Contents of Lecture 9

- Depth First Search for DCE (Dead Code Elimination)
- Liveness Analysis for DCE
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- Control Dependencies
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Two Simple Forms of Dead Code Elimination

```
#include <stdio.h>

int main(void)
{
        int     a;

        a = 1;
        a = a + 2;

        goto L;

        printf("a = %d\n", a);

L:
        return 0;
}
```

- DFS
- Liveness Analysis

Limitations of DCE Based on Liveness Analysis

```
for (i = 0; i < n; ++i)
        a = a + i * i;
return;
```

- The variable a is live in the loop but will not affect program output.
- The loop should be deleted but it cannot be using DCE based on liveness.

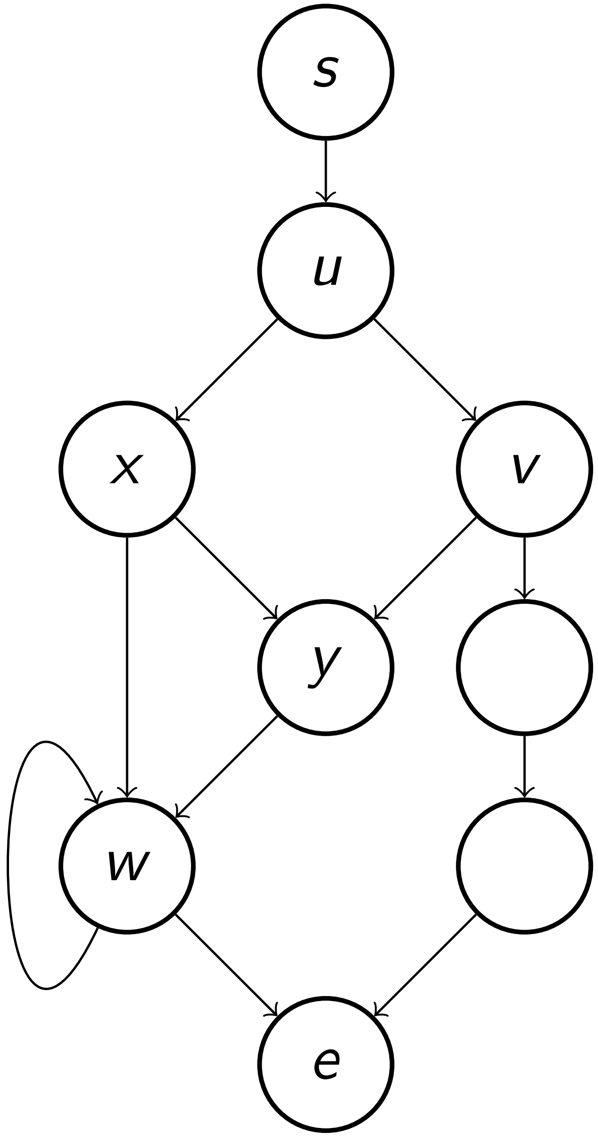
DCE Based on Observable Output

- The correct approach to DCE is to delete all code which cannot affect the observable output.
- In each function, some instructions are marked as **live**, e.g. calls to `printf`, and are put in a worklist.
- Then, recursively, each instruction which provides input to a live instruction is also marked as live and put on the worklist.
- Eventually no new instructions are marked as live and all other instructions can be deleted (but read more about branches first!).
- Instructions initially marked live include: function calls, memory writes, and return instructions, and in **vcc** additionally the **put** and **get** instruction.
- Why did it take more than 30 years to discover this form of DCE?

