Motivation for using optimizing compilers

Control flow analysis
- Dominance analysis
- Loop analysis

Scalar optimizations on SSA Form
- Copy propagation
- Global value numbering
- Partial redundancy elimination
- Operator strength reduction
- Constant propagation
- Dead code elimination

Instruction scheduling

Register allocation

Optimizations for Java and the HotSpot JVM
Motivation for Using Optimizing Compilers

- **Execution time / energy reduction**: possible speedups due to compiler optimization depend on the application and the architecture (e.g. pipeline, SIMD, caches, multicore).

Example: SPEC CPU2000 benchmark gzip on a Power machine: PowerMac Quad G5/2.5 GHz with similar cores to the IBM Power4 plus SIMD (the first multicore chip)

<table>
<thead>
<tr>
<th>Compiler</th>
<th>Opt level</th>
<th>Execution time</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBM XL</td>
<td>max opt</td>
<td>135 s</td>
</tr>
<tr>
<td>GCC 4.7.2</td>
<td>max opt</td>
<td>145 s</td>
</tr>
<tr>
<td>GCC 4.7.2</td>
<td>no opt</td>
<td>494 s</td>
</tr>
</tbody>
</table>

- Increase **programmer productivity** by knowing
  - what the compiler can optimize faster and better than himself/herself, and
  - compilers’ limitations and how to write code that helps them to do better automatic optimization.
Some Optimization Goals

- Redundancy Elimination
- Operator Strength Reduction – replace slow instructions with faster
- Control Flow Optimization
  - For example place basic blocks in an order which reduces the number of dynamically executed branches.
  - Inline functions to eliminate function call overhead.
- Memory Hierarchy Optimization
  - Register allocation
  - Locality optimizations using e.g. tiling.
  - Cut functions into two halves and put hot halves together in cache blocks or virtual pages.
- Pipeline Optimizations through instruction scheduling.
- Parallelization
  - SIMD
  - Multicore
a = u + v;
if (a > b) {
    y = u;
} else {
    a = u - v;
    b = a - 1;
}
y = a * b;
Control-flow graph: Basic Blocks and Branches

Basic block: sequence of instructions with no label or branch
CFG: directed graph with basic blocks as nodes and branches as edges
Control-Flow Graph: the CFG View

Special nodes:
- the first node is called $s$ — start
- the last node is called $e$ — exit
Consider a control flow graph $G(V, E, s, e)$ and two vertices $u, v \in V$.
If every path from $s$ to $v$ includes $u$ then $u$ dominates $v$, written $u \gg v$.
For example 1 dominates itself, 2, 3, 4, and $e$. 
Predecessors of a dominated vertex

- If the edge \((v, w) \in E\) of a graph \((V, E)\) then \(v\) is a predecessor of \(w\).
- Consider any two vertices \(u, v \in V\) and \(u \neq v\). Then we have:
  
  \[ u \succ v \iff u \succ p_i; \quad \forall p_i \in \text{pred}(v). \]

if not \(u \succ 2\) then it cannot be true that \(u \succ v\)

\(u\) must dominate each predecessor of \(v\) to be able to dominate \(v\).

Since \(u\) dominates every \(p_i\) it must be on every path to \(v\) and therefore dominate \(v\).
procedure compute_dominance

\[ \text{dom}(s) \leftarrow \{s\} \]

for each \( w \in V - \{s\} \) do

\[ \text{dom}(w) \leftarrow V \]

\( \text{change} \leftarrow \text{true} \)

while \( \text{change} \) do

\( \text{change} \leftarrow \text{false} \)

for each \( w \in V - \{s\} \) do

\( \text{old} \leftarrow \text{dom}(w) \)

\[ \text{dom}(w) \leftarrow \{w\} \cup \bigcap_{p \in \text{pred}(w)} \text{dom}(p) \]

if \( \text{old} \neq \text{dom}(w) \)

\( \text{change} \leftarrow \text{true} \)

end
$\text{dom}(w) \leftarrow \{w\} \cup \bigcap_{p \in \text{pred}(w)} \text{dom}(p)$

<table>
<thead>
<tr>
<th>vertex</th>
<th>init.</th>
<th>1st iter.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>${0}$</td>
<td>${0} = {0}$</td>
</tr>
<tr>
<td>1</td>
<td>$V$</td>
<td>${1} \cup {0} = {0, 1}$</td>
</tr>
<tr>
<td>2</td>
<td>$V$</td>
<td>${2} \cup {0, 1} = {0, 1, 2}$</td>
</tr>
<tr>
<td>3</td>
<td>$V$</td>
<td>${3} \cup ({0, 1, 2} \cap {0, 1}) = {0, 1, 3}$</td>
</tr>
<tr>
<td>4</td>
<td>$V$</td>
<td>${4} \cup {0, 1, 3} = {0, 1, 3, 4}$</td>
</tr>
<tr>
<td>5</td>
<td>$V$</td>
<td>${5} \cup ({0, 1, 3, 4} \cap V) = {0, 1, 3, 4, 5}$</td>
</tr>
<tr>
<td>6</td>
<td>$V$</td>
<td>${6} \cup {0, 1, 2} = {0, 1, 2, 6}$</td>
</tr>
<tr>
<td>7</td>
<td>$V$</td>
<td>${7} \cup {0, 1, 2, 6} = {0, 1, 2, 6, 7}$</td>
</tr>
</tbody>
</table>
An Example Control Flow Graph 2(3)

\[
dom(w) \leftarrow \{w\} \cup \bigcap_{p \in \text{pred}(w)} \dom(p)
\]

