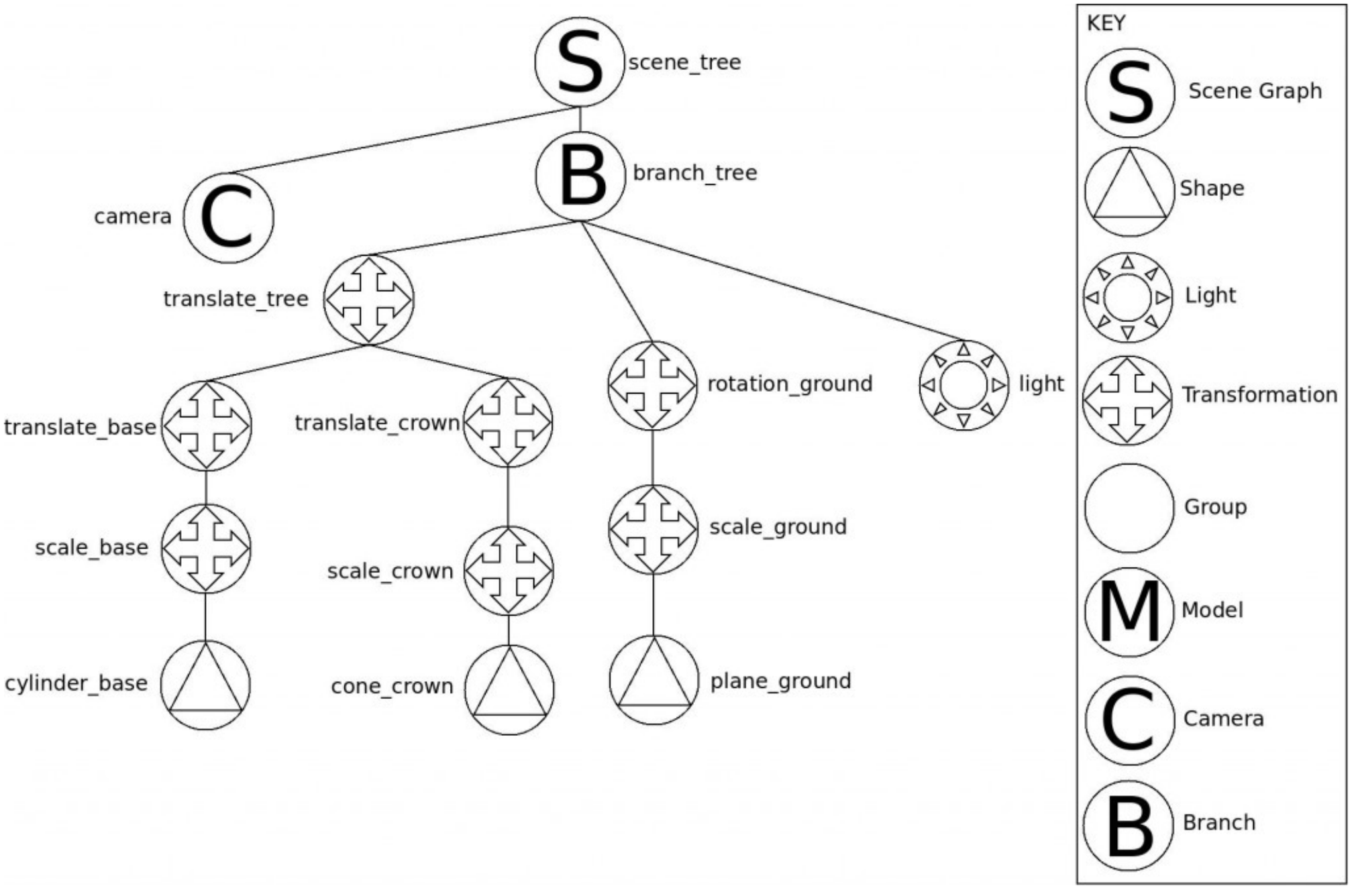


Scene Graph Example



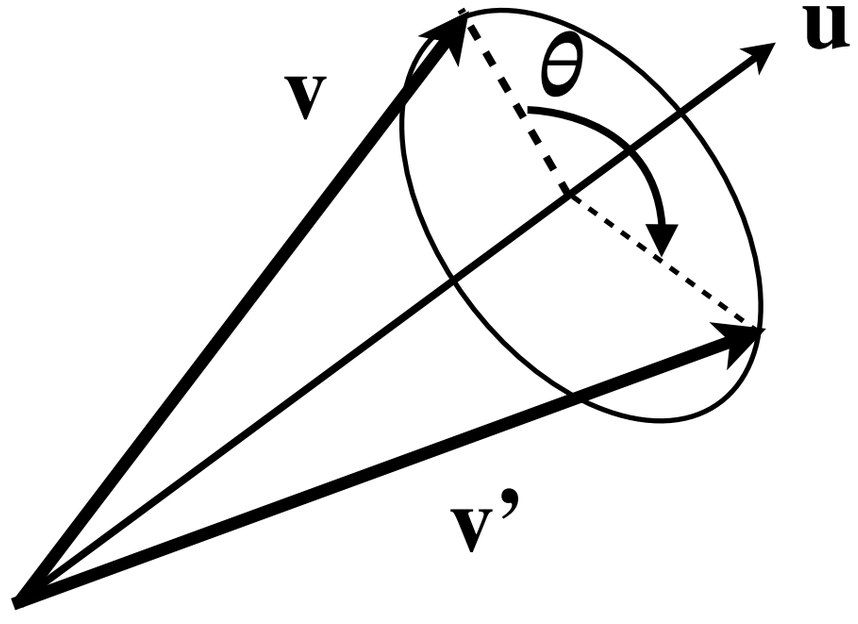
http://what-when-how.com/advanced-methods-in-computer-graphics/scene-graphs-advanced-methods-in-computer-graphics-part-1/

Scene Graph Example



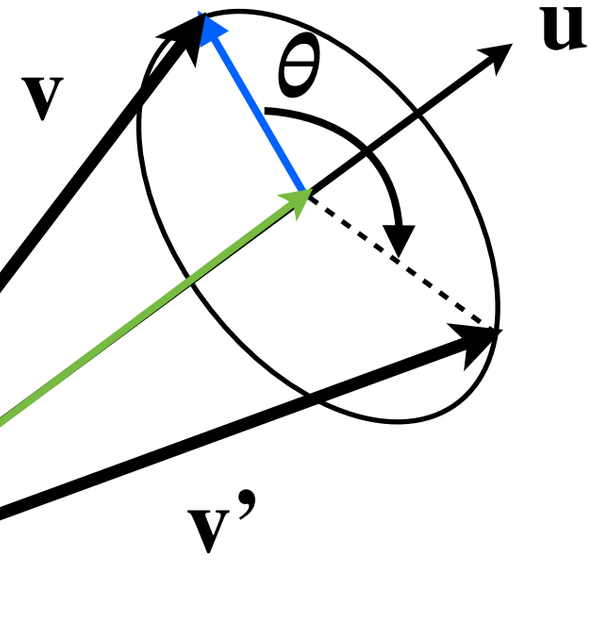
Rotation around arbitrary vector

- Rotate vector **v** θ degrees around the vector **u**



Rotation around arbitrary vector

- Rotate vector **v** θ degrees around the vector **u**
  - Find plane orthogonal to **u**
  - Find component of **v** in this plane
  - Rotate the projection