SSA and DCE

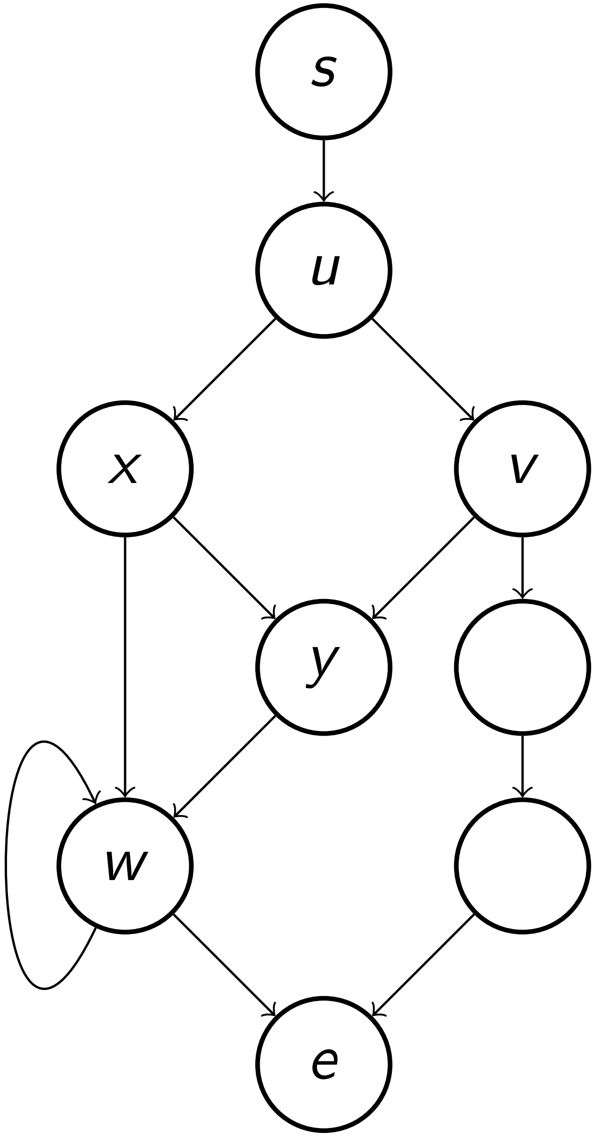
- The main reason why it was not invented earlier is that the other approaches usually were sufficient.
- With SSA Form, however, it's more likely there will be lots of instructions, in particular ϕ-functions, which remain after other optimizations.
- For example, operator strength reduction explicitly copies and modifies the strongly connected components in the SSA Graph of induction variables, which can leave a lot of work to DCE.
- The article in Transactions on Programming Languages and Systems (TOPLAS) which presented SSA Form also presented the DCE algorithm we will study.

Conditional Branches



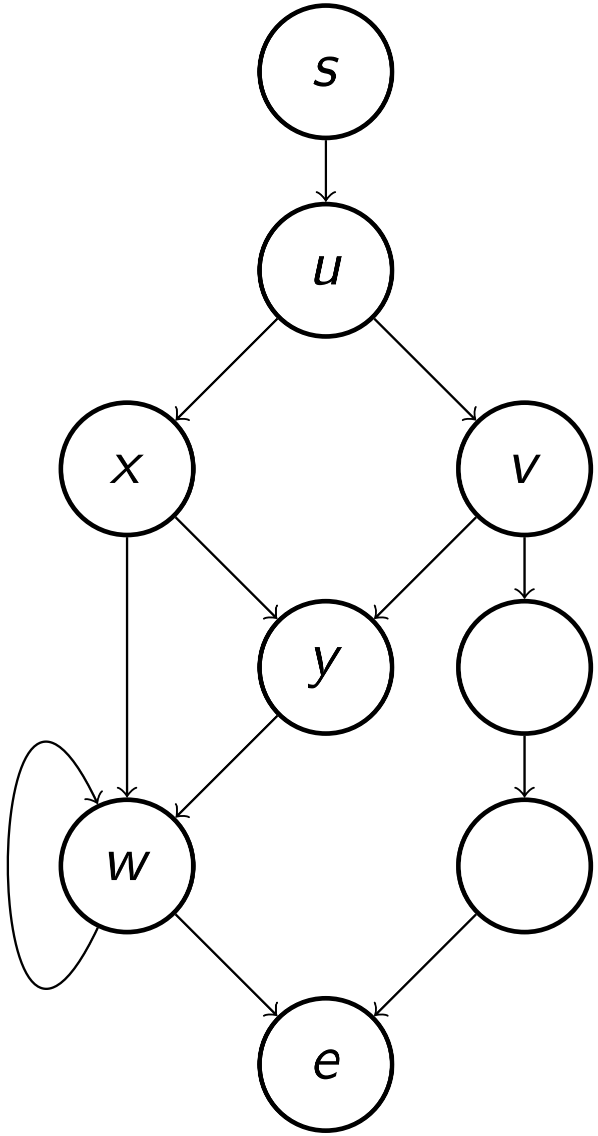
- Assume there is a live instruction in vertex x.
- The DCE algorithm must assure execution actually reaches x exactly as the original program would.
- Therefore some conditional branch instructions (and the instructions providing their input etc) which branch to x must also be marked live.
- In this example the branch in u controls whether x certainly will be executed.
- For vertex w, the vertices which can control that w will be executed are u, v, and w.

