A New Kinematic and Dynamic Direct Adaptive Fuzzy Control of Constrained Mobile Wheeled Vehicles

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Abstract— *In this paper a new kinematic and dynamic adaptive Fuzzy control is applied to a trajectory tracking problem of a constrained vehicle with two indipendent wheels. The vehicle dynamics and kinematics are completely unknown. A dynamical and kinematical adaptive control provide to on line estimation of the dynamic and kinematic parameters of the vehicle model. Moreover the parameters of the kinematic control law are obtained using a Fuzzy controller and they are time varying and dependent on tracking errors. The stabililty of the kinematic and dynamic adaptive Fuzzy control system and the convergence of tracking errors to zero are proved using Lyapunov's method and Barbalat's Lemma. The effectiveness and efficiency of the proposed controller are demonstrated through simulation studies.*

1. Introduction

The control of wheeled vehicles with nonholonomic constraints has been the subject of considerable research effort over the last few years. In [1], [12] a kinematic model of a vehicle approximated by a unicycle model is proposed. The dynamic model of a mobile platform with two independent wheels was dealt with in [5], [7], [18]. In [21], [22] a decoupled dynamic model of a vehicle is proposed using the Lagrange Euler approach. References [1],[5],[7],[8],[12],[17],[18] deal with algorithms for the synthesis of a kinematic controller. The kinematic controller is essential, in order to grant the vehicle movement along the direction of the orientation. This is well known in literature as a "nonholonomic constraint" i.e. a force able to move the vehicle along the direction of orientation without slipping [18]. Any perturbation can be suitably compensated with a stable control system, where a kinematic controller applies suitable speed signals to the vehicle. Since the mobile platform can be controlled by the torque applied to the motors, it is necessary to insert a dynamic model into the control system. Therefore, "backstepping" methodologies were treated in [7], [11], [18], [24]. Backstepping method is very important since we aim to convert a speed control (kinematic control) into a torque control (dynamic control) for the actual physical cart. In the literature this allows to convert the steering system commands to torques which make use the mass, the friction and the dynamic parameters of the vehicle. The traditional backstepping method implies the knowledge of kinematic and dynamic parameters. Assuming that these parameters are not well known, in [5] Wavelet Networks are used; in [8],[18],[23],[24], techniques of "adaptive backstepping" are proposed. Fuzzy systems are often adopted for various applications [16]. The modern literature is focussing on developing a less heuristic theory of Fuzzy systems. In [4] and [21] an analysis of stability of a Fuzzy Mamdani control system is proposed. In [2],[3],[10],[13],[14],[20] a Fuzzy controller applied to mobile wheeled vehicles is proposed exclusively referring to a kinematic model. In [19] a stable dynamical Neuro Fuzzy controller for industrial manipulators versus Lyapunov's theory and Barbalat's Lemma is proposed.

In this paper in order to continue this research line, a new Fuzzy kinematic and dynamic adaptive control for trajectory tracking of nonholonomic vehicles is proposed; the asymptotic stability of the control system is proved using Lyapunov's method and Barbalat's Lemma. This paper is organized as follows. In Section II the kinematic and dynamic model of a vehicle with nonholonomic constraints is developed. In Section III a new adaptive fuzzy speed control and an adaptive dynamic controller based on adaptive backstepping are developed from the vehicle kinematics and dynamics in order to grant the asymptotic stability of the position error. Using a kinematical and dynamical adaptive control, kinematic and dynamic parameters of the vehicle may be unknown. Moreover the constants parameters of kinematic control law are obviated using

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a new fuzzy kinematic controller. By using the fuzzy inference mechanism, the parameters of the kinematic controller depend on tracking errors. In section IV some simulations results illustrate the effectiveness of the adaptive fuzzy tracking control law of the present work with respect to the classical controller without fuzzy mechanism.

2. Kinematic and dynamic model of a constrained vehicle

Let us consider a mobile vehicle (see Fig. 1) with generalized coordinates **q,** subject to *m* constraints. The well known dynamic model in generalized coordinates is:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \boldsymbol{\tau}_d = \mathbf{E}(\mathbf{q})\boldsymbol{\tau} - \mathbf{A}^{\mathbf{T}}(\mathbf{q})\boldsymbol{\lambda} \quad (1)$$

where **M(q)** is a *nxn* symmetric, positive definite matrix; $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is a *nxn* centripetal Coriolis matrix; $\boldsymbol{\tau}_d$ is a *nx1* bounded unknown disturbance including unstructured and unmodeled dynamics; $\boldsymbol{\tau}$ is a *nx1* vector including torques applied to right and left wheel; **A(q)** is a *mxn* matrix of nonholonomic constraints [11,18] and λ is a *mx1* vector. Supposing that the *m* constraints are time invariant, we obtain:

$$\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0} \quad (2)$$

Let **S(q)** be a full rank matrix *nx(n-m)* made up by a set of smooth and linearly independent vectors spanning the null space of **A(q),** i.e.,

$$\mathbf{A}(\mathbf{q})\mathbf{S}(\mathbf{q}) = \mathbf{0} \quad (3)$$

where **S(q)** is the Jacobian matrix. It is possible to find a *(n-m)* **v** velocity vector as follows:

$$\mathbf{v}^{\mathbf{T}} = (u,\omega) \quad (4)$$

where u and ω are respectively the linear and angular body-fixed (*Xc,Yc*) velocities. We indicate with (x_0, y_0) the P_0 coordinates (see Fig. 1) in an inertial cartesian frame (*X,Y*) and with ϕ the vehicle orientation with respect to the inertial basis. In this way we focus only on three variables *x,y,*ϕ excluding the angular velocity of the right wheel ($\dot{\theta}_r$) and of the left wheel ($\dot{\theta}_l$). We indicate with *r* (see Fig. 1) the ray of the wheels. Let P_c be the mass center of the vehicle, which is on the X-axis, and let *d* the distance from P_0 to P_c. For the later description, m_c is the mass of the vehicle without the driving wheels, m_w is the mass of each driving wheels, I_c, I_w and I_m are the inertia moments of the body around a vertical axis through P_0, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. The relation ship between (u,ω) and $(\dot{\theta}_r,\dot{\theta}_l)$ is the following:

$$\begin{pmatrix}\dot{\theta}_r\\ \dot{\theta}_l\end{pmatrix} = \begin{pmatrix}\frac{1}{r} & \frac{b}{r}\\ \frac{1}{r} & \frac{-b}{r}\end{pmatrix}\begin{pmatrix}u\\ \omega\end{pmatrix} \quad (5)$$

Equations (5) can be rewritten as follows:

$$\boldsymbol{\eta} = \begin{pmatrix}\frac{1}{r} & \frac{b}{r}\\ \frac{1}{r} & \frac{-b}{r}\end{pmatrix}\mathbf{v} \quad (6)$$

where:

$$\boldsymbol{\eta}^{\mathbf{T}} = (\dot{\theta}_r \;\; \dot{\theta}_l) \quad (7)$$

Therefore, the model of the vehicle can be described employing five generalized coordinates:

$$\mathbf{q}^{\mathbf{T}} = (x_0, y_0, \phi, \theta_r, \theta_l) \quad (8)$$

In the condition of pure rolling and non-slipping, the vehicle can only move in the direction normal to the driving wheels axis. So we have three constraint equations:

$$\begin{aligned}&\dot{x}_0\sin\phi - \dot{y}_0\cos\phi = 0\\ &\dot{x}_0\cos\phi + \dot{y}_0\sin\phi + b\dot{\phi} = r\dot{\theta}_r\\ &\dot{x}_0\cos\phi + \dot{y}_0\sin\phi - b\dot{\phi} = r\dot{\theta}_l\end{aligned} \quad (9)$$

The constraints (9) can be written in *pfaffian* form (2). From equation (3), a Jacobian matrix **S(q)** can be defined as follows:

$$\mathbf{S}^{\mathbf{T}}(\mathbf{q}) = \begin{pmatrix} cb\cos\phi & cb\sin\phi & c & 1 & 0\\ cb\cos\phi & cb\sin\phi & -c & 0 & 1\end{pmatrix} \quad (10)$$

where *c=r/2b*.

But from Frobenius theorem [22], one of three constraints (9) is holonomic. In fact from (9) we can write:

$$\phi = c(\theta_r - \theta_l) \quad (11)$$

Therefore from (8) two new vectors of generalized coordinates can be defined:

$$\mathbf{q}_1^{\mathbf{T}} = (x_0, y_0, \theta_r, \theta_l) \quad (12)$$

or

$$\mathbf{q}_2^{\mathbf{T}} = (x_0, y_0, \phi) \quad (13)$$

In cases (12) and (13) **A(q)** matrix presents only nonholonomic constraints. In particular, in case (12) a new Jacobian matrix is obtained as follows:

$$\mathbf{S}_1^{\mathbf{T}}(\mathbf{q}_1) = \begin{pmatrix} c^2b\cos(\theta_r - \theta_l) & c^2b\sin(\theta_r - \theta_l) & 1 & 0\\ c^2b\cos(\theta_r - \theta_l) & c^2b\sin(\theta_r - \theta_l) & 0 & 1\end{pmatrix} \quad (14)$$

In this way the kinematic model results:

$$\dot{\mathbf{q}}_1 = \mathbf{S}_1(\mathbf{q}_1)\boldsymbol{\eta} \quad (15)$$

By applying (15), the dynamical model (1) in body-fixed coordinates (*Xc, Yc*) can be written as follows:

$$\overline{\mathbf{M}}\dot{\boldsymbol{\eta}} + \overline{\mathbf{V}}_{\mathbf{m}}(\boldsymbol{\eta})\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\tau} - \mathbf{S}_1^{\mathrm{T}}\boldsymbol{\tau}_d$$

$$\overline{\mathbf{M}} = \begin{pmatrix} b^2c^2m + c^2I + I_w & b^2c^2m - c^2I \\ b^2c^2m - c^2I & b^2c^2m + c^2I + I_w \end{pmatrix}$$