<table>
<thead>
<tr>
<th>vertex</th>
<th>1st iter.</th>
<th>2nd iter.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>{0}</td>
<td>same</td>
</tr>
<tr>
<td>1</td>
<td>{0, 1}</td>
<td>same</td>
</tr>
<tr>
<td>2</td>
<td>{0, 1, 2}</td>
<td>same</td>
</tr>
<tr>
<td>3</td>
<td>{0, 1, 3}</td>
<td>same</td>
</tr>
<tr>
<td>4</td>
<td>{0, 1, 3, 4}</td>
<td>same</td>
</tr>
<tr>
<td>5</td>
<td>{0, 1, 3, 4, 5}</td>
<td>{5} \cup ({0, 1, 3, 4} \cap {0, 1, 2, 6, 7})</td>
</tr>
<tr>
<td>6</td>
<td>{0, 1, 2, 6}</td>
<td>same</td>
</tr>
<tr>
<td>7</td>
<td>{0, 1, 2, 6, 7}</td>
<td>same</td>
</tr>
</tbody>
</table>

After the third iteration also \(\dom(5) = \{0, 1, 5\}\) will remain the same and the algorithm terminates.
\[ \text{dom}(w) \leftarrow \{w\} \cup \bigcap_{p \in \text{pred}(w)} \text{dom}(p) \]

<table>
<thead>
<tr>
<th>vertex</th>
<th>3rd iter. ( \text{dom}(w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>{0}</td>
</tr>
<tr>
<td>1</td>
<td>{0, 1}</td>
</tr>
<tr>
<td>2</td>
<td>{0, 1, 2}</td>
</tr>
<tr>
<td>3</td>
<td>{0, 1, 3}</td>
</tr>
<tr>
<td>4</td>
<td>{0, 1, 3, 4}</td>
</tr>
<tr>
<td>5</td>
<td>{0, 1, 5}</td>
</tr>
<tr>
<td>6</td>
<td>{0, 1, 2, 6}</td>
</tr>
<tr>
<td>7</td>
<td>{0, 1, 2, 6, 7}</td>
</tr>
</tbody>
</table>
Immediate dominators

- The set $\text{dom}(w)$ is a total order.
- In other words: if $u, v \in \text{dom}(w)$ then either $u \gg v$ or $v \gg u$.
- We can order all vertices in $\text{dom}(w)$ to find the ”closest” dominator of $w$.
- First let $S \leftarrow \text{dom}(w) - \{w\}$.
- Consider any two vertices in $S$.
- Remove from $S$ the one which dominates the other. Repeat.
- The only remaining vertex in $S$ is the immediate dominator of $w$.
- We write the immediate dominator of $w$ as $\text{idom}(w)$.
- Every vertex, except $s$, has a unique immediate dominator.
- We can draw the immediate dominators in a tree called the dominator tree.
The Dominator Tree of Example CFG 1(2)

<table>
<thead>
<tr>
<th>vertex</th>
<th>$\text{dom}(w) - {w}$</th>
<th>$\text{idom}(w)$</th>
<th>how to find idom</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\emptyset$</td>
<td>-</td>
<td>has no idom</td>
</tr>
<tr>
<td>1</td>
<td>${0}$</td>
<td>0</td>
<td>only 0</td>
</tr>
<tr>
<td>2</td>
<td>${0, 1}$</td>
<td>1</td>
<td>remove 0</td>
</tr>
<tr>
<td>3</td>
<td>${0, 1}$</td>
<td>1</td>
<td>remove 0</td>
</tr>
<tr>
<td>4</td>
<td>${0, 1, 3}$</td>
<td>3</td>
<td>remove 0,1</td>
</tr>
<tr>
<td>5</td>
<td>${0, 1}$</td>
<td>1</td>
<td>remove 0</td>
</tr>
<tr>
<td>6</td>
<td>${0, 1, 2}$</td>
<td>2</td>
<td>remove 0,1</td>
</tr>
<tr>
<td>7</td>
<td>${0, 1, 2, 6}$</td>
<td>6</td>
<td>remove 0,1,2</td>
</tr>
</tbody>
</table>
The Dominator Tree of Example CFG 2(2)

**CFG**

```
0
  
1
  
2

3

6

4

7

5
```

<table>
<thead>
<tr>
<th>w</th>
<th>idom(w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

**DT**

```
0

1

2

3

5

6

4

7
```
The LT algorithm is the standard algorithm for computing the dominator tree.
It was completed in 1979 by Robert Tarjan and his PhD student Thomas Lengauer at Stanford.
Thomas Lengauer is the brother of Christian Lengauer whose group in Passau has developed many high order transformations which are now being implemented in GCC.
The LT algorithm calculates the immediate dominators in a clever way and is based on insights from depth first search.
It’s easy to implement but complex to prove correct.
Loop Analysis Using Dominance: More About DFS
Loops can be found by exploiting cycle arcs.

In a **natural loop**, one vertex called the header dominates all vertices in the loop.

Suppose there is a cycle arc \((v, u)\) such as \((3, 1)\) above.

Then, if \(u \gg v\) we know that \(u\) is a natural loop header.

We can search backwards from \(v\) and include everything we find to the loop, stopping at \(u\).