Post Dominance and the Reverse Control Flow Graph



- A vertex v **postdominates** u if every path from u to the vertex e contains v, and we write it $v \lesseqqgtr u$ (not standard notation).
- A vertex v **strictly postdominates** u if $v \underline{\ll} u$ and $v \neq u$, and we write it $v \ll u$.
- Thus we have $w \underline{\ll} x$ and $w \underline{\ll} y$.
- The reverse control flow graph is the control flow graph with the direction of each edge reversed, where s and e have switched roles, and is written **RCFG**.
- Post dominance can be computed as dominance in the RCFG.

Control Dependence



- A non-null path has at least one edge.
- A vertex v is control dependent on vertex u, written $u\ \delta^c\ v$ if
  1. there exists a non-null path from u to v and v postdominates every vertex on the path after u, and
  2. v does not strictly postdominate u.
- The set of vertices which are control dependent on u is denoted $CD(u)$ and the set of vertices a vertex v is control dependent on is denoted $CD^{-1}(v)$.

Lemma 2.34, page 103

Lemma

Assume $v \in succ(u)$ and there is a path $p = (v_0 = v, v_1, ..., v_k = w)$ from v to w. Then $w \underline{\ll} v \Leftrightarrow w \underline{\ll} v_i$ for every vertex v_i on p.

Proof.

Let us show $\Rightarrow$ first. Assume therefore in contradiction that there exists some $0 < i < k$ such that $w \underline{\not\ll} v_i$. Thus there exists a path from v_i to e which does not include w. Then there is a path from v to v_i to e which avoids w which is a contradiction. Hence $w \underline{\ll} v_i$.
Since v is on the path, $\Leftarrow$ follows directly. $\square$

Dominance Frontiers

- Recall that the *dominance frontier* of a vertex u is the set of vertices v such that u dominates a predecessor of v but does not strictly dominate v:
$$DF(u) \stackrel{\text{def}}{=} \{v | (\exists p \in pred(v))\ u \geqq p \wedge u \not\gg v\}.$$
- With Lemma 2.34 we can simplify the definition of control dependence and show that it is equivalent to dominance frontiers in the reverse control flow graph.
- First the simplified definition: a vertex v is control dependent on $u \in CD^{-1}(v)$ if v postdominates a successor of u but does not strictly postdominate u:
$$CD^{-1}(v) \stackrel{\text{def}}{=} \{\ u \mid (\exists s \in succ(u))\ \wedge\ v \leqq s\ \wedge\ v\ \not\ll\ u\}$$