$$\overline{\mathbf{V}}_{\mathbf{m}}(\boldsymbol{\eta}) = \begin{pmatrix} 0 & 2bc^3dm_c(\dot{\theta}_r - \dot{\theta}_l) \\ -2bc^3dm_c(\dot{\theta}_r - \dot{\theta}_l) & 0 \end{pmatrix} \tag{16}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \boldsymbol{\tau} = \begin{pmatrix} \tau_r \\ \tau_l \end{pmatrix}$$

where:

$$m = m_c + 2m_w; \quad I = m_c d^2 + I_c + 2m_w b^2 + 2I_m \tag{17}$$

The $\overline{\mathbf{M}}$ and $\overline{\mathbf{V}}_{\mathbf{m}}$ matrices are respectively the Inertia and Coriolis matrices in body-fixed (*Xc Yc*) coordinate system (see Fig. 1). The vector τ has in his components the torque applied to the right and left wheel.

In case (13) the kinematic model is:

$$\begin{pmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} cb\cos\phi & cb\cos\phi \\ cb\sin\phi & cb\sin\phi \\ c & -c \end{pmatrix} \begin{pmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{pmatrix} \tag{18}$$

From (4) and (13) and by substituting equation (6) into (18), we also can write:

$$\dot{\mathbf{q}}_2 = \mathbf{S}_2(\mathbf{q}_2)\mathbf{v}$$

$$\mathbf{S}_2(\mathbf{q}_2) = \begin{pmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{pmatrix} \tag{19}$$

$$\mathbf{v}^{\mathrm{T}} = (u, \omega) \quad \mathbf{q}_2^{\mathrm{T}} = (x_0, y_0, \phi)$$

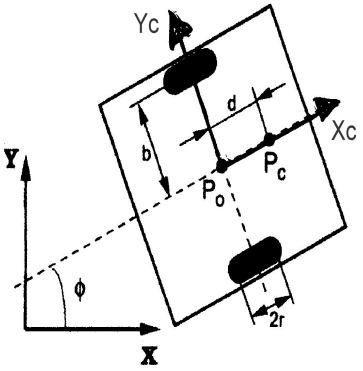


Figure 1. Nonholonomic vehicle.

3. Adaptive fuzzy kinematic and dynamic trajectory control of mobile vehicles

Let the velocity and position of a reference vehicle be:

$$\begin{aligned} \dot{x}_r &= u_r \cos\phi_r \\ \dot{y}_r &= u_r \sin\phi_r \\ \dot{\theta}_r &= \omega_r \end{aligned} \tag{20}$$

were u_r>0 for all t is the reference linear velocity and ω_r is the reference angular velocity. Then the tracking position error between the reference position $(x_r, y_r, \phi_r)^T$ and the actual position $(x, y, \phi)^T$ can be expressed in the vehicle local frame (*Xc, Yc*) as [7]:

$$\mathbf{e} = \begin{pmatrix} e_x \\ e_y \\ e_\phi \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \phi_r - \phi \end{pmatrix} \tag{21}$$

Differentiating equation system (21) and by applying two auxiliary inputs:

$$u_1 = -u_c + u_r \cos e_3; \quad u_2 = \omega_r - \omega_C \tag{22}$$

where u_c and ω_c are kinematic control laws, we can write:

$$\begin{pmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\phi \end{pmatrix} = \begin{pmatrix} 0 & \omega_c & 0 \\ -\omega_c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_\phi \end{pmatrix} + \begin{pmatrix} 0 \\ u_r \sin e_\phi \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{23}$$

Now we propose a new fuzzy kinematic control law as follows:

$$\begin{aligned} u_c &= u_r \cos e_\phi + k_x(t)e_x \\ \omega_c &= \omega_r + u_r(k_y(t)e_y + k_\phi(t)\sin e_\phi) \end{aligned} \tag{24}$$

This control law depends on the error vector (21) and on kinematic parameters:

$$\mathbf{K}^{\mathrm{T}} = \left(k_x(t), k_y(t), k_\phi(t)\right) \tag{25}$$

Parameters (25) are provided by a fuzzy controller and depend on the error vector (21) and time. The used controller is a Mandani controller. The input is error vector (21), while the output is vector (25). Input-output membership functions, set of rules and defuzzification methods have been obtained through tests based on experimental simulations on the classical kinematic controller [7]. The membership function of e_x is shown in Fig 2. Three normalized membership functions have been defined. Linguistic labels of rules are:

S=Small; M=Medium; H=High.

Analogously, the membership functions of e_y and e_ϕ have been obtained. Figure 3 shows the defuzzification function of K_x; analogously defuzzyfication functions of K_y and K_ϕ have been obtained. It is possible to observe that K_x , K_y and K_ϕ are positive numbers for

any tracking error value. The fuzzy rules are shown in Table 1.

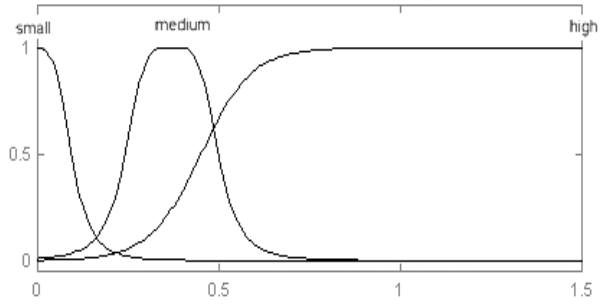


Fig. 2. Membership functions of e_x.