Due to \(u \gg v\) we cannot go wrong and miss \(u\).
A variable is only assigned to by one unique instruction
That instruction dominates all the uses of the assigned value
We introduce a new variable name at each assignment
SSA Form is the key to elegant and efficient scalar optimization algorithms
Invented by IBM Research Yorktown Heights in New York

But what to do when paths from different assignments join???
In node e: if we came from node x we let $a_2 \leftarrow a_0$ and if we came from node y we let $a_2 \leftarrow a_1$. This operation is called the $\phi$-function.
Our Example Translated to SSA Form

\[
a_0 = u + v \\
a_0 > b ??
\]

\[
y = u \\
a_1 = u - v \\
b = a_1 - 1
\]

\[
a_2 \leftarrow \phi(a_0, a_1) \\
y_0 = a_2 + v
\]
A Function Translated to SSA Form

- We insert a $\phi$-function where the paths from two different assignments of the same variable join.
- With the $\phi$-function, each definition dominates its uses.
Copy Propagation

\[ x_0 = a_0 + b_0; \]
\[ \text{if (...) \{ ...; } \]
\[ y_0 = x_0; \quad \text{/** COPY */} \]
\[ \text{if (...) \{ ...; } \]
\[ c_0 = y_0 + 1; \quad \text{/** USE */} \]
\[ c_0 = x_0 + 1; \]

- With SSA Form we can know that it is correct to replace \( y_0 \) with \( x_0 \)
- The values of \( x_0 \) and \( y_0 \) do not change after the definition (in a static sense)
A function is translated to SSA Form in the following steps

1. Compute the dominator tree \( DT \) of the function.
2. Compute the dominance frontier of each vertex in the CFG.
3. Insert \( \phi \)-functions.
4. Rename variables while traversing the dominator tree.
We want to insert a $\phi$-function where two paths from assignments meet.

This formulation of the problem was difficult to use to find an efficient algorithm.

The following is a trick which makes it easier to answer the question of where to insert $\phi$-functions:

**Trick:** Every variable is given a assignment in the start vertex.

That is, a variable $x$ is given an assignment $x_0$ in the start vertex.

No assembler code is produced for the assignment though.
Why would \( x_0 \) help???

With the assignment to \( x_0 \) we can see that two paths from assignments join in the vertices with \( x_1 \).

Therefore each of them needs a \( \phi \)-function.

Another way to see this is that these vertices are just outside what is dominated by the vertex with \( x_1 = \).
We need to insert a $\phi$-function in every vertex which is just outside what is dominated by a vertex with an assignment.

"Just outside" is called the **dominance frontier** of a vertex $u$.

It is written $DF(u)$.

$DF(u) = \{ v \mid \exists p \in pred(v), \ u \gg p, \ u \not\gg v \}$.

In words: if $u$ dominates a predecessor of $v$ but does not dominate $v$ strictly, then $v$ is in the dominance frontier of $u$.

After the dominator tree is found, the dominance frontier for each vertex is computed.

Each local variable and compiler-generated temporary is inspected: for each vertex $u$ with an assignment to the variable, a $\phi$-function is inserted in $DF(u)$.

Note that a $\phi$-function is an assignment — which also needs $\phi$-functions in the dominance frontier of its vertex.
The new version $x_1$ is pushed on the stack of $x$.

The vertex with $x_4$ is a child in the $DT$ and is inspected next.

The new version $x_4$ is pushed on the stack of $x$.

Each $\phi$-function in a successor vertex gets one of its operands replaced with $x_4$ from the current top of the stack.

The vertex with $x_4$ has no child in the $DT$ and $x_4$ is popped from the stack.

$x_1$ becomes the top of the stack and is used next.
The basic idea when translating from SSA Form is to replace the $\phi$-functions with copy statements in the predecessor vertices.
Translation from SSA Form

- $a_1 = u_3 + v_1$
- $a_1 > b_4$ ??
- $a_3 = a_1$
- $b_6 = b_4$
- $a_2 = u_3 - v_2$
- $b_5 = a_2 - 1$
- $a_3 = a_2$
- $b_6 = b_5$
- $y_1 = a_3 * b_6$

- It’s thus necessary to have a vertex to insert the copy statements into!
- Without the leftmost vertex, there is an edge from a vertex with multiple successors to a vertex with multiple predecessors and such an edge is called a **critical edge**.
- Critical edges are removed by inserting an extra empty vertex.
- This is done before dominance analysis.
The $\phi$-functions are parallel copy statements.

Conceptually all $\phi$-functions are executed concurrently by first reading all operands and then writing all destinations.

So what will go wrong here with a "naive" translation from SSA Form?
Translation from SSA Form

What is wrong?

*a_2 = a_1*
*b_2 = b_1*
*a_2 = b_2*
*b_2 = a_2*
Translation from SSA Form

\[
\begin{align*}
    a_2 &= a_1 \\
    b_2 &= b_1 \\
    x &= a_2 \\
    a_2 &= b_2 \\
    b_2 &= x
\end{align*}
\]

The value of \( a_2 \) must be saved before being overwritten!
a = 1;
b = 2;
if (a < b)
c = 3;
else
  c = 4;
put(c);

- Each variable can be either
  - Unknown
  - Constant
  - Non-constant
- Iterative dataflow analysis is performed to determine whether a variable is constant and in that case which constant.
- All branches (i.e. paths in a function) are assumed to be executable.
- Since c cannot be both 3 and 4 it’s assumed to be nonconstant.
Constant Propagation with Conditional Branches

a = 1;
b = 2;
if (a < b)
    c1 = 3;
else
    c2 = 4;
c3 = phi(c1, c2);
put(c3);

- Based on SSA Form.
- Invented at IBM Research and published 1991.
- Recall Kildall’s algorithm assumed every branch was executable.
- This algorithm assumes that nothing is executable except the start vertex.
- The function is interpreted and the constant expressions are propagated.
- The interpretation proceeds until no new knowledge about constants can be found.
Key Idea with $\phi$-functions

Thanks to SSA Form, one statement and variable is analyzed at a time.