Introduce unit vector notation: $\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{|\mathbf{u}|}$

$$\mathbf{v}' = \cos(\theta)\,(\mathbf{v} - (\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}) + \sin(\theta)\,(\hat{\mathbf{u}} \times \mathbf{v}) + (\hat{\mathbf{u}} \cdot \mathbf{v})\ \hat{\mathbf{u}}$$

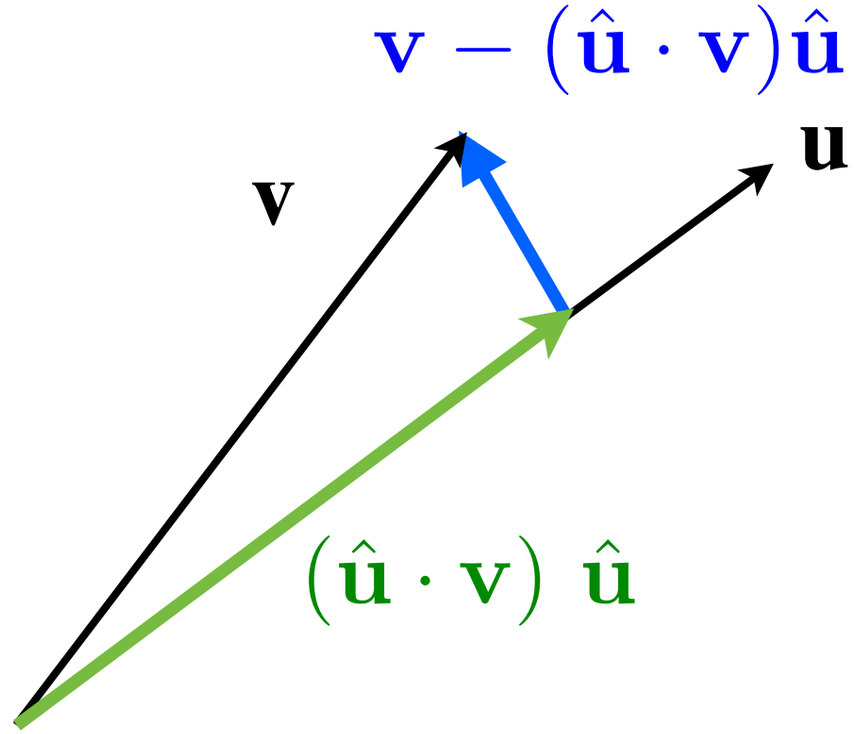
Rotation around arbitrary vector

- Find plane orthogonal to **u**
  - Decompose **v** into one component along **u** (projection on **u**):

$$(\hat{\mathbf{u}} \cdot \mathbf{v})\ \hat{\mathbf{u}}$$

... and another component perpendicular to **u**

$$\mathbf{v} - (\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}$$

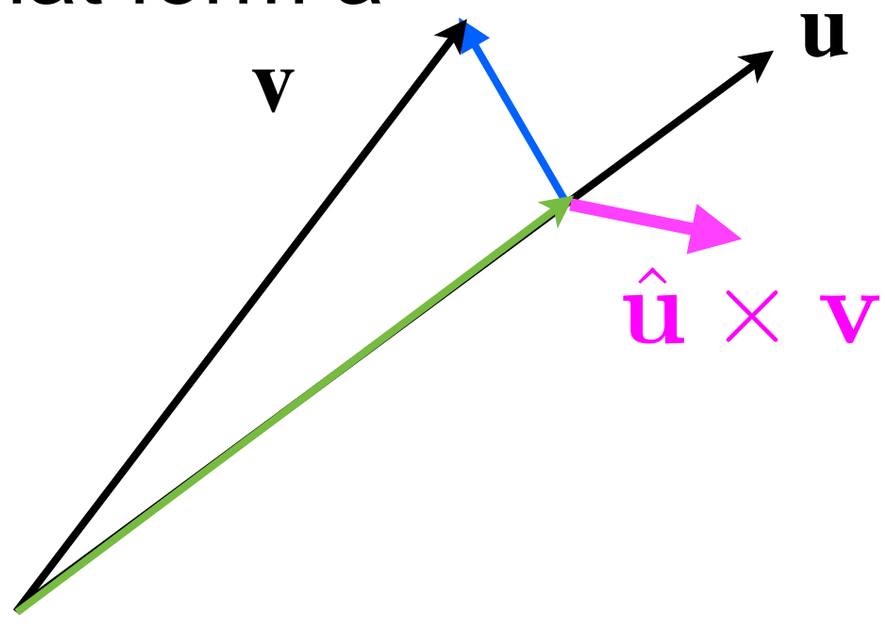


Rotation around arbitrary vector

- Find plane orthogonal to **u**
  - Find vector that is perpendicular to both **u** and **v** (use cross product)
  - We now have two vectors that form a plane orthogonal to **u**

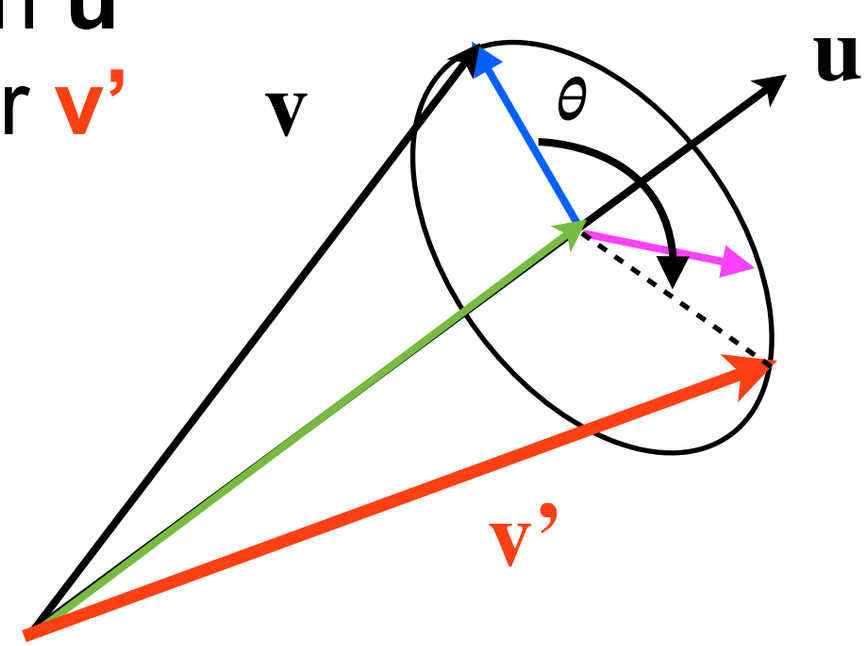
$\mathbf{v} - (\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}$ and $\hat{\mathbf{u}} \times \mathbf{v}$

$\mathbf{v} - (\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}$



Rotation around arbitrary vector

- Rotate within this plane
  - Rotate using the two basis vectors $\mathbf{v} - (\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}$ and $\hat{\mathbf{u}} \times \mathbf{v}$ in the plane
  - Add in the projection of **v** on **u** $(\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}$ to get the final vector **v'**



$$\mathbf{v'} = \cos(\theta)(\mathbf{v} - (\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}) + \sin(\theta)\ (\hat{\mathbf{u}} \times \mathbf{v}) + (\hat{\mathbf{u}} \cdot \mathbf{v})\hat{\mathbf{u}}$$

Homogeneous Coordinates

Homogeneous Coordinates

- Transformations of vectors and points

$$\mathbf{v}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$P' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