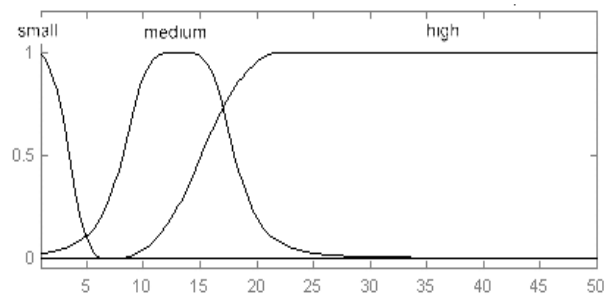


Fig.3. Membership functions of Kx.

TABLE I
CONTROLLER RULES

Nº RULES	E_X	E_Y	E_Φ	K_X	K_Y	K_Φ
1	S	S	S+	S	S	S
2	S	M	S+	S	M	S
3	S	H	S+	M	H	S
4	M	S	S+	M	S	S
5	M	M	S+	M	M	S
6	M	H	S+	M	H	S
7	H	S	S+	H	M	S
8	H	M	S+	H	M	S
9	H	H	S+	H	H	S
10	S	S	M+	M	M	M
11	S	M	M+	M	M	M
12	S	H	M+	M	H	M
13	M	S	M+	M	M	M
14	M	M	M+	M	M	M
15	M	H	M+	M	H	M
16	H	S	M+	H	M	M
17	H	M	M+	H	M	M
18	H	H	M+	H	H	M
19	S	S	OPP	M	M	H
20	S	M	OPP	M	M	H
21	S	H	OPP	M	H	H
22	M	S	OPP	M	M	H
23	M	M	OPP	M	M	H
24	M	H	OPP	M	H	H
25	H	S	OPP	H	M	H
26	H	M	OPP	H	M	H
27	H	H	OPP	H	H	H
28	S	S	M-	M	M	M
29	S	M	M-	M	M	M
30	S	H	M-	M	H	M
31	M	S	M-	M	M	M
32	M	M	M-	M	M	M
33	M	H	M-	M	H	M
34	H	S	M-	H	M	M
35	H	M	M-	H	M	M
36	H	H	M-	H	H	M
37	S	S	S-	S	S	S
38	S	M	S-	S	M	S
39	S	H	S-	M	H	S
40	M	S	S-	M	S	S
41	M	M	S-	M	M	S
42	M	H	S-	M	H	S
43	H	S	S-	H	M	S
44	H	M	S-	H	M	S
45	H	H	S-	H	H	S

Table 1: Fuzzy rules

From (19), (21), (23) and (24) the model of the closed loop system is the following:

$$\dot{\mathbf{e}} = \left[\dot{e}_x \dot{e}_y \dot{e}_\phi\right]^T = \begin{bmatrix} (\omega_r + u_r(k_y(t))e_y + k_\phi(t)\sin e_\phi)e_y - k_x(t)e_x \\ -(\omega_r + u_r(k_y(t)e_y + k_\phi(t)\sin e_\phi)e_x + u_r \sin e_\phi \\ -u_r(k_y(t)e_y + k_\phi(t)\sin e_\phi) \end{bmatrix} \tag{26}$$

Theorem III.1: Let the kinematic model (19), fuzzy kinematic control laws (24), the linear reference velocity u_r positive and also:

$$\mathbf{K}^T(\mathbf{e}(t)) = \begin{pmatrix} k_x(t) & k_y(t) & k_\phi(t) \end{pmatrix} = \mathbf{0} \Leftrightarrow \mathbf{e} = \mathbf{0}$$

$$0 \le k_x(t) \le k_{x\max} \quad ; \; 0 \le k_y(t) \le k_{y\max}$$

$$0 \le k_\phi(t) \le k_{\phi\max} \quad ; \left(\frac{dk_y}{d\mathbf{e}}\right)^T \left(\frac{d\mathbf{e}}{dt}\right) > 0 \; . \tag{27}$$

Then the equilibrium state of the non autonomous closed loop system (26) is asymptotically stable.

Proof: since **K(e)** (cf eq. 25) is equal to zero if only if **e** is equal to zero, the equilibrium state of the closed loop system (26) is the origin of the state space. The system (26) is non autonomous. The following Lyapunov function is chosen:

$$V_0 = \frac{1}{2}(e_x^2 + e_y^2) + (1 - \cos e_\phi)g(t) \tag{28}$$

where :

$$g(t) = 1/k_y(t) \tag{29}$$

and for hypothesis:

$$k_y(t) > 0 \Rightarrow \mathrm{g(t)} > 0 \; \forall t \tag{30}$$

Therefore Lyapunov function (28) is positive definite.