Equivalence of CD in CFG and DF in RCFG

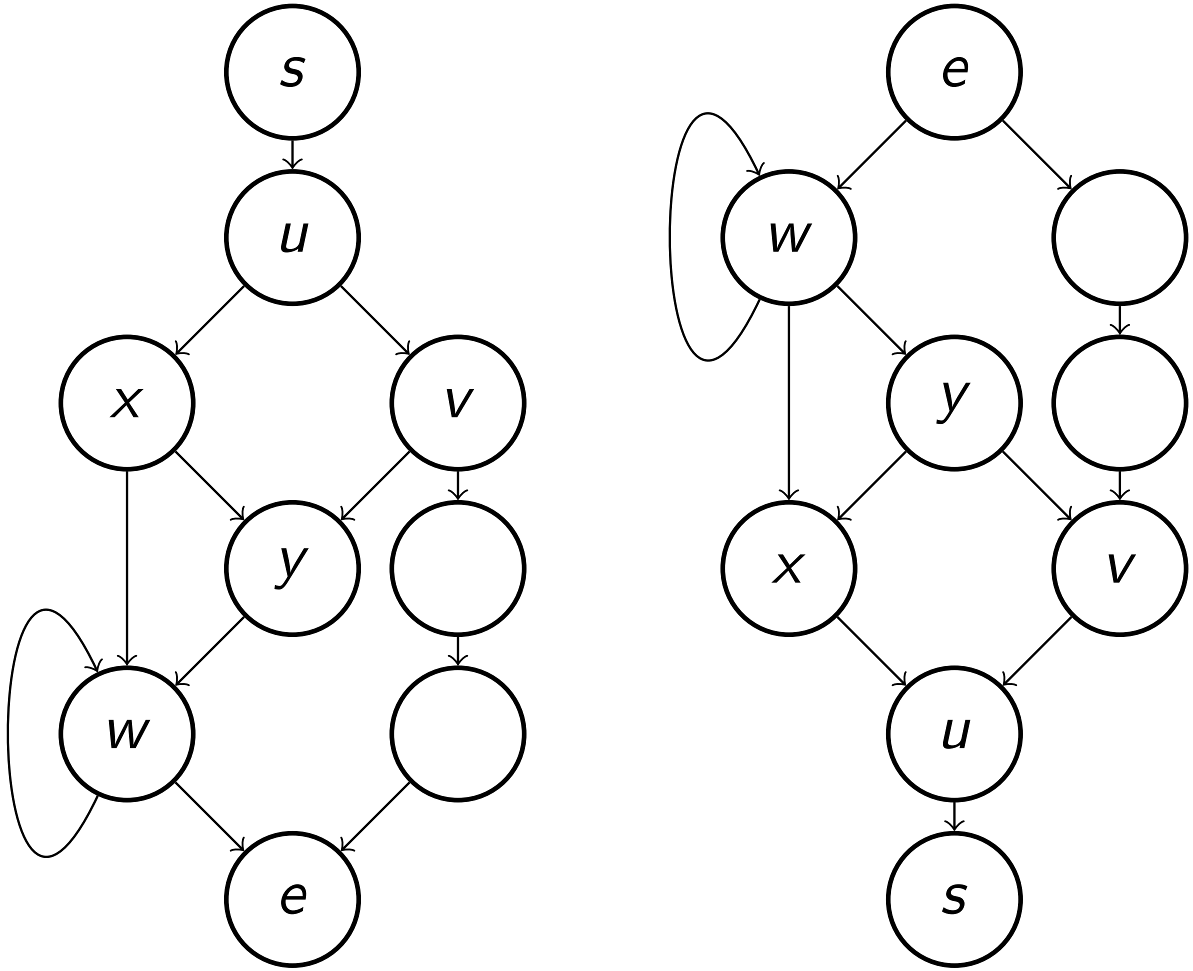
Theorem

$u \; \delta^c \; v$ in CFG $\Leftrightarrow u \in DF(v)$ in RCFG.

Proof.

This follows from Lemma 2.34, since $u \; \delta^c \; v$ in CFG means v postdominates a successor of u but does not strictly postdominate u, which in RCFG means v dominates a predecessor of u but v does not strictly dominate u, ie $u \in DF(v)$. $\square$

Example CFG and RCFG

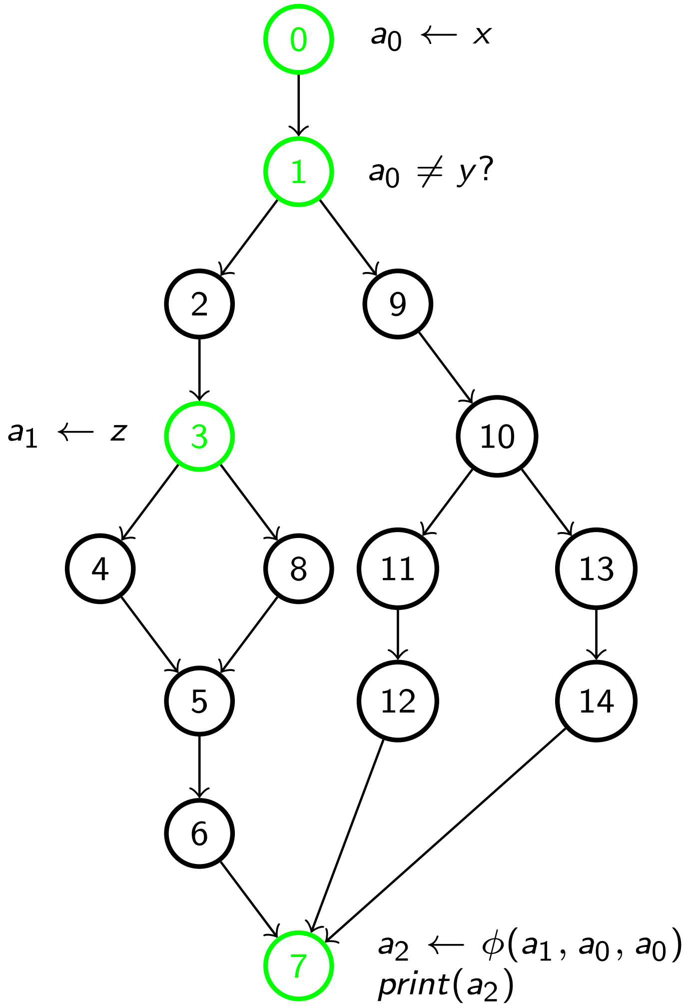


The DCE Algorithm

```
procedure eliminate_dead_code(G)
    for each statement s do
        if (s is prelive) {
            live(s) ← true
            add s to worklist
        } else
            live(s) ← false
    worklist ← prelive
    while (worklist ≠ ∅) do {
        take s from worklist
        v ← vertex(s)
        live(v) ← true
        for each source operand ω of s do {
            t ← def(ω)
            if (not live(t)) {
                live(t) ← true
                add t to worklist
            }
        }
        for each vertex u ∈ CD^{-1}(v) do {
            t ← multiway branch of u
            if (not live(t)) {
                live(t) ← true
                add t to worklist
            }
        }
    }
    for each statement s do
        if (not live(s)  and s ∉ {label, branch})
            delete s from vertex(S)
    simplify(G)
end
```