- Rotations and scalings can be multiplied together, but translations are a special case
- We want a unified representation

Vectors and points

- Remember: vectors are not points

  - A vector: $\mathbf{v} = (v_x, v_y, v_z)$

  - A point $P = P_o + (P_x, P_y, P_z)$

    origin + vector

Homogeneous Coordinates

- Add a fourth dimension
  - The coordinate is 1 for points, 0 for vectors
    - Vector: $\mathbf{v} = (v_x, v_y, v_z, 0)$
    - Point: $P = (P_x, P_y, P_z, 1)$
- All affine transformations can be represented as matrix multiplications in homogeneous coordinates

Homogeneous Coordinates

- Transformation

$$\mathbf{v} = (v_x, v_y, v_z, 0)$$

$$P = (P_x, P_y, P_z, 1)$$

Point

$$P' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} & t_x \\ a_{10} & a_{11} & a_{12} & t_y \\ a_{20} & a_{21} & a_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix}$$

Vector

$$\mathbf{v}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} & \cdot \\ a_{10} & a_{11} & a_{12} & \cdot \\ a_{20} & a_{21} & a_{22} & \cdot \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$$

Homogeneous Coordinates

- All transforms, including translations are expressed as 4x4 matrices
- This is the format graphics hardware APIs (OpenGL, Direct 3D) specify transformations
- In upcoming lecture, we will see that homogeneous coordinates are also useful for projections

Translation

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translate point:

$$\mathbf{T}P = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} P_x + t_x \\ P_y + t_y \\ P_z + t_z \\ 1 \end{bmatrix}$$

Translate vector:

$$\mathbf{T}\mathbf{v} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$$

Vectors are unmodified by translations!

Rotation and Scaling

- Rotation

$$\mathbf{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Scaling

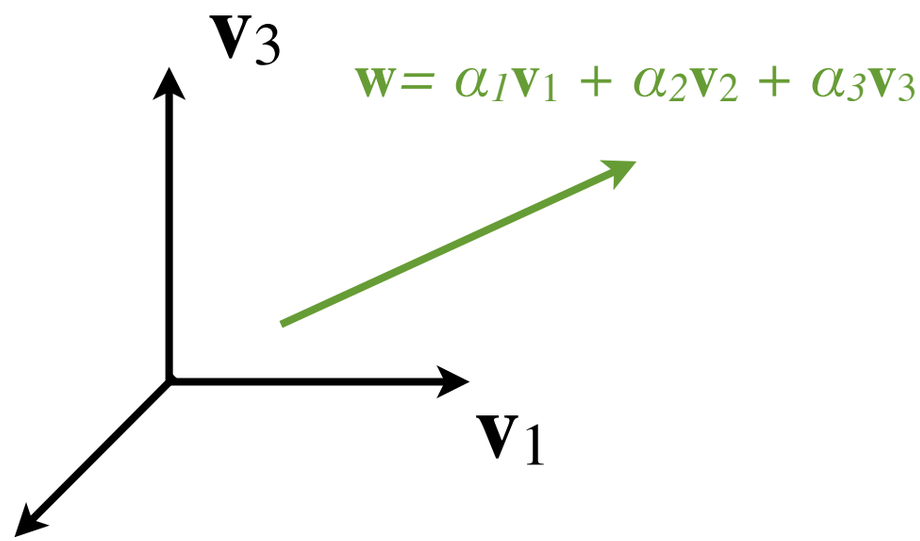
$$\mathbf{S} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordinate Systems

Basis and Frame

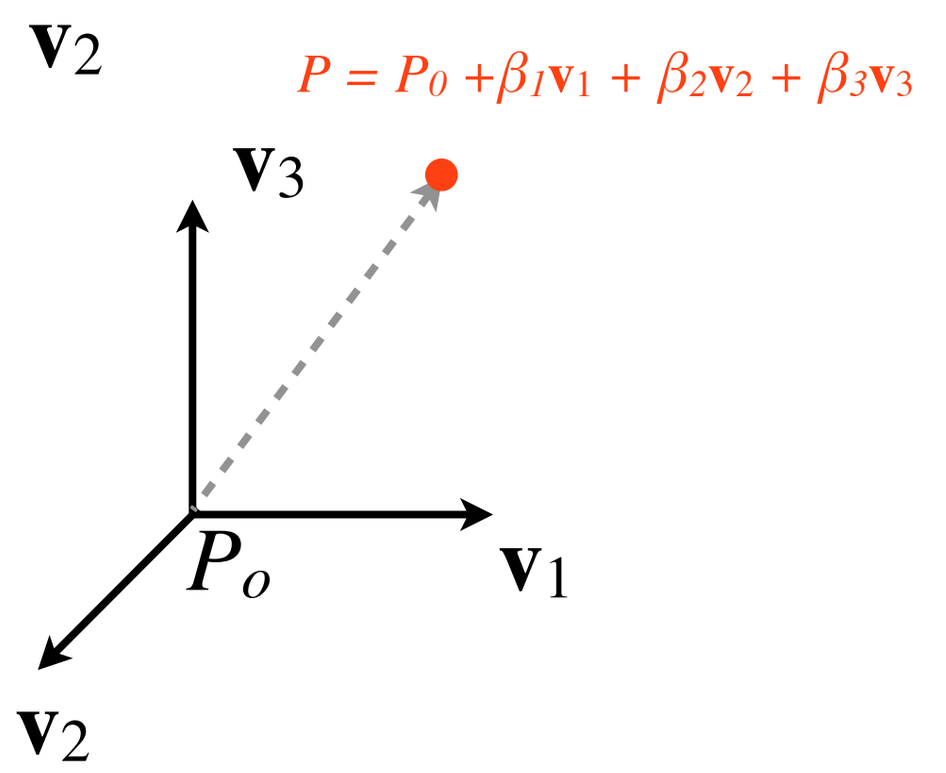
- Basis
  - Any 3D **vector** $\mathbf{w}$ can be expressed in terms of three linearly independent vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$

$$\mathbf{w} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \alpha_3 \mathbf{v}_3$$



- Frame
  - Basis vectors + origin(P_o)
    - A 3D **point** P expressed as

$$P = P_o + \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 + \beta_3 \mathbf{v}_3$$



Point in Euclidean Frame

origin basis vectors

$$P = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \alpha_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{bmatrix}$$ coordinate vector