At a $\phi$-function, if any operand is nonconstant the result is nonconstant, and if any two constants have different values the result also is nonconstant.

However, operands corresponding to branches which we don’t think will be executed can be ignored for the moment.

While interpreting the program we may later realize that the branch in fact might be executed and then the $\phi$-function will be re-evaluated.

We can ignore $c_2$ and let $c_3$ be 3.
The name is due to each expression, e.g. \( t_i \leftarrow a + b \), is given a number, essentially a hash-table index.

In subsequent occurrences of \( t_j \leftarrow a + b \) it is checked whether the statement can be changed to \( t_j \leftarrow t_i \).

This is a very old optimization technique with one version that is performed during translation to SSA Form and other versions when the code already is on SSA Form.

There are obviously older versions used before SSA Form but we will not look at them.
An expression $a + b$ is **redundant** if it is evaluated multiple times with identical values of the operands.

Eliminating redundant expressions is a very important optimization goal.

There are different approaches to redundancy elimination, including:

1. Hash-Based Value Numbering
2. Global Value Numbering
3. Common Subexpression Elimination
4. Code Motion out of Loops
5. Partial Redundancy Elimination

We will look at 1, 2, and 5.
Example 1

In vertex 1 the expression $a_0 + b_0$ is first computed.

The redundant occurrences of $a_0 + b_0$ can easily be removed.

On SSA Form we simply check that the variable versions are the same in the current and previous occurrence.
Example 2

The occurrences in vertices 3 and 4 cannot mistakenly be regarded as useful due to mismatching variable versions.
Example 3

Obviously there are no redundant expressions here.

We could perhaps save memory by computing $a_0 + b_0$ in vertex 1 but that is not a goal for redundancy elimination.

Which data structure should we use for performing value numbering during translation to SSA Form?
int h(int a, int b)
{
    int x, y;
    x = 1;
    y = 1;
    do {
        a = a + b;
        x = x + a;
        y = y + a;
    } while (a > 0);
    return x + y;
}

int h(int a, int b)
{
    int x, y;
    x = 1;
    do {
        a = a + b;
        x = x + a;
    } while (a > 0);
    return x + x;
}
Partial Redundancy Elimination, or PRE, can eliminate both **full** and **partial** redundancies.

- **Full redundancies**: when the expression is available from all predecessor basic blocks.
- **Partial redundancies**: when the expression is only available from some but not all predecessor basic blocks.
- **Partial redundancies** also covers loops, i.e. PRE can move code out from loops.
PRE was invented by Morel and Renvoise in 1979.
Then Fred Chow in his PhD thesis at Stanford from 1983 (with John Hennessy as supervisor) improved it.
In 1992 Knoop et al. published a version of PRE which is optimal in the sense of minimizing register pressure. They called their algorithm Lazy Code Motion.
It was stated by a famous researcher that PRE cannot be done on SSA Form since SSA Form involves variables while PRE involves expressions.
"Cannot" is dangerous to state in public...
In 1999 Kennedy and Chow and others at SGI published the SSA formulation of Lazy Code Motion and called it SSAPRE.
We will look at a simpler version of it and then note that there exists a faster implementation.
Limitations of Value Numbering

Both hash-based and global value numbering can optimize the full redundancy in vertex 1. None of them can optimize the partial redundancy in vertex 3.
The Key Idea of SSAPRE

- We create $\Phi$-functions for the hypothetical variable $h$.
- After SSAPRE, $\Phi$-functions become normal $\phi$-functions and they are really the same (different notation to distinguish between them only).
- By inserting the expression $a + b$ at $\Phi$-operands with the value $\bot$ ("bottom"), the partial redundancy in vertex 3 becomes a full redundancy and can be eliminated.
double a[N];

for (i = 0; i < N; ++i)
    x += a[i];

while (p < end)
    x += *p++;

- The most important purpose is to rewrite the code to the left into the code to the right.
- C/C++ compilers are required to make it possible to use the address of the array element after the last declared element.
- Typically, in total one extra byte might be wasted in memory due to this.
- It’s not one extra byte per array but rather per memory segment.
Invalid C Code

double a[N];
double* p = &a[N];

for (i = N-1; i >= 0; --i)
    x += a[i];

while (--p >= a)
    x += *p;

- In the last iteration p == &a[-1] in the comparison.
- The compiler is not required to make that address valid.
- The code to the right triggers undefined behavior if performed by the programmer.
Another Name for OSR

OSR is also known as Induction Variable Elimination

do {
    x = x + a[i];
    i = i + 1;
} while (i < N);

do {
    s = i * 4;
    t = load a+s;
    x = x + t;
    i = i + 1;
} while (i < N);
The primary goal is to get rid of the multiplication

\[
\text{do } \{ \\
\quad s = i \times 4; \\
\quad t = \text{load a+s;}
\quad x = x + t; \\
\quad i = i + 1; \\
\text{ } \} \text{ while (i < N);} \\
\]

