Contents of Lecture 9

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Two Simple Forms of Dead Code Elimination

```c
#include <stdio.h>

int main(void)
{
    int a;
    a = 1;
    a = a + 2;
    goto L;
    printf("a = %d\n", a);

L:    return 0;
}
```

- DFS
- Liveness Analysis
for (i = 0; i < n; ++i)
    a = a + i * i;
return;

- The variable a is live in the loop but will not affect program output.
- The loop should be deleted but it cannot be using DCE based on liveness.
The correct approach to DCE is to delete all code which cannot affect the observable output.

In each function, some instructions are marked as live, e.g. calls to `printf`, and are put in a worklist.

Then, recursively, each instruction which provides input to a live instruction is also marked as live and put on the worklist.

Eventually no new instructions are marked as live and all other instructions can be deleted (but read more about branches first!).

Instructions initially marked live include: function calls, memory writes, and return instructions, and in VCC additionally the put and get instruction.

Why did it take more than 30 years to discover this form of DCE?
The main reason why it was not invented earlier is that the other approaches usually were sufficient.

With SSA Form, however, it’s more likely there will be lots of instructions, in particular $\phi$-functions, which remain after other optimizations.

For example, operator strength reduction explicitly copies and modifies the strongly connected components in the SSA Graph of induction variables, which can leave a lot of work to DCE.

The article in Transactions on Programming Languages and Systems (TOPLAS) which presented SSA Form also presented the DCE algorithm we will study.
Conditional Branches

- Assume there is a live instruction in vertex $x$.
- The DCE algorithm must assure execution actually reaches $x$ exactly as the original program would.
- Therefore some conditional branch instructions (and the instructions providing their input etc) which branch to $x$ must also be marked live.
- In this example the branch in $u$ controls whether $x$ certainly will be executed.
- For vertex $w$, the vertices which can control that $w$ will be executed are $u$, $v$, and $w$. 

A vertex \( v \) **postdominates** \( u \) if every path from \( u \) to the vertex \( e \) contains \( v \), and we write it \( v \preceq u \) (not standard notation).

A vertex \( v \) **strictly postdominates** \( u \) if \( v \preceq u \) and \( v \neq u \), and we write it \( v \ll u \).

Thus we have \( w \preceq x \) and \( w \preceq y \).

The reverse control flow graph is the control flow graph with the direction of each edge reversed, where \( s \) and \( e \) have switched roles, and is written \( \text{RCFG} \).

Post dominance can be computed as dominance in the RCFG.
A non-null path has at least one edge.

A vertex $v$ is control dependent on vertex $u$, written $u \delta^c v$ if

1. there exists a non-null path from $u$ to $v$ and $v$ postdominates every vertex on the path after $u$, and
2. $v$ does not strictly postdominate $u$.

The set of vertices which are control dependent on $u$ is denoted $CD(u)$ and the set of vertices a vertex $v$ is control dependent on is denoted $CD^{-1}(v)$. 
Lemma 2.34, page 103

**Lemma**

Assume $v \in \text{succ}(u)$ and there is a path $p = (v_0 = v, v_1, ..., v_k = w)$ from $v$ to $w$. Then $w \preceq v \iff w \preceq v_i$ for every vertex $v_i$ on $p$.

**Proof.**

Let us show $\Rightarrow$ first. Assume therefore in contradiction that there exists some $0 < i < k$ such that $w \preceq v_i$. Thus there exists a path from $v_i$ to $e$ which does not include $w$. Then there is a path from $v$ to $v_i$ to $e$ which avoids $w$ which is a contradiction. Hence $w \preceq v_i$.

Since $v$ is on the path, $\Leftarrow$ follows directly.
Recall that the *dominance frontier* of a vertex \( u \) is the set of vertices \( v \) such that \( u \) dominates a predecessor of \( v \) but does not strictly dominate \( v \):

\[
DF(u) \overset{\text{def}}{=} \{ v \mid (\exists p \in \text{pred}(v)) \ u \succcurlyeq p \land u \succ v \}.
\]

With Lemma 2.34 we can simplify the definition of control dependence and show that it is equivalent to dominance frontiers in the reverse control flow graph.

First the simplified definition: a vertex \( v \) is control dependent on \( u \in CD^{-1}(v) \) if \( v \) postdominates a successor of \( u \) but does not strictly postdominate \( u \):

\[
CD^{-1}(v) \overset{\text{def}}{=} \{ u \mid (\exists s \in \text{succ}(u)) \land v \preccurlyeq s \land v \preceq u \}.
\]
Theorem

\[ u \overset{\delta^c}{\rightarrow} v \text{ in CFG } \iff u \in DF(v) \text{ in RCFG}. \]

Proof.