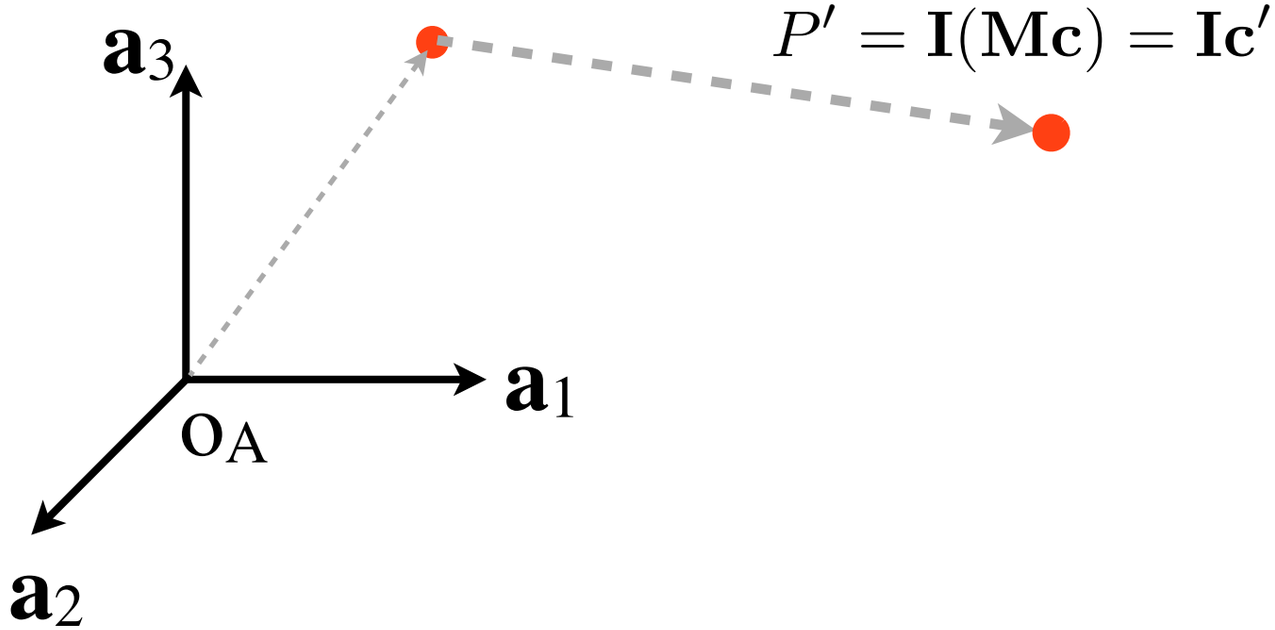
$$= \mathbf{Ic}$$

Coordinate Frames

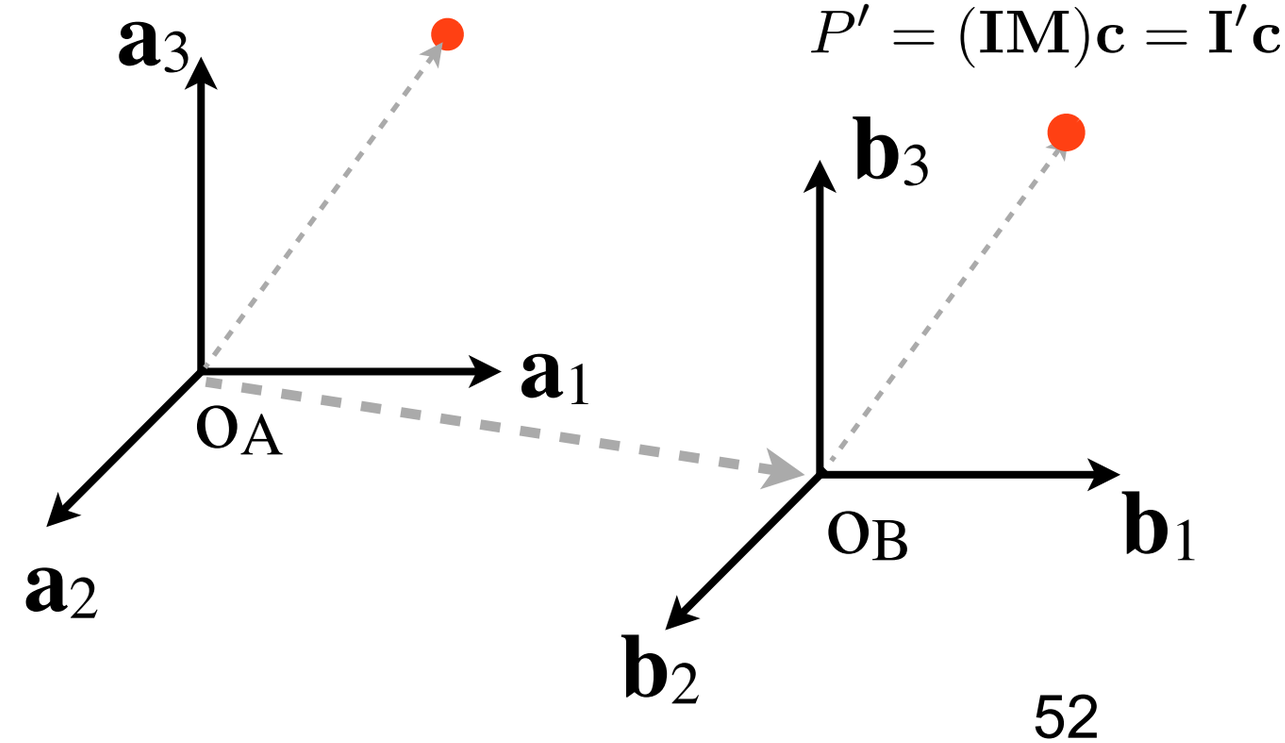
- Point P in frame $\mathbf{I}$ (Euclidean frame) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{bmatrix} = \mathbf{Ic}$
- Now, transform P with matrix $\mathbf{M}$ $P' = \mathbf{IMc}$
- **Two interpretations**:
  1. Transform the point in the frame $\mathbf{I}$ $P' = \mathbf{I}(\mathbf{Mc}) = \mathbf{Ic}'$
  2. Transform the frame itself $P' = (\mathbf{IM})\mathbf{c} = \mathbf{I}'\mathbf{c}$