Simplifying the CFG after DCE

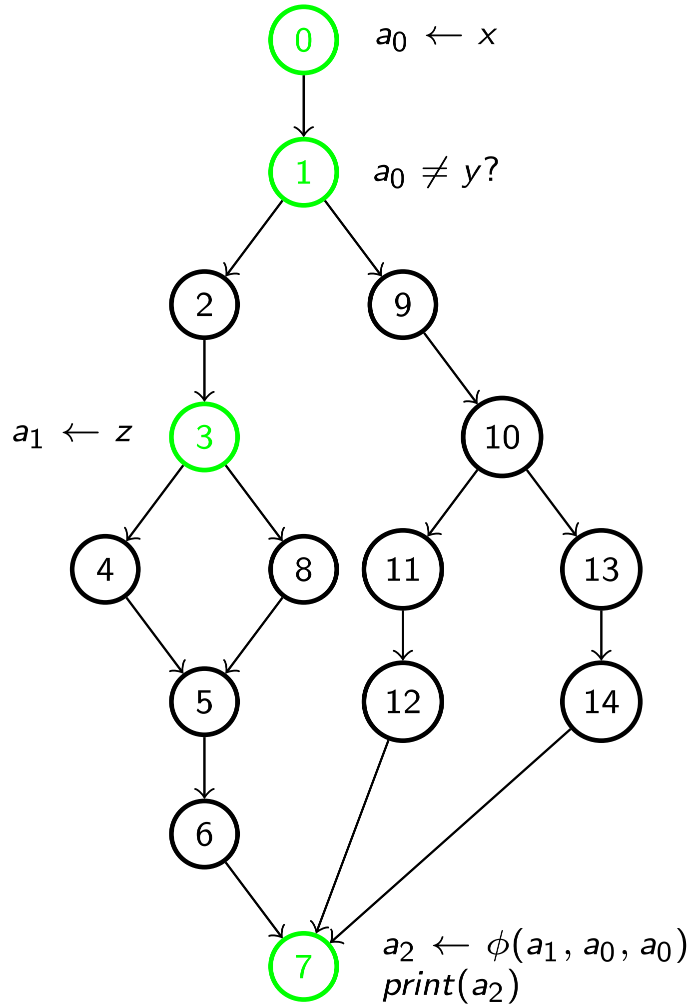
```
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← ipdom(v) /* idom in RCFG */
            while (not live(w))
                w ← ipdom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update ϕ-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end
```



- Green denotes live vertices

Processing 0

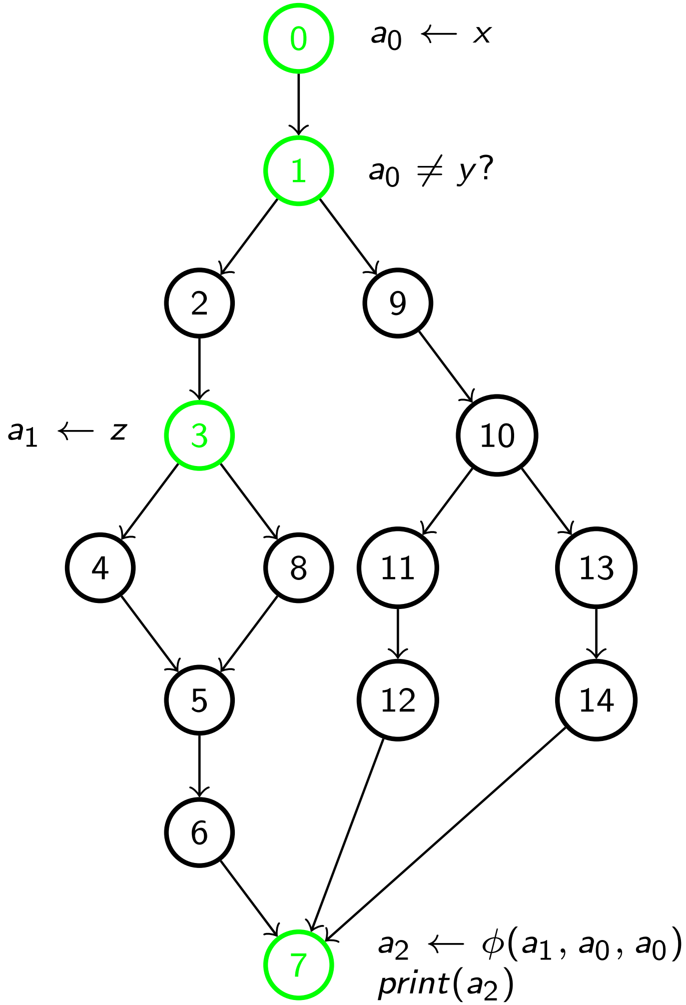
procedure *simplify* (G)
    live $(e) \leftarrow$ true
    modified $\leftarrow$ false
    for each vertex $u \in G$ **do** {
        if (**not** *live* (u))
            continue
        for each $v \in$ *succ* (u) **do** {
            if (*live* (v))
                continue
            $w \leftarrow$ *ipdom* (v) /* idom in RCFG */
            while (**not** *live* (w))
                $w \leftarrow$ *ipdom* (w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update ϕ-functions in w if necessary
            modified $\leftarrow$ true
        }
    }
    if (*modified*) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end