This follows from Lemma 2.34, since \( u \overset{\delta^c}{\rightarrow} v \) in CFG means \( v \) postdominates a successor of \( u \) but does not strictly postdominate \( u \), which in RCFG means \( v \) dominates a predecessor of \( u \) but \( v \) does not strictly dominate \( u \), ie \( u \in DF(v) \).
Example CFG and RCFG
The DCE Algorithm

```plaintext
procedure eliminate_dead_code(G)
  for each statement s do
    if (s is prelive) {
      live(s) ← true
      add s to worklist
    } else
      live(s) ← false
  worklist ← prelive
  while (worklist ≠ ∅) do {
    take s from worklist
    v ← vertex(s)
    live(v) ← true
    for each source operand ω of s do {
      t ← def(ω)
      if (not live(t)) {
        live(t) ← true
        add t to worklist
      }
    }
  }
  for each vertex u ∈ CD−1(v) do {
    t ← multiway branch of u
    if (not live(t)) {
      live(t) ← true
      add t to worklist
    }
  }
  for each statement s do
    if (not live(s) and s ∉ {label, branch})
      delete s from vertex(S)
  simplify(G)
end`
```
Simplifying the CFG after DCE

procedure simplify(G)
live(e) ← true
modified ← false
for each vertex \( u \in G \) do {
    if (not live(u))
        continue
    for each \( v \in \text{succ}(u) \) do {
        if (live(v))
            continue
        w ← ipdom(v) /* idom in RCFG */
        while (not live(w))
            w ← ipdom(w)
        replace \((u, v)\) with \((u, w)\)
        update the branch in \( u \) to its new target \( w \)
        update \( \phi \)-functions in \( w \) if necessary
        modified ← true
    }
}
if (modified) {
    delete vertices from \( G \) which now have become unreachable
    update dominator tree \( DT \)
}
end

Green denotes live vertices
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← idom(v) /* idom in RCFG */
            while (not live(w))
                w ← idom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update φ-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end

only successor is live.
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← ipdom(v) /* idom in RCFG */
            while (not live(w))
                w ← ipdom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update φ-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end

2 is dead. Nearest live is 3.
procedure `simplify(G)`

\[ \text{live}(e) \leftarrow \text{true} \]

\[ \text{modified} \leftarrow \text{false} \]

\[ \text{for each vertex } u \in G \text{ do} \{
\]

\[ \text{if } (\text{not live}(u)) \]

\[ \text{continue} \]

\[ \text{for each } v \in \text{succ}(u) \text{ do} \{
\]

\[ \text{if live}(v)) \]

\[ \text{continue} \]

\[ w \leftarrow \text{ipdom}(v) /* \text{idom in RCFG} */ \]

\[ \text{while (not live}(w)) \]

\[ w \leftarrow \text{ipdom}(w) \]

\[ \text{replace } (u, v) \text{ with } (u, w) \]

\[ \text{update the branch in } u \text{ to its new target } w \]

\[ \text{update } \phi \text{-functions in } w \text{ if necessary} \]

\[ \text{modified} \leftarrow \text{true} \]

\[ \} \]

\[ \} \]

\[ \text{if (modified)} \{ \]

\[ \text{delete vertices from } G \text{ which now have become unreachable} \]

\[ \text{update dominator tree } DT \]

\[ \} \]

end

- 2 is dead. Nearest live is 3.
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← ipdom(v) /* idom in RCFG */
            while (not live(w))
                w ← ipdom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update φ-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end

9 is dead. Nearest live is 7.
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex $u \in G$ do {
        if (not live(u))
            continue
        for each $v \in \text{succ}(u)$ do {
            if (live(v))
                continue
            $w \leftarrow \text{idom}(v)$ /* idom in RCFG */
            while (not live(w))
                $w \leftarrow \text{idom}(w)$
            replace $(u, v)$ with $(u, w)$
            update the branch in $u$ to its new target $w$
            update $\phi$-functions in $w$ if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from $G$ which now have become unreachable
        update dominator tree $DT$
    }
end

Must fix $\phi(a)$ in 7.
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex $u \in G$ do {
        if (not live(u))
            continue
        for each $v \in \text{succ}(u)$ do {
            if (live(v))
                continue
            $w ← \text{ipdom}(v)$ /* idom in RCFG */
            while (not live(w))
                $w ← \text{ipdom}(w)$
            replace $(u, v)$ with $(u, w)$
            update the branch in $u$ to its new target $w$
            update $\phi$-functions in $w$ if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from $G$ which now have become unreachable
        update dominator tree $DT$
    }
end

Later remove one $(3, 7)$!
Keep only live vertices.