Coordinate Frames

1. Transform the point in the frame $\mathbf{I}$

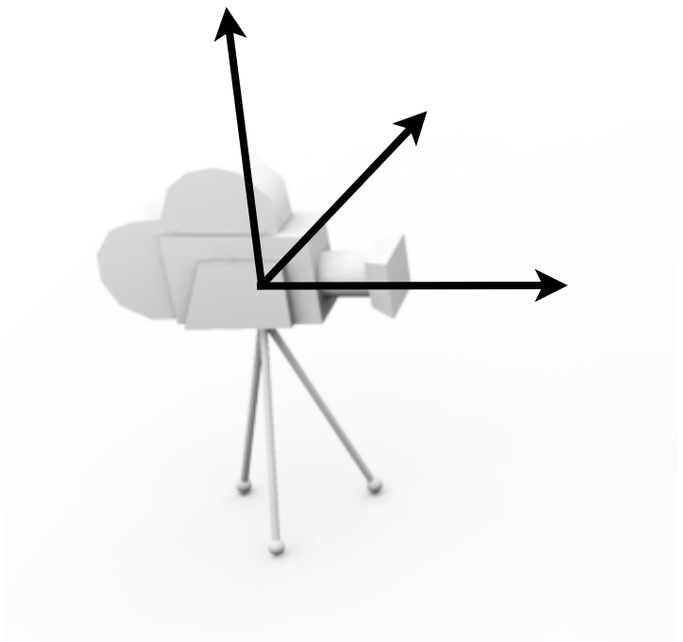


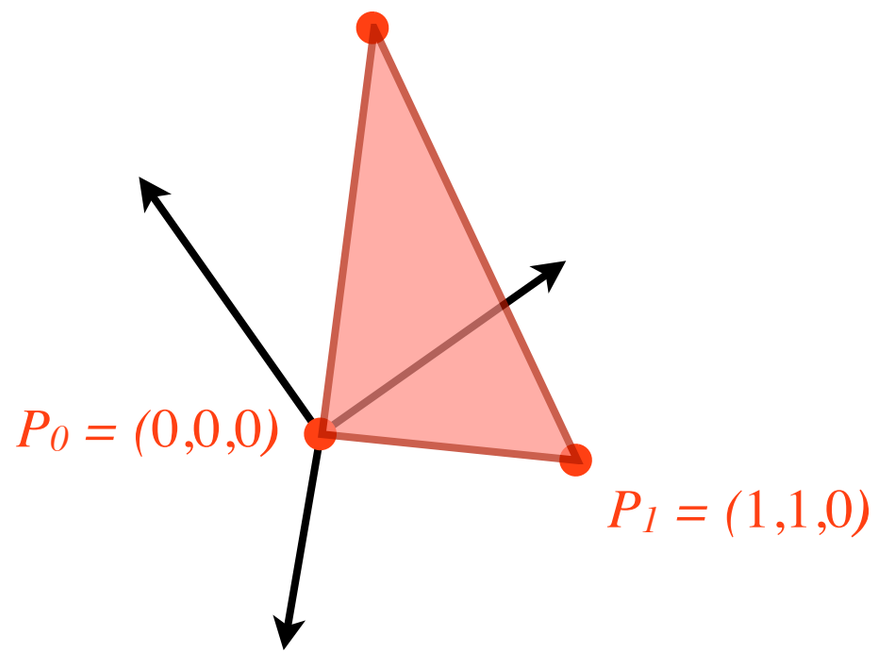
2. Transform the frame itself



Change of Coordinate System

- Problem at hand:
  - Given an object represented in a local coordinate system, how to express the object's coordinates from the cameras point of view?





Change of Coordinate System

- Point in frame $\mathbf{A}$

$$P_A = \mathbf{A}\mathbf{c}_A$$

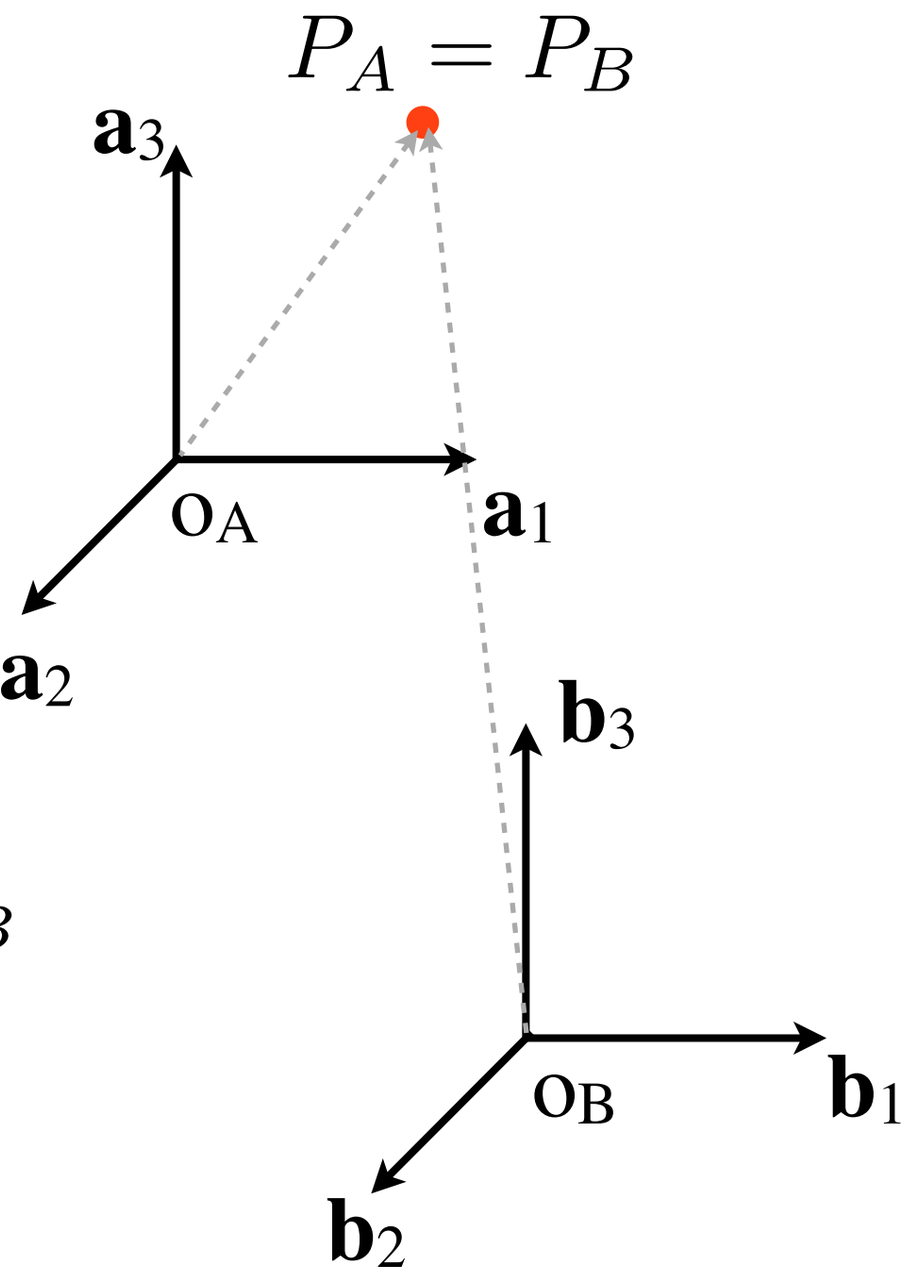
- Point in frame $\mathbf{B}$

$$P_B = \mathbf{B}\mathbf{c}_B$$

- Same point:

$$P_A = P_B \implies \mathbf{c}_A = \mathbf{A}^{-1}\mathbf{B}\mathbf{c}_B$$

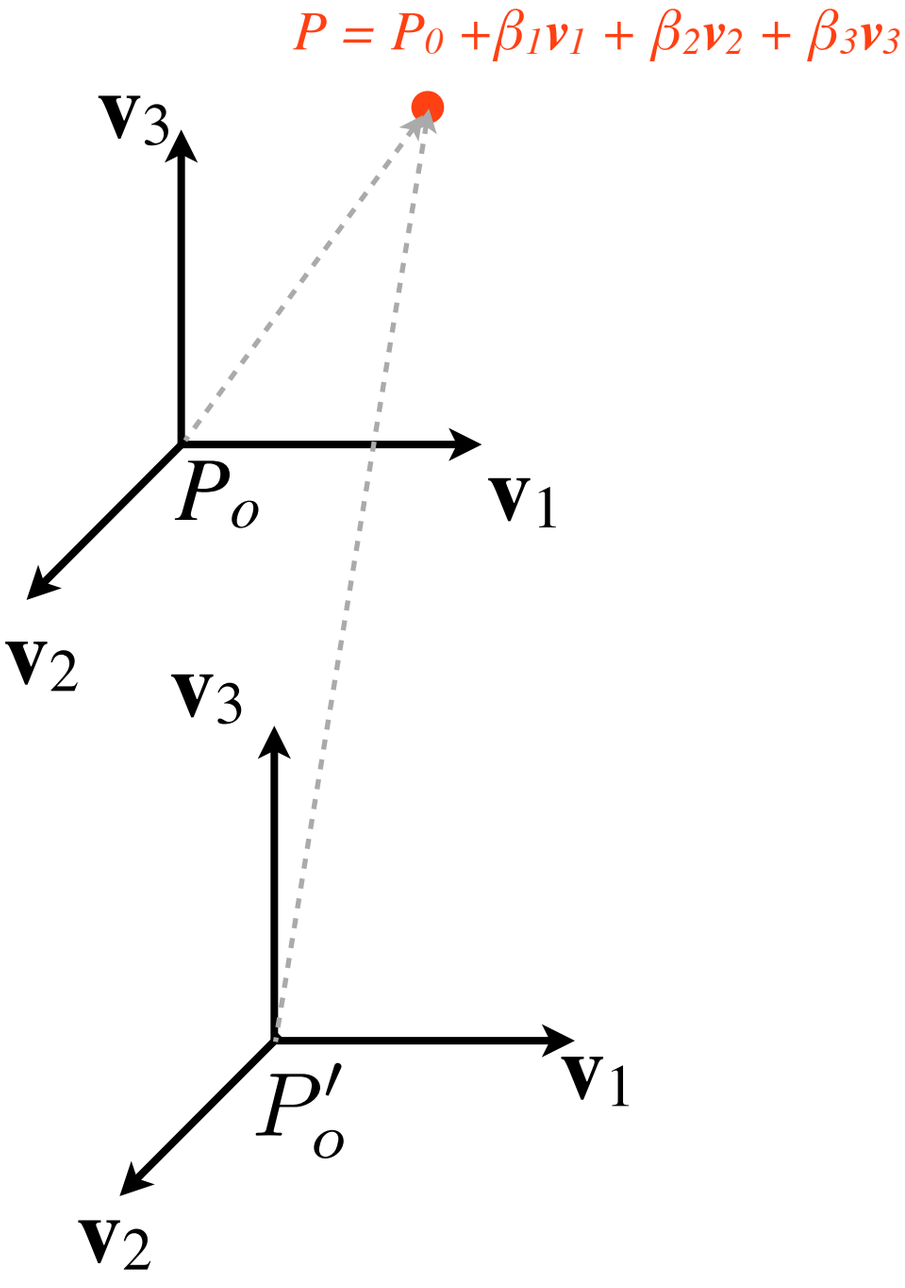
- If $\mathbf{A}$ is the Euclidean basis $\quad \mathbf{c}_A = \mathbf{B}\mathbf{c}_B$