- \(i\) is a \textit{basic} induction variable
- Classes of \textit{dependent} induction variables: \(j \leftarrow b \times i + c, \ i \) is a basic IV
- \(s \leftarrow 4 \times i + 0\)
do {
    s = i * 4;
    t = load a+s;
    x = x + t;
    i = i + 1;
}

} while (i < N);

s = 4 * i;

} while (i < N);

- Initialize the dependent IV before the loop
- Increment the dependent IV just after the basic IV is incremented
- Maybe we can get rid of the basic IV now?
s = 4 * i;
do {
    t = load a+s;
    x = x + t;
    i = i + 1;
    s = s + 4;
} while (i < N);

m = 4 * N;
s = 4 * i;
do {
    t = load a+s;
    x = x + t;
    s = s + 4;
} while (s < m);

- \( s = i \times b + c \) (we have \( b = 4 \) and \( c = 0 \))
- \( i = \frac{s-c}{b} \)
- \( i < N \Rightarrow \frac{s-c}{b} < N \Rightarrow s < N \times b + c \), if \( b > 0 \)
double a[N];

for (i = 0; i < N; ++i)
    x += a[i];
We first find all strongly connected components of the SSA graph.

We want to copy the SCC of $i$ and modify the copy for $t_1$.

Therefore we want to have processed $i$ before processing $t_1$.

Let us start with $x$. 
Processing of $x_0$

- $SCC_0 = \{x_0\}$. Empty stack.
- Nodes processed in a SCC are green.
- Next processing $x_1$. 

\[ \phi(x_0, x_2) \]
\[ \phi(i_0, i_2) \]
\[ x_1 \]
\[ x_2 \]
\[ x_0 \]
\[ t_1 \]
\[ t_2 \]
\[ i_1 \times 8 \]
\[ M[a + t_1] \]
\[ i_1 + 1 \]
\[ i_0 \]
\[ i_2 \]
\[ 0 \]

jonasskeppstedt.net (js@cs.lth.se)  Optimizing Compilers  2020  55 / 104
Processing of $x_1$ and $x_2$

- $x_1$ and $x_2$ are pushed and then the search continues with $t_2$.
- Nodes on the stack are red.
- Next processing $t_2$. 
Processing of $t_2$

Next processing $t_1$. 
Processing of $t_1$

- $x_0 \rightarrow 0$ to $t_1$
- $x_1 \rightarrow \phi(x_0, x_2)$ to $t_1$
- $x_2 \rightarrow x_1 + t_2$
- $i_1 \times 8$ to $M[a + t_1]$
- $t_2$
- $i_0 \rightarrow 0$
- $i_0 \rightarrow \phi(i_0, i_2)$
- $i_1 \rightarrow i_1 + 1$
- $i_2$

*Next processing $i_2$.***
Processing of $i_2$ and $i_1$

- Next processing $i_0$. 
Processing of $i_0$

- $SCC_1 = \{i_0\}$
- Next more processing in $i_2$. 
Classifying $\text{SCC}_2 = \{i_1, i_2\}$

- $\text{SCC}_2 = \{i_1, i_2\}$
- $\text{SCC}_2$ is an **induction variable** due it consists of a $\phi$-function and an add with a **region constant**.
- A region constant is not modified in a loop, i.e. it’s a number or its definition strictly dominates the loop header.
Replacing $i_1 \times 8$

- $SCC_3 = \{ t_1 \}$
- $SCC_3$ is a multiplication of an induction variable and a region constant.
- Therefore $SCC_3$ is replaced by a modified copy of $SCC_2$ with $\phi(i)$. 
Modifying a Copy of $\text{SCC}_2$ to Compute $t_1$

- $\text{SCC}_4 = \{y_1, y_2\}$
- Due to the replacement, the assignment to $t_1$ becomes dead code.
- There is a very beautiful algorithm to remove $t_1$ and other dead code that we will look at soon.
Also $a + t_1$ can be Replaced

- Due to Tarjan’s algorithm we can start in any node and be sure we have already processed the operand nodes, when a variable’s definition is going to be replaced.
- Not only multiplications but also some additions can be replaced, but we don’t show this in the example.
#include <stdio.h>

int main(void)
{
    int    a;

    a = 1;
    a = a + 2;
    goto L;

    printf("a = %d\n", a);

    L:
    return 0;
}
Depth First Search and Dominance Analysis

- DFS from the start vertex visits all basic blocks reachable from the start vertex, obviously.
- All other vertices are removed before performing dominance analysis.
- For some minor modifications of the control flow graph an existing dominator tree can be updated.
- In general, it’s easier and probably faster to recompute the dominator tree from scratch, according to some researchers who tried to update the DT.
Limitations of DCE Based on Liveness Analysis

for (i = 0; i < n; ++i)
    a = a + i * i;
return;

- The variable a is live in the loop but will not affect program output.
- The loop should be deleted but it cannot be using DCE based on liveness.
The correct approach to DCE is to delete all code which cannot affect the observable output.

In each function, some instructions are marked as live, e.g. calls to `printf`, and are put in a worklist.

Then, recursively, all instructions which provide input to a live instruction is marked as live and put on the worklist.

Eventually no new instructions are marked as live and all other instructions can be deleted (but read more about branches first!).

Instructions initially marked live include: function calls, memory writes, and return instructions.