The time derivative of (28) is:

$$\dot{V}_0 = e_x\dot{e}_x + e_y\dot{e}_y + \dot{e}_\phi \sin e_\phi g(t) + (1 - \cos e_\phi)\dot{g}(t) \tag{31}$$

where:

$$\dot{g}(t) = \frac{d}{dt}\left[\frac{1}{k_y(t)}\right] = -\frac{dk_y/dt}{k_y^2(t)} = -\frac{(dk_y/d\mathbf{e})^T(d\mathbf{e}/dt)}{k_y^2(t)} \tag{32}$$

By substituting (26) into (31), it yields:

$$\dot{V}_0 = -k_x(t)e_x^2 - u_r k_\phi(t)\sin^2(e_\phi)g(t) + (1 - \cos e_\phi)\dot{g}(t) \tag{33}$$

Under the hypothesis of the theorem, function (31) is negative semidefinite, because it do not depends on e_y error. Since it results:

$$V_0 = \frac{1}{2}(e_x^2 + e_y^2) + (1 - \cos e_\phi)g(t) \le \le \frac{1}{2}(e_x^2 + e_y^2) + (1 - \cos e_\phi)g_{\max} \tag{34}$$

where:

$$g_{\max} = \max[g(t)], \tag{35}$$

then the (28) is a decrescent function. Therefore vector (21) is bounded and the equilibrium state of the closed loop system (26) is stable. It is also possible to calculate the second time derivative of Lyapunov function (28). Since the second time derivative of (28) depends on bounded variables, it is a bounded function. Therefore function (33) is uniformly continuous. From Lyapunov-like version of Barbalat's Lemma [9], it yields:

$$\lim_{t\to\infty} \dot{V}_0(t) = 0 \tag{36}$$

From equations (33) and (36), e_x and e_ϕ converge to zero. From equations (26), $\dot{e}_y$ function converges to zero. Therefore the steady state error e_y is constant. It results:

$$\dot{e}_\phi(\infty) = -u_r k_y(\infty)\bar{e}_y \tag{37}$$

where $\bar{e}_y$ is the steady state value of e_y . Since e_ϕ converges to zero, e_y converges to zero. We observe that k_y converges to zero if $\bar{e}_y$ converges to zero. Therefore the equilibrium point of the closed loop system (26) is asymptotically stable (Q.E.D.).

The control surfaces of fuzzy inference mechanism are chosen (see Fig. 4) so that the hypothesis on k_y (cf. eqs. 27) can be verified.

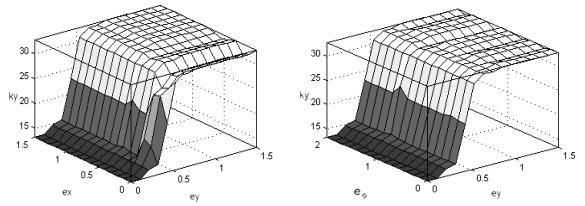


Fig.4. k_y versus e_x and e_y , k_y versus e_ϕ and e_y

From Fig. 4 we can observe that the control surfaces are continuous function and it is results:

$$\left(\frac{dk_y}{d\mathbf{e}}\right)^T \left(\frac{d\mathbf{e}}{dt}\right) \cong \frac{dk_y}{de_y}\frac{de_y}{dt} > 0 \quad \forall t\,.$$

Now, from (5), (6), (15) and (23) it results :

$$\frac{d}{dt}\begin{pmatrix} e_x \\ e_y \\ e_\phi \end{pmatrix} = \dot{\theta}_r \begin{bmatrix} -\frac{r}{2} + \frac{r}{2b}e_y \\ -\frac{r}{2b}e_x \\ -\frac{r}{2b} \end{bmatrix} + \dot{\theta}_l \begin{bmatrix} -\frac{r}{2} - \frac{r}{2b} \\ \frac{r}{2b}e_x \\ \frac{r}{2b} \end{bmatrix} + \begin{bmatrix} u_r\cos(e_\phi) \\ u_r\sin(e_\phi) \\ \omega_r \end{bmatrix} \tag{38}$$

We set as follows:

$$\alpha = 1/r;\ \beta = b/r \tag{39}$$

By considering the estimation errors of the kinematic parameters (39):

$$\hat{\alpha} = \bar{\alpha} - \alpha;\ \hat{\beta} = \bar{\beta} - \beta \tag{40}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the estimated kinematic parameters, it yields:

$$\frac{d}{dt}\begin{pmatrix} e_x \\ e_y \\ e_\phi \end{pmatrix} = \left(1 + \frac{\hat{\alpha}}{\alpha}\right) u_c \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \left(1 + \frac{\hat{\beta}}{\beta}\right) \omega_c \begin{bmatrix} e_y \\ -e_x \\ -1 \end{bmatrix} + \tag{41}$$

$$+ \begin{bmatrix} u_r\cos(e_\phi) & u_r\sin(e_\phi) & \omega_r \end{bmatrix}^{\mathrm{T}}$$

We obtain the following theorem:

Theorem III.2: Let the kinematic model (19) and fuzzy kinematic control law (24). If the linear reference and angular velocity are bounded functions and the angular velocity reference converges to zero, by choosing of adaptive kinematic control law:

$$\dot{\bar{\alpha}} = \gamma e_x u_c \quad \dot{\bar{\beta}} = \delta \frac{\omega_c \sin(e_\phi)}{K_y(t)} \tag{42}$$

$$\gamma, \delta > 0$$

the components of the vector $(e_x, e_y, e_\phi)^T$ of the closed loop system (41) converge to zero.