Change of Coordinate System

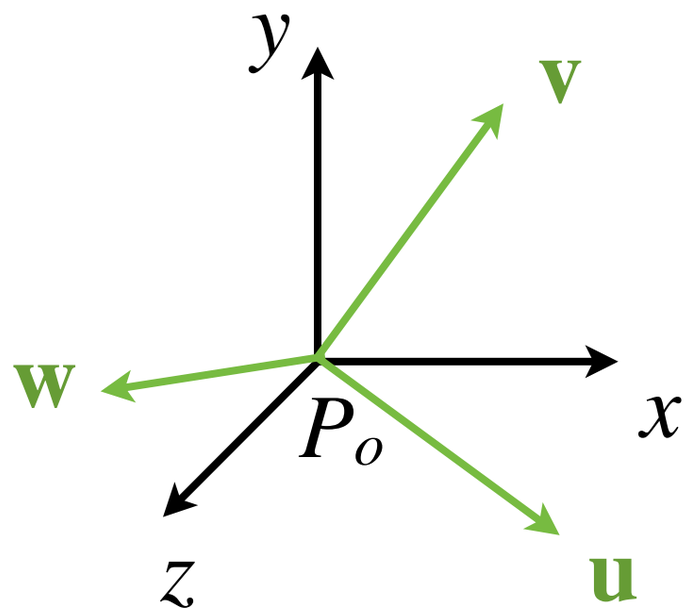
- Translation
  - Simply change origin of coordinate frame
  - Basis vectors are not affected by pure translation

$$\begin{aligned} P &= P_o + \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 + \beta_3 \mathbf{v}_3 \\ &= P'_o + \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 + \beta_3 \mathbf{v}_3 \end{aligned}$$



Change of Coordinate System

- Rotation and scaling
  - Express $(\mathbf{u}, \mathbf{v}, \mathbf{w})$ in the basis $(1,0,0)^T$, $(0,1,0)^T$ and $(0,0,1)^T$

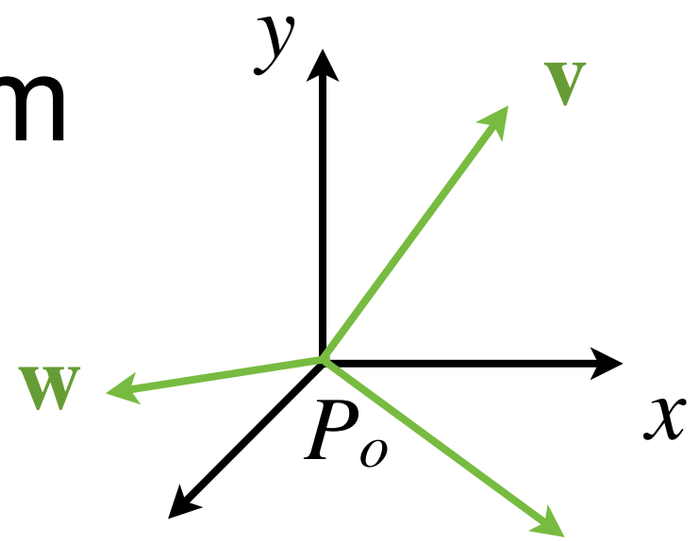


$$\begin{aligned} \mathbf{u} &= (u_x, u_y, u_z)^T \\ \mathbf{v} &= (v_x, v_y, v_z)^T \\ \mathbf{w} &= (w_x, w_y, w_z)^T \end{aligned}$$

$$\begin{bmatrix} | & | & | \\ \mathbf{u} & \mathbf{v} & \mathbf{w} \\ | & | & | \end{bmatrix}_{3\times3} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}_{3\times3}$$

Change of Coordinate System

- The matrix $\mathbf{M}$ transforms **from** the coordinate system $(1,0,0)^T$, $(0,1,0)^T$ and $(0,0,1)^T$ **to** the system $(\mathbf{u}, \mathbf{v}, \mathbf{w})$

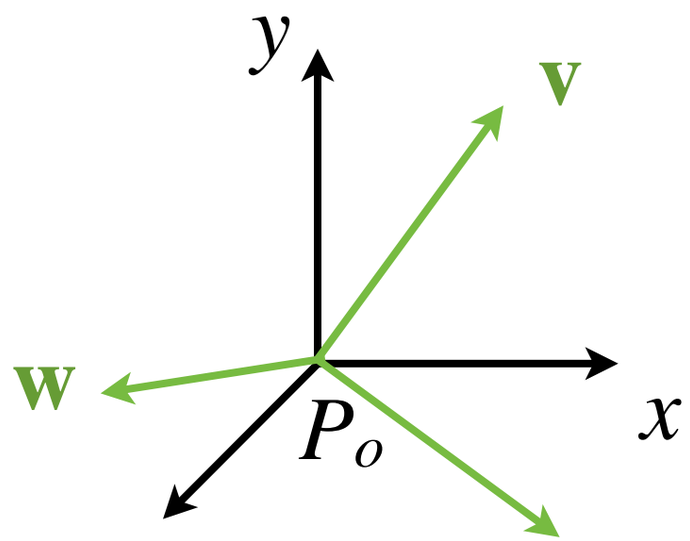


- Example:

$$\mathbf{M} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} \qquad \mathbf{M} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{u}$$

Change of Coordinate System

- The matrix $\mathbf{M}^{-1}$ transforms from $(\mathbf{u}, \mathbf{v}, \mathbf{w})$ to the Euclidean system $(1,0,0)^T$, $(0,1,0)^T$ and $(0,0,1)^T$