Why did it take more than 30 years to invent this obvious approach to DCE?
The main reason why it was not invented earlier is that the other approaches usually were sufficient.

With SSA Form, however, it’s more likely there will be lots of instructions, in particular $\phi$-functions, which remain after other optimizations.

For example, operator strength reduction explicitly copies and modifies the strongly connected components in the SSA Graph of induction variables, which can leave a lot of work to DCE.

The article in ACM Transactions on Programming Languages and Systems (TOPLAS) which presented SSA Form also presented the DCE algorithm we will study.
Assume there is a live instruction in vertex $x$.

The DCE algorithm must assure execution actually reaches $x$ exactly as the original program would.

Therefore some conditional branch instructions (and the instructions providing their input etc) which branch to $x$ must also be marked live.

In this example the branch in $u$ controls whether $x$ certainly will be executed.

The vertices that control whether $w$ will be executed are $u$, $v$, and $w$ itself.
The DCE Algorithm

```plaintext
procedure eliminate_dead_code(G)
    for each statement s do
        if (s is prelive) {
            live(s) ← true
            add s to worklist
        } else
            live(s) ← false
    worklist ← prelive
    while (worklist ≠ ∅) do {
        take s from worklist
        v ← vertex(s)
        live(v) ← true
        for each source operand ω of s do {
            t ← def(ω)
            if (not live(t)) {
                live(t) ← true
                add t to worklist
            }
        }
        for each vertex v ∈ CD⁻¹(vertex(s)) do {
            t ← multiway branch of v
            if (not live(t)) {
                live(t) ← true
                add t to worklist
            }
        }
    }
    for each statement s do
        if (not live(s) and s /∈ {label, branch})
            delete s from vertex(S)
    simplify(G)
```

Simplifying the CFG after DCE

procedure simplify(G)  
  live(e) ← true  
  modified ← false  
  for each vertex u ∈ G do {  
    if (not live(u))  
      continue  
    for each v ∈ succ(u) do {  
      if (live(v))  
        continue  
      w ← ipdom(v) /* idom in RCFG */  
      while (not live(w))  
        w ← ipdom(w)  
      replace (u, v) with (u, w)  
      update the branch in u to its new target w  
      update ϕ-functions in w if necessary  
      modified ← true  
    }  
  }  
  if (modified) {  
    delete vertices from G which now have become unreachable  
    update dominator tree DT  
  }
end
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← idom(v) /* idom in RCFG */
            while (not live(w))
                w ← idom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update φ-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end

Only successor is live.
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u)) continue
        for each v ∈ succ(u) do {
            if (live(v)) continue
            w ← ipdom(v) /* idom in RCFG */
            while (not live(w))
                w ← ipdom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update $\phi$-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end

2 is dead. Nearest live is 3.
procedure simplify(\(G\))

\(\text{live}(e) \leftarrow \text{true}\)

\(\text{modified} \leftarrow \text{false}\)

for each vertex \(u \in G\) do {

if (not \(\text{live}(u)\))
continue

for each \(v \in \text{succ}(u)\) do {

if (\(\text{live}(v)\))
continue

\(w \leftarrow \text{ipdom}(v)\) /* idom in RCFG */

while (not \(\text{live}(w)\))

\(w \leftarrow \text{ipdom}(w)\)

replace \((u, v)\) with \((u, w)\)

update the branch in \(u\) to its new target \(w\)

update \(\phi\)-functions in \(w\) if necessary

\(\text{modified} \leftarrow \text{true}\)
}

if (\(\text{modified}\)) {

delete vertices from \(G\) which now have become unreachable
update dominator tree \(\text{DT}\)
}

end

---

2 is dead. Nearest live is 3.
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← ipdom(v) /* idom in RCFG */
            while (not live(w))
                w ← ipdom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update \( \phi \)-functions in w if necessary
        }
        modified ← true
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end
procedure simplify(G)
  live(e) ← true
  modified ← false
  for each vertex u ∈ G do {
    if (not live(u))
      continue
    for each v ∈ succ(u) do {
      if (live(v))
        continue
      w ← ipdom(v) /* idom in RCFG */
      while (not live(w))
        w ← ipdom(w)
      replace (u, v) with (u, w)
      update the branch in u to its new target w
      update φ-functions in w if necessary
      modified ← true
    }
  }
  if (modified) {
    delete vertices from G which now have become unreachable
    update dominator tree DT
  }
end

Must fix φ(a) in 7.
procedure simplify($G$)
    $live(e) \leftarrow \text{true}$
    modified $\leftarrow \text{false}$
    for each vertex $u \in G$ do {
        if (not $live(u)$) continue
        for each $v \in \text{succ}(u)$ do {
            if ($live(v)$) continue
            $w \leftarrow \text{idom}(v)$ /* idom in RCFG */
            while (not $live(w)$)
                $w \leftarrow \text{idom}(w)$
            replace $(u, v)$ with $(u, w)$
            update the branch in $u$ to its new target $w$
            update $\phi$-functions in $w$ if necessary
            modified $\leftarrow \text{true}$
        }
    }
    if (modified) {
        delete vertices from $G$ which now have become unreachable
        update dominator tree $DT$
    }
end
A variable $x$ is **live** at a point $p$ (instruction) if it may be used in the future without being assigned to.