- Only successor is live.

Processing 1: Edge $(1, 2)$

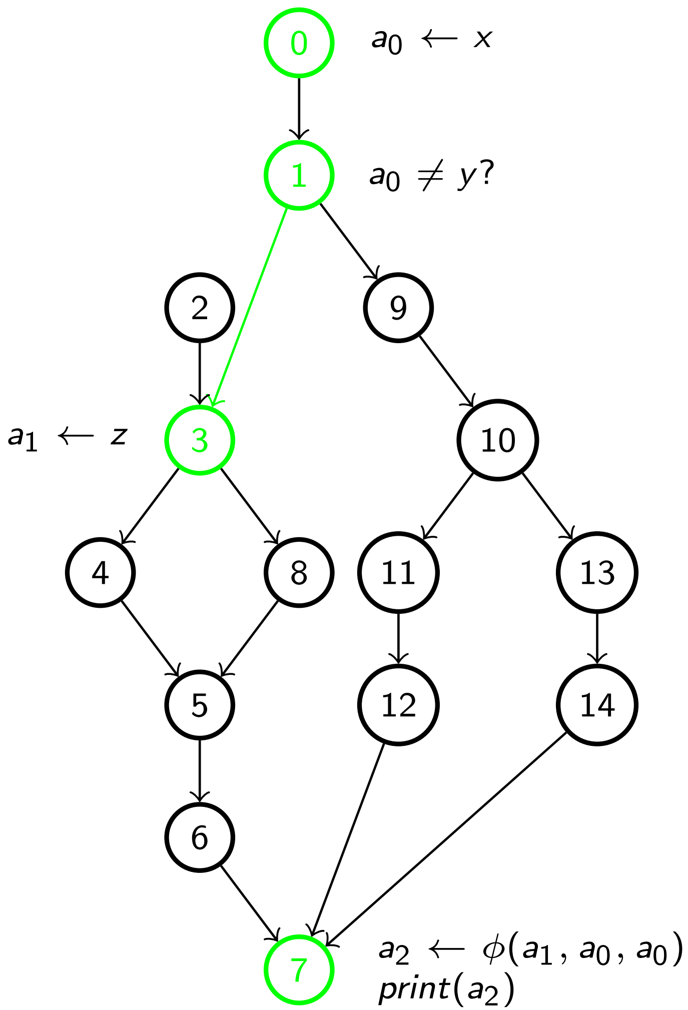
```
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← ipdom(v) /* idom in RCFG */
            while (not live(w))
                w ← ipdom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update ϕ-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end
```



- 2 is dead. Nearest live is 3.