- Example:

$$\mathbf{M}\begin{bmatrix}1\\0\\0\end{bmatrix} = \mathbf{u}$$

$$\begin{aligned}\mathbf{Mx} &= \mathbf{u}\\ \mathbf{M}^{-1}\mathbf{Mx} &= \mathbf{M}^{-1}\mathbf{u}\\ \mathbf{x} &= \mathbf{M}^{-1}\mathbf{u}\end{aligned}$$

$$\mathbf{M}^{-1} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}^{-1}$$

Change of Coordinate System

- Align origins
  - Translation
- Create rotation matrix from basis vectors
  - Express each basis vector of one coordinate system as a linear combination of the basis vectors of the other coordinate system

Orthonormal Basis

- Remember: If **uvw** is an orthonormal basis, we have: $\mathbf{M}^{-1} = \mathbf{M}^T$

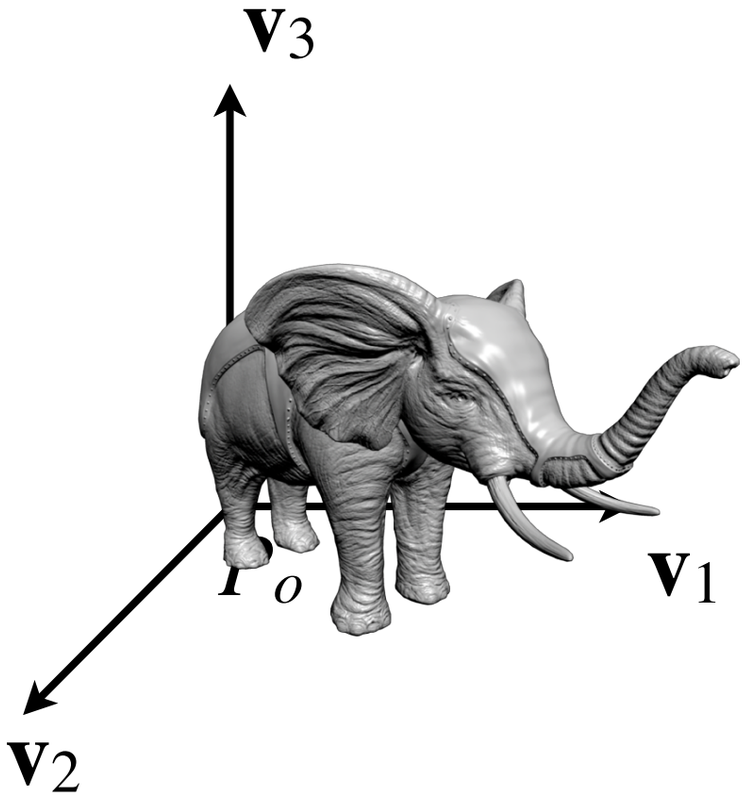
$$\mathbf{M} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}$$

$$\mathbf{M}^{-1} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}^{-1} = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}$$

Common Coordinate Systems in Computer Graphics

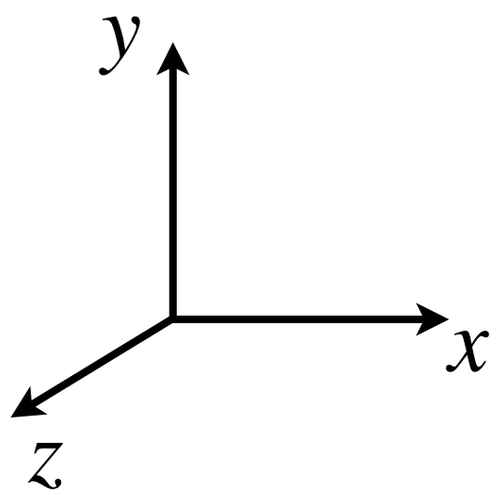
Object coordinates

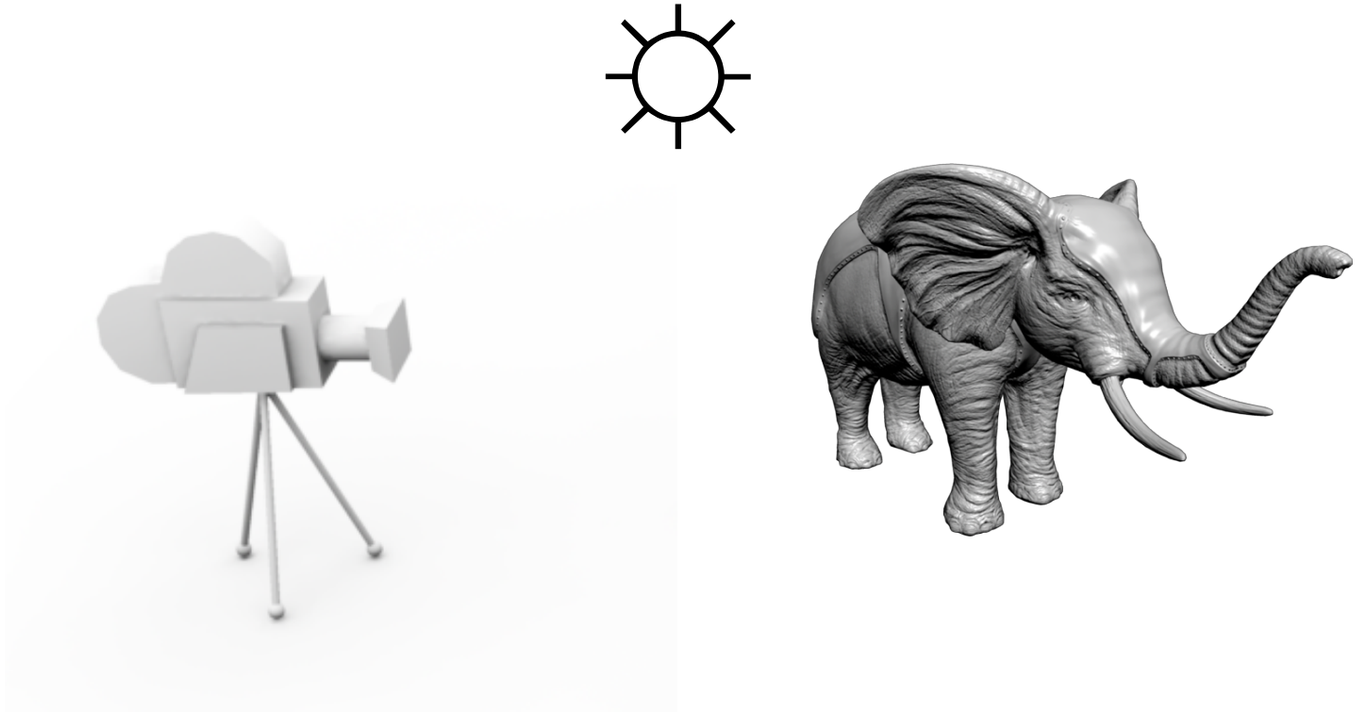
- The local coordinate system the model is defined in
- Often called “Object space”, “Model space” or “Model coordinates”