- $a$ is live from the function start and up to and including the add, and then after $S_3$ and up to and including the negation.
- $b$ is live from the start and up to and including the subtraction.
- $c$ is live from $S_1$ and up to and including the multiplication.
An Example of Graph Coloring

a = 1  
b = a + 2  
c = a - b  
d = c  
e = d + 1  
f = d - e

Which variables cannot use the same register?  
How many registers are needed?
The Interference Graph

 live = use(i) ∪ (live - {def(i)})

1. Initially live = out = \{c, f\}.
2. def(f): add edge (c, f).
   live = \{c, d, e\}.
3. def(e): add edges (e, c), (e, d).
   live = \{c, d\}.
4. def(d): add edge (d, c).
   live = \{c\}.
5. def(c): no new edge.
   live = \{a, b\}.
6. def(b): add edge (a, b).
   live = \{a\}.
7. def(a): no new edge. live = \emptyset.
This interference graph needs three colors.
Can we use fewer colors?
c and d have the same value so they can use the same register!

It is done using a technique called register coalescing.

Register coalescing is an example of node merging.

Register coalescing needs a minor modification to the construction of the interference graph.
Consider an interference graph $IG$ and a number of available colors $K$.

Assume the $IG$ can be colored with $K$ colors and there is a node $v \in IG$ with fewer than $K$ neighbors.

Since $v$ has fewer than $K$ neighbors there must be at least one unused color left for $v$.

Therefore we can remove $v$ from the $IG$ without affecting the colorability of $IG$.

We remove $v$ from $IG$ and push $v$ on a stack.

Then we proceed looking for a new node with fewer than $K$ neighbors.

Assume the original $IG$ was colorable and all its nodes have been pushed on the stack.

Then each node is popped and re-inserted into $IG$ and given a color which no neighbor has.
The number of neighbors of a node $v$ is denoted its **degree**, or $\text{deg}(v)$.

When there is no node with $\text{deg}(v) < K$ a variable is selected for spilling.

Spilling means that a variable will reside in memory instead of being allocated a register.

Through spilling the $IG$ eventually will become empty, obviously.

Heuristics are used to decide which variable (i.e. node) to spill.

The expected number of memory accesses removed by allocating a variable is calculated, and this count is typically divided by a ”size” of the node.

By size is meant the number of vertices or instructions that the register would be reserved in for that variable, and hence cannot be used for any other variable.
a = b + c;

... 

d = a + c;

----------

t1 = b + c;
a = t1;

...

t2 = a;
d = t2 + a;

- On a RISC machine where operands cannot be in memory a new tiny live range is created at each original memory access of the spilled variable.
- These tiny live ranges should never be spilled.
- The rewriting is done after all nodes have been removed from the interference graph.
- If there was spilling the algorithm is re-executed.
- Eventually it will terminate and three iteration almost always suffice.
The purpose of instruction scheduling is to improve performance by reducing the number of pipeline stalls suffered during execution.

The following example illustrates the concept, where the right column is the scheduled code.

Due to instructions only are scheduled within one basic block, only a limited improvement is achieved — the fsub and stf are not helped at all.

```
ldf t2, a, t1
ldf t3, b, t1
fadd t4, t2, t3
ldf t5, c, t1
ldf t6, d, t1
fmul t7, t5, t6
fsub t8, t3, t7
stf t8, e, t1
```

```
ldf t2, a, t1
ldf t3, b, t1
ldf t5, c, t1
ldf t6, d, t1
fadd t4, t2, t3
fmul t7, t5, t6
fsub t8, t3, t7
stf t8, e, t1
```
The goal of instruction scheduling is to reduce pipeline stall and this is achieved by separating the producer and consumer.

This separation makes it more difficult to perform register allocation.

**Question:** Which of instruction scheduling and register allocation should be performed first?

**Answer:** Instruction scheduling because register allocation would create unnecessary constraints for the scheduler, and advanced instruction scheduling would be seriously limited with already assigned registers.

If register allocation results in spill code, the instruction scheduler is usually run a second time in order to separate the load instructions from the uses of the loaded register.
The left schedule needs three floating point registers and the right schedule one more.

```plaintext
ldf  f2,ra,ri
ldf  f3,rb,ri
fadd f2,f2,f3
ldf  f3,rc,ri
ldf  f4,rd,ri
ldf  f5,rd,ri
ldf  f4,rc,ri
fadd f2,f2,f3
fmul f3,f3,f4
fmul f4,f4,f5
fsub f2,f2,f3
fsub f2,f2,f4
stf  f2,re,ri
stf  f2,re,ri
```
Consider the following loop and assume there are true dependencies from $A$ to $B$ and from $B$ to $C$.

```c
void h()
{
    int i;
    for (i = 0; i < 100; ++i) {
        A;
        B;
        C;
    }
}
```

Due to list scheduling only works with one basic block, it cannot improve this loop.

Such loops are of course extremely common.
Let us take instructions from three iterations and interleave them.

First we need to execute instructions from the first two iterations in a prologue.

<table>
<thead>
<tr>
<th>cycle</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>A₀</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>B₀</td>
<td>A₁</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>C₀</td>
<td>B₁</td>
<td>A₂</td>
</tr>
<tr>
<td>3</td>
<td>A₃</td>
<td>C₁</td>
<td>B₂</td>
</tr>
<tr>
<td>4</td>
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<td>A₄</td>
<td>C₂</td>
</tr>
<tr>
<td>5</td>
<td>C₃</td>
<td>B₄</td>
<td>A₅</td>
</tr>
<tr>
<td>6</td>
<td>A₆</td>
<td>C₄</td>
<td>B₅</td>
</tr>
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</tr>
<tr>
<td>8</td>
<td>C₆</td>
<td>B₇</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>C₇</td>
<td></td>
</tr>
</tbody>
</table>

Assume for illustration only 8 iterations are executed.