Processing 1: Edge $(1, 2)$

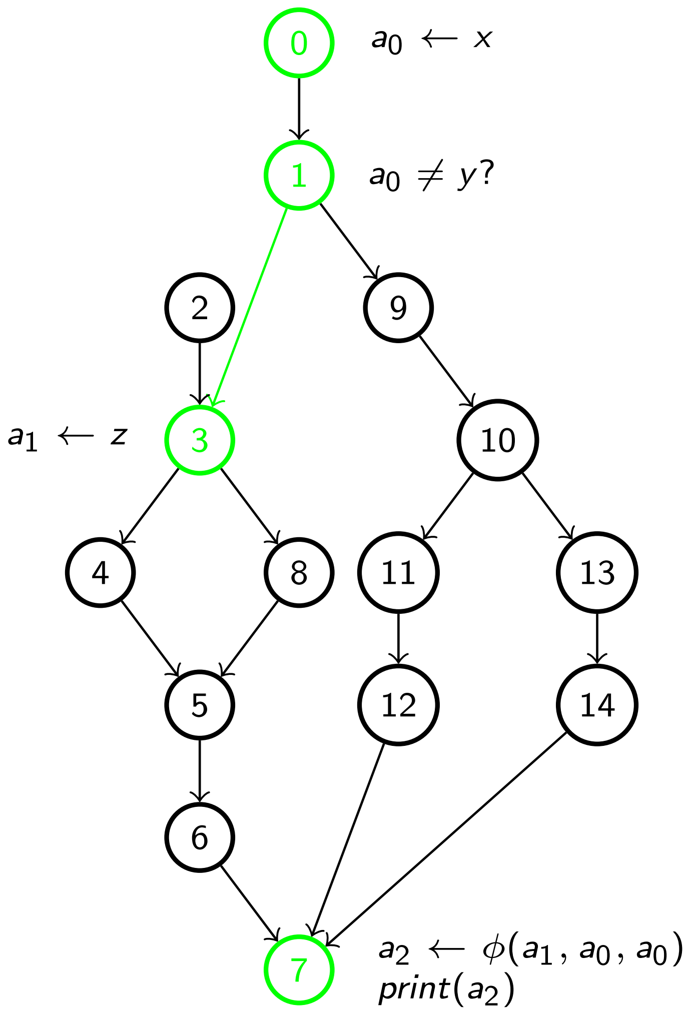
```
procedure simplify(G)
    live(e) ← true
    modified ← false
    for each vertex u ∈ G do {
        if (not live(u))
            continue
        for each v ∈ succ(u) do {
            if (live(v))
                continue
            w ← ipdom(v) /* idom in RCFG */
            while (not live(w))
                w ← ipdom(w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update ϕ-functions in w if necessary
            modified ← true
        }
    }
    if (modified) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end
```



- 2 is dead. Nearest live is 3.

Processing 1: Edge $(1, 9)$

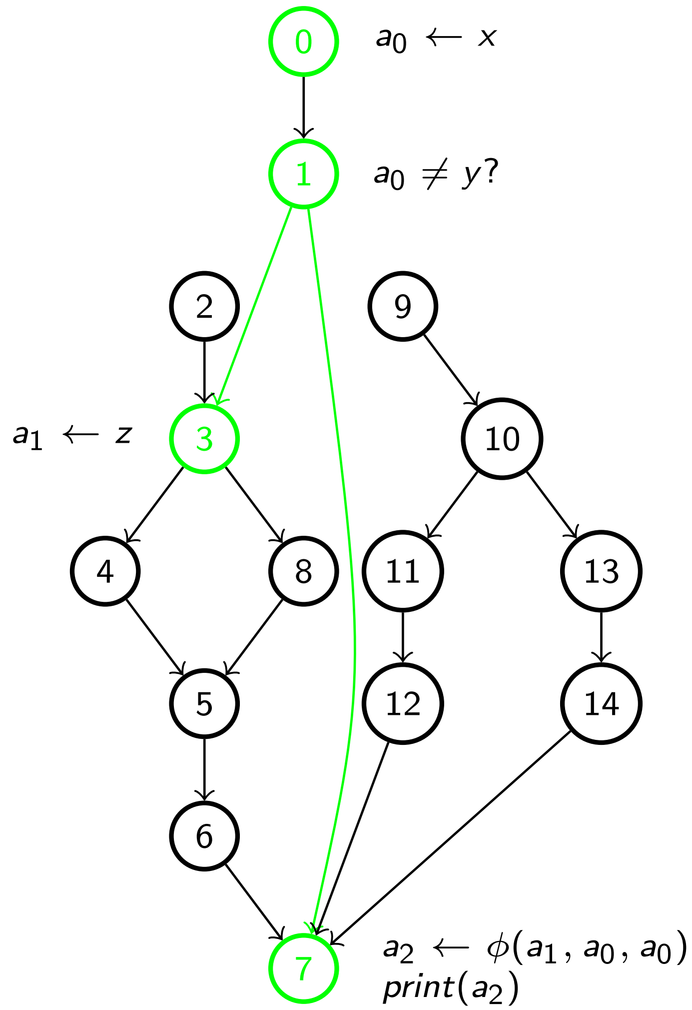
```
procedure simplify(G)
        live(e) ← true
        modified ← false
        for each vertex u ∈ G do {
                if (not live(u))
                        continue
                for each v ∈ succ(u) do {
                        if (live(v))
                                continue
                        w ← ipdom(v) /* idom in RCFG */
                        while (not live(w))
                                w ← ipdom(w)
                        replace (u, v) with (u, w)
                        update the branch in u to its new target w
                        update φ-functions in w if necessary
                        modified ← true
                }
        }
        if (modified) {
                delete vertices from G which now have become unreachable
                update dominator tree DT
        }
end
```