World coordinates

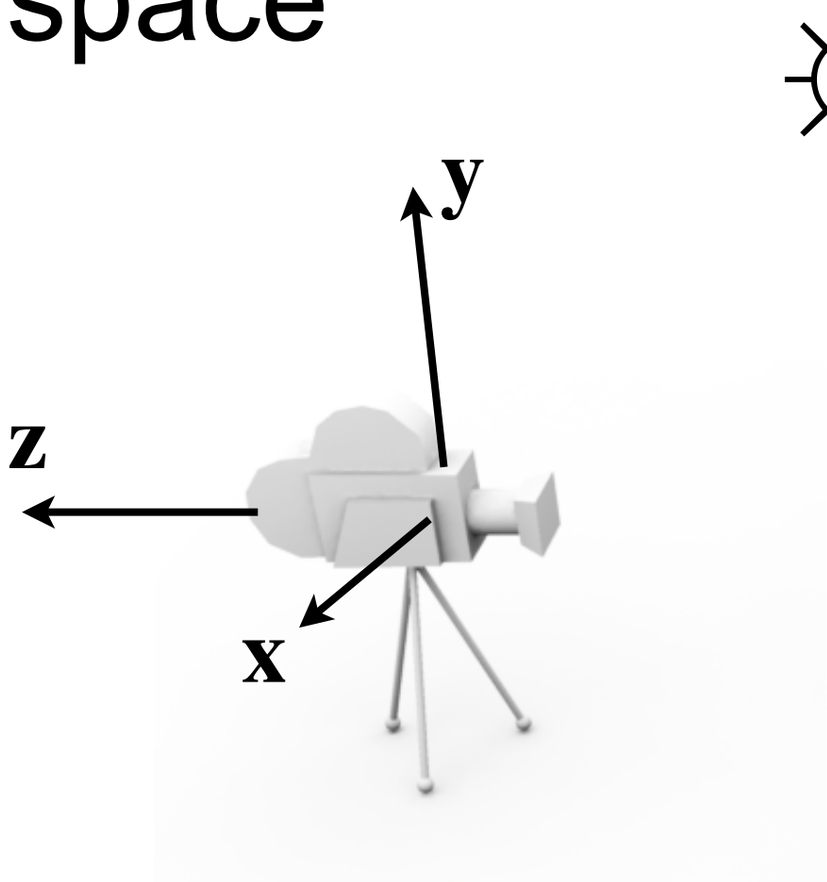
- Reference system in which all objects, lights and the camera are positioned
- Often referred to as “World space”

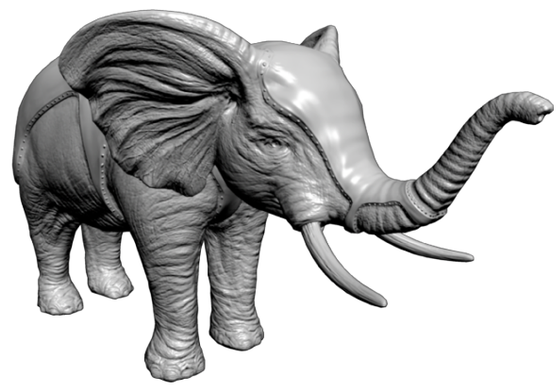




Eye (view) coordinates

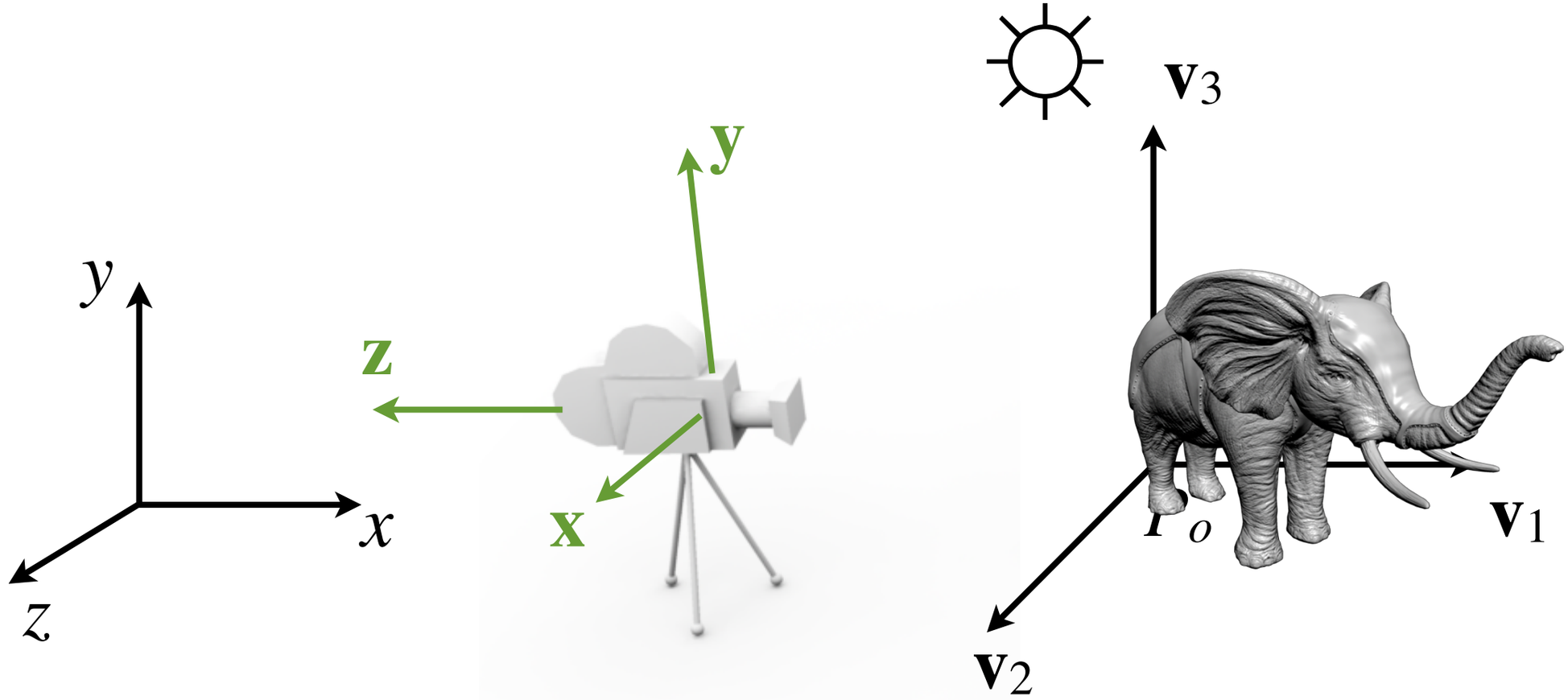
- A coordinate system with the camera at center, looking along negative z
- "Camera space"





Change coordinate system

- Model or World matrix: From model to world
- View matrix: From world to view
- ModelView matrix: From model to view



World View Object/Model

Coordinate Systems

- Clip coordinates
  - after projection matrix has been applied
- Normalized Device Coordinates (NDC)
  - Projected position on screen (x,y) : [-1,1]x[-1,1]
- Window (screen) coordinates
  - Pixel position

These coordinate systems will be explained in detail in a later lecture. For now: Note the names

Next

- Wednesday Seminar
  - Assignment 1 - Solar System
- Lab 0 - set up - optional
- Next week
  - Monday lecture - Parametric Surfaces & Animation
  - Labs - Assignment approval sessions
    - Sign up now!

VFX Breakdowns

- DNEG Avengers: Endgame
    - https://youtu.be/pTffQIFFYR8?si=OVicNXn7F8RpTN4-
- Framestore Avengers: Infinity War
    - https://youtu.be/V5mS7BHmZJI?si=w9KiX6xuDu_ayTQ1

Extras

- Graphics pipeline matrices

  - http://www.realtimerendering.com/udacity/?load=demo/unit7-view-pipeline.js