Proof: for the model (41) and (42) an extended state vector can be defined:

$$\bar{\mathbf{e}}^{\mathbf{T}} = \left(e_x, e_y, e_\phi, \hat{\alpha}, \hat{\beta}\right) \tag{43}$$

The following Lyapunov function can be chosen as follows:

$$V_1 = V_0 + \frac{1}{2\gamma\alpha}\hat{\alpha}^2 + \frac{1}{2\delta\beta}\hat{\beta}^2 \tag{44}$$

$$\gamma, \delta > 0$$

where V_0 is given by (28). Since V_0 is positive definite (cf. theorem III.1) , it is obvius that V_1 is positive definite. From (24), (41), the time derivative of function (44) results:

$$\dot{V}_1 = \dot{V}_0 + \frac{\hat{\alpha}}{\gamma\alpha}(\dot{\bar{\alpha}} - \gamma e_x u_c) + \frac{\hat{\beta}}{\delta\beta}(\dot{\bar{\beta}} - \delta\frac{\omega_c \sin(e_\phi)}{K_y(t)}) \tag{45}$$

where u_c and ω_c are given by (24) and $\dot{V}_0$ is given by (32). Function (45) is negative semidefined if only equations (42) are verified . In this case it results:

$$\dot{V}_1 = \dot{V}_0 \tag{46}$$

Since the function (45) do not depends on e_y component, it is negative semidefined. Therefore the closed loop system (41) is stable and the components of the state vector (43) are bounded (cf. theorem III.1). It is also possible to calculate the second time derivative of Lyapunov function (44). Since it depends on bounded variables, from Barbalat's Lemma it results:

$$\lim_{t\to\infty} \dot{V}_1(t) = 0 \tag{47}$$

Therefore, from (45), (46) and (47), e_x and e_ϕ converge to zero (cf. theorem III.1). But from (41) and (24) it results:

$$\dot{e}_\phi = -\left(1+\frac{\hat{\beta}}{\beta}\right)(\omega_r + u_r K_y(t) e_y + K_\phi(t)\sin(e_\phi)) + \omega_r \tag{48}$$

Since for hypotesys the linear reference, the angular velocity and the components of vector state (43) are bounded, $\ddot{e}_\phi$ is bounded. Therefore $\dot{e}_\phi$ is uniformly continuous. Since e_ϕ converges to zero, from Barbalat's Lemma [9], $\dot{e}_\phi$ converges to zero. Therefore from (48) e_y converges to zero only if the angular velocity reference ω_r converges to zero (Q.E.D.).

Now, by using of (6), (24), (39), (40) and (42) we can write the adaptive fuzzy kinematic control law in terms of angular velocity of left ($\dot{\theta}_{lc}$)and right ($\dot{\theta}_{rc}$) wheels:

$$\boldsymbol{\eta}_\mathbf{c} = \begin{pmatrix} \dot{\theta}_{rc} \\ \dot{\theta}_{lc} \end{pmatrix} = \begin{pmatrix} \overline{\alpha} & \overline{\beta} \\ \overline{\alpha} & -\overline{\beta} \end{pmatrix} \begin{pmatrix} u_c \\ \omega_c \end{pmatrix} \tag{49}$$

For theorem III.1 and theorem III.2, employing the adaptive fuzzy kinematic control law (49), the closed loop control system of the nonholonomic vehicle is asymptotically stable.

Now we can design an adaptive controller for the dynamic model (16). Preliminarily from (16) we can write as follows [18]:

$$\overline{\mathbf{M}}\dot{\boldsymbol{\eta}} + \overline{\mathbf{V}}_\mathbf{m}(\boldsymbol{\eta})\boldsymbol{\eta} = \mathbf{Y}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\mathbf{p} \tag{50}$$

where:

$$\mathbf{Y}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \begin{pmatrix} \ddot{\theta}_r & \ddot{\theta}_l & (\dot{\theta}_r - \dot{\theta}_l)\dot{\theta}_l \\ \ddot{\theta}_l & \ddot{\theta}_r & -(\dot{\theta}_r - \dot{\theta}_l)\dot{\theta}_r \end{pmatrix}$$

$$\mathbf{p} = (p_1 p_2 p_3)^T = \begin{pmatrix} b^2c^2m + c^2I + I_w \\ b^2c^2m - c^2I \\ 2bc^3 dm_c \end{pmatrix} \tag{51}$$