- 9 is dead. Nearest live is 7.

Processing 1: Edge $(1, 9)$

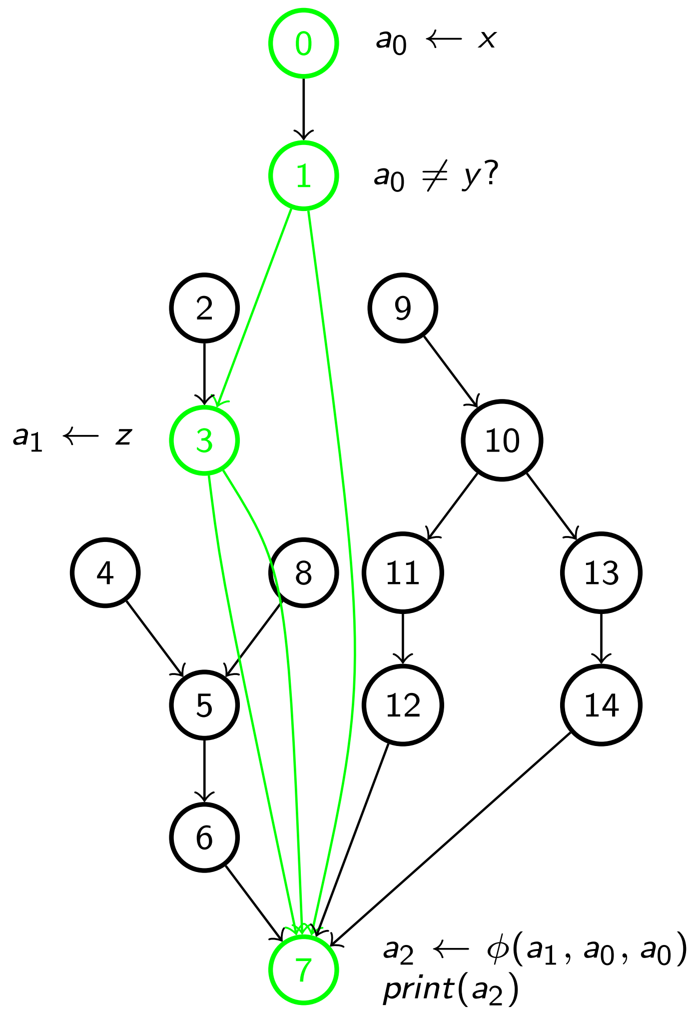
```
procedure simplify(G)
        live(e) ← true
        modified ← false
        for each vertex u ∈ G do {
                if (not live(u))
                        continue
                for each v ∈ succ(u) do {
                        if (live(v))
                                continue
                        w ← ipdom(v) /* idom in RCFG */
                        while (not live(w))
                                w ← ipdom(w)
                        replace (u, v) with (u, w)
                        update the branch in u to its new target w
                        update φ-functions in w if necessary
                        modified ← true
                }
        }
        if (modified) {
                delete vertices from G which now have become unreachable
                update dominator tree DT
        }
end
```



- Must fix $\phi(a)$ in 7.

Result of Processing 3

procedure *simplify* (G)
    live $(e) \leftarrow$ true
    modified $\leftarrow$ false
    for each vertex $u \in G$ **do** {
        if (**not** *live* (u))
            continue
        for each $v \in$ *succ* (u) **do** {
            if (*live* (v))
                continue
            $w \leftarrow$ *ipdom* (v) /* idom in RCFG */
            while (**not** *live* (w))
                $w \leftarrow$ *ipdom* (w)
            replace (u, v) with (u, w)
            update the branch in u to its new target w
            update ϕ-functions in w if necessary
            modified $\leftarrow$ true
        }
    }
    if (*modified*) {
        delete vertices from G which now have become unreachable
        update dominator tree DT
    }
end



- Later remove one $(3, 7)$!
- Keep only live vertices.