For example $A₃$ denotes instruction $A$ in iteration 3.

After a steady-state with $2 \times 3$ iterations there is an epilogue.

Consider instruction $B₃$. While it waits for $A₃$, the CPU can also execute $C₁$ and $B₂$, assuming a pipelined superscalar CPU.
List Scheduled Execution

- Each iteration is completed before the next starts.
- The height of an iteration is the number of clock cycles it takes.
Parallelism with Modulo Scheduling

A new iteration is started before the current has completed.

We wish to start the next iteration as early as possible.

If we start the next iteration the same clock cycle, we need a multicore with one core per loop iteration.
All normal optimizations are applicable to OOP as well.

Virtual function calls, i.e. calls through a pointer to an unknown method limits optimization opportunities.

Therefore, it is important to find calls which must refer to a specific method.

Sometimes that can be done by only analyzing the type hierarchy, but at other times the assignments must be tracked.

It is of course not always possible to find which method is called statically.

There are function pointers in C as well, and they can sometimes be analyzed using symbol table information (number and types of parameters) plus tracking assignments.
The Java byte code is used for several languages other than Java:

- Scala
- Ruby
- Python
- Lisp
- Scheme
HotSpot

- The HotSpot virtual machine originates from the Strongtalk virtual machine for the Smalltalk language.
- It was used by Sun research for the Self language.
- The first release as a Java virtual machine was in 1999.
- It is the default virtual machine from Sun/Oracle since Java 1.3.
- Hotspot is written in C++ some assembler, and consist of 250,000 lines.
- Due to HotSpot is partly written in assembler it has triggered the IcedTea project based on HotSpot but without assembler code. Available for example for Power and ARM processors and others.
The JVM is a stack machine.

This means a byte code instruction pops operands from a stack and pushes the result back to the stack.

At about the same time as the JVM was designed Bell Labs also designed a virtual machine (for their Inferno operating system) which instead is a register-based virtual machine.

Register-based virtual machines are easier to produce faster code for, and therefore HotSpot translates the byte code to that.
Execution of a method starts by interpreting the byte code and after the execution count of the method has reached a limit, optimization is used.

The whole method is optimized.

Different optimization levels are used depending on whether the JVM is for desktops (clients) or servers.

Servers are expected to run for longer time and enables more time-consuming optimizations.

In addition to the method invocation counter, there are loop iteration counters which also can trigger optimization.
The optimization can make guesses and perform better optimizations as long as the guesses are correct.

For this, runtime checks are inserted to validate the guesses.

If a guess was wrong, the method is deoptimized and interpreted again, but can be optimized later.

Deoptimization can also be needed after a new class has been loaded.
First the control flow graph of a method is constructed by inspecting the byte codes.

Then the instructions of a basic block are created by simulating the JVM execution stack.

The stack-based execution model of the JVM is thus replaced with the SSA representation.

This is called the HIR representation, or the high-level intermediate representation.

Client JVM optimizations on SSA Form include:
  - Constant folding
  - Value numbering
  - Inlining
Low-level intermediate representation

- Not SSA Form
- Essentially symbolic assembler code, as in Bell Labs’ Inferno
- Unlimited number of machine registers before register allocation
The server JVM also uses SSA Form.

In addition to the control flow graph, control and data dependencies are analyzed.

Additional optimizations include:

- Constant propagation
- Dead code elimination
- Instruction scheduling
- Graph coloring register allocation
- Loop unrolling
- Loop invariant code motion
If you are interested in optimizing compilers, there is the course EDA230 in September.

It is focused on SSA Form and you will start with a subset C compiler which first compiles and then simulates the input C program.

There you will implement:

- Lengauer-Tarjan dominance analysis
- Translation to/from SSA Form
- Constant propagation on SSA Form
- Dead code elimination on SSA Form
### Optimizing Compilers Hall of Fame at LTH

<table>
<thead>
<tr>
<th>Year</th>
<th>Group</th>
<th>Programme</th>
<th>Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>Martin Nitsche</td>
<td>Math. Göttingen</td>
<td>33526</td>
</tr>
<tr>
<td>2011</td>
<td>Linus Åkesson</td>
<td>PhD/CS</td>
<td>112805</td>
</tr>
<tr>
<td>2010</td>
<td>Joakim Andersson/Jon Steen</td>
<td>D</td>
<td>126616</td>
</tr>
<tr>
<td>2009</td>
<td>Manfred Dellkrantz/Jesper Öqvist</td>
<td>D</td>
<td>950</td>
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<tr>
<td>2008</td>
<td>Jonas Paulsson</td>
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<tr>
<td>2007</td>
<td>Björn Carlin/Hans Gylling</td>
<td>π/D</td>
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<td>2006</td>
<td>Fredrik Nilsson</td>
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<td>Mats Mattsson</td>
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<td>Bo Do/Per Fransson</td>
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<tr>
<td>2000</td>
<td>Per Cederberg</td>
<td>PhD/Robotics</td>
<td>10768</td>
</tr>
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</table>

- The benchmark has sometimes been modified over the years and for 2011 there was a new one.