Also from (18) we have:

$$\begin{pmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \frac{r}{2}\cos(\phi) & \frac{r}{2}\cos(\phi) \\ \frac{r}{2}\sin(\phi) & \frac{r}{2}\sin(\phi) \\ \frac{r}{2b} & -\frac{r}{2b} \end{pmatrix} \begin{pmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{pmatrix} = \begin{pmatrix} \frac{r}{2}\cos(\phi) \\ \frac{r}{2}\sin(\phi) \\ \frac{r}{2b} \end{pmatrix} \dot{\theta}_r + \begin{pmatrix} \frac{r}{2}\cos(\phi) \\ \frac{r}{2}\sin(\phi) \\ -\frac{r}{2b} \end{pmatrix} \dot{\theta}_l$$

$$= \begin{pmatrix} \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r}{2} \\ \frac{r}{2b} \end{pmatrix} \dot{\theta}_r + \begin{pmatrix} \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{r}{2} \\ \frac{r}{2b} \end{pmatrix} \dot{\theta}_l =$$

$$= \boldsymbol{\Sigma}_1 \boldsymbol{\theta}_1 \dot{\theta}_r + \boldsymbol{\Sigma}_2 \boldsymbol{\theta}_2 \dot{\theta}_l \tag{52}$$

To finish this work we reformulate the theorem on the adaptive backstepping technique [7], [8], [18], [23].

Theorem III.3: Let the dinamical model (16) and the kinematical model (18). The properties (50), (51), and (52) are verified. The linear and angular velocity reference are bounded functions and the angular velocity reference converges to zero. By choosing the adaptive control law of kinematic parameters (42), the adaptive fuzzy kinematic control law (49) and the following adaptive dynamical control law:

$$\boldsymbol{\tau} = \overline{\mathbf{B}}^{-1}\left(-\mathbf{K}_\mathbf{d}\tilde{\boldsymbol{\eta}} + \mathbf{Y}\hat{\mathbf{p}} - \left(\frac{\partial V_1}{\partial \mathbf{q}}\hat{\mathbf{S}}\right)^\mathbf{T}\right)$$
$$\dot{\hat{\boldsymbol{\theta}}}_\mathbf{i} = \boldsymbol{\Lambda}_i\left(\frac{\partial V_1}{\partial \mathbf{q}}\boldsymbol{\Sigma}_i\right)\tilde{\eta}_i \quad \mathrm{i} = 1...2 \;; \dot{\hat{\mathbf{p}}} = -\boldsymbol{\Psi}\mathbf{Y}^T\tilde{\boldsymbol{\eta}} \tag{53}$$

where $\hat{\boldsymbol{\theta}}_i$ is the estimation of $\boldsymbol{\theta}_i$ (cf. eq. 52), $\mathbf{Y}$ and $\mathbf{p}$ are given by (51), $\hat{\mathbf{p}}$ is the estimation of dynamical parameters of $\mathbf{p}$ vector, $\boldsymbol{\Sigma}_i$ (i=1,2) matrices are given by (52), V_1 is given by (44), $\hat{\mathbf{S}}$ is the Jacobian matrix (cf eq. 18) and it depends on estimated kinematic parameters $\hat{\boldsymbol{\theta}}_\mathbf{i}$ for i=1,2, $\tilde{\boldsymbol{\eta}}$ is given by:

$$\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta}_\mathbf{c} - \boldsymbol{\eta} = (\tilde{\eta}_1, \tilde{\eta}_2)^T \tag{54}$$

where $\boldsymbol{\eta}_\mathbf{c}$ is given by (49) and $\boldsymbol{\eta}$ is the dynamical velocity vector of model (16), $\mathbf{K}_\mathbf{d}$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Lambda}_i$ are simmetric and positive definite matrices with appropriate dimensions, then the fuzzy adaptive kinematic and dynamic control system of nonholonomic vehicle is asymptotically stable.

4. Tests of simulation

The dynamical and kinematical models of an nonholonomic vehicle shown in fig. 1 are proposed in (16) and (18) respectively. The inference fuzzy mechanism is proposed in table 1. The fuzzy adaptive kinematical and dynamical control laws are proposed in (24), (42), (49) and (53). In this section we present some simulations of the proposed control system. Besides, the control system of this paper is compared to classical controller without fuzzy inference mechanism. All the simulations are developed in Matlab environment. The kinematical and dynamical parameters of the mobile vehicle are chosen as:

$$b = 0.75m;\ d = 0.3m;\ r = 0.15m;\ m_c = 30kg$$
$$m_w = 1kg;\ I_c = 15.6;\ I_w = 0.005;\ I_m = 0.0025.$$

The parameters of the kinematic controller (24) are the output of the fuzzy mechanism (cf. table (1)). The parameters of the fuzzy kinematical and dynamical adaptive fuzzy controller are:

$$\gamma = 0.005;\ \delta = 20.75;\ K_d = 5;\ \psi = 5\ .$$

With regard to the classical controller without fuzzy inference mechanism, the parameters of the kinematic controller are chosen as:

$K_x = K_y = K_\phi = 5$

The initial conditions for the reference position are:

$(x(0), y(0), \phi(0)) = (0,0,3.48rad)$

The initial conditions for the actual vehicle position are:

$(x(0), y(0), \phi(0)) = (-30,20,4.36rad)$.

In Fig. 5 the reference trajectory in (x,y) plane is shown.

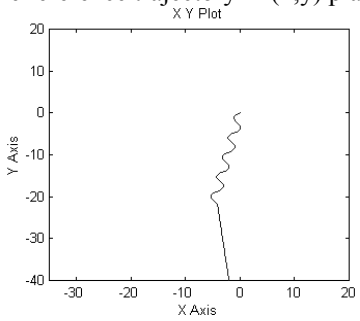


Fig. 5. Reference trajectory y[m], x[m].

In Fig. 6 and 7 the tracking errors of the adaptive fuzzy control system of this paper are shown. In Fig. 8 and 9 the tracking errors of the classical controller without fuzzy mechanism are shown. Fig. 10 and 11 show the estiomated kinematical parameters $\overline{\alpha}$, $\overline{\beta}$ (cf. eqs. 40 and 42) and the dynamical parameters of $\hat{\mathbf{p}}$ vector (cf. eqs. 50 and 51) respectively.

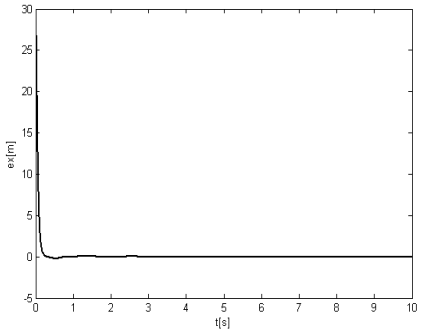


Fig. 6. Tracking error of Fuzzy control e_x[m].

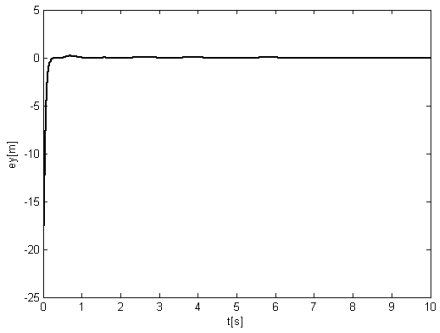


Fig. 7. Tracking error of Fuzzy control e_y[m].

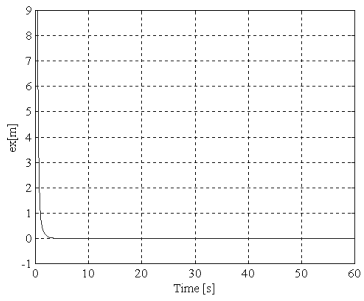


Fig.8. Tacking error without fuzzy e_x[m].

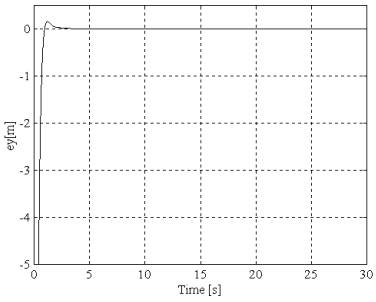


Fig.9. Tracking error without fuzzy e_y[m]

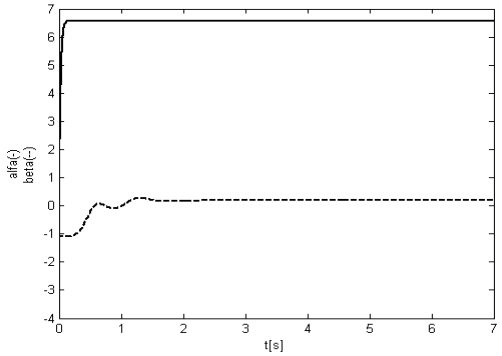


Fig.9. Kinematical parameters ($-\alpha$, --β).

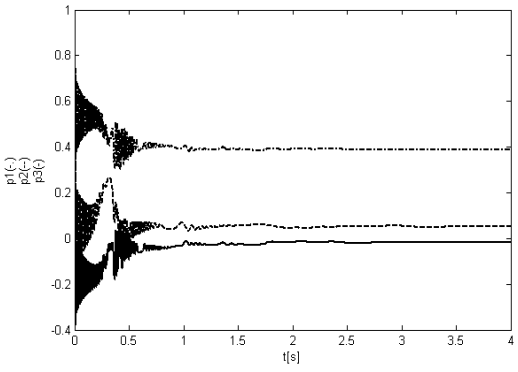


Fig. 10. Dynamical parameters (-.p1, --p2, -p3).

From Fig. 6-9 it obvius that the tracking errors resulting from the fuzzy adaptive control described in this work converge to zero more rapidly than the tracking errors of the classical one without fuzzy mechanism. From Fig. 10 and 11 we observe that the adaptive control is direct because the estimated parameters are not phisical values, but the steady state parameters are constant and the tracking errors of fig. 6 and 7 converge to zero.

5. Conclusions

In this paper we have presented a new kinematic and dynamic fuzzy adaptive control applied to a trajectory tracking of constrained vehicles. In particular a fuzzy mechanism inference determines the parameters of the kinematic controller. Therefore these parameters are time varying. An adaptive mechanism on line estimates the unknown dynamical and kinematical parameters of the vehicle model. Our concepts are formulated to proof the convergence to zero of the tracking errors through well known Lyapunov's theory and Barbalat's Lemma. Simulations in Matlab environment have shown that the adaptive fuzzy control system of this paper reduces the speed of the response with respect to classical controller without Fuzzy inference mechanism and the results of on line estimation of the unknown parameters are very good.

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