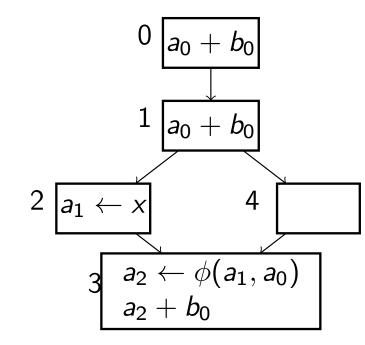
- What can PRE achieve?
- Partial Redundancy Elimination History
- Key ideas in SSAPRE from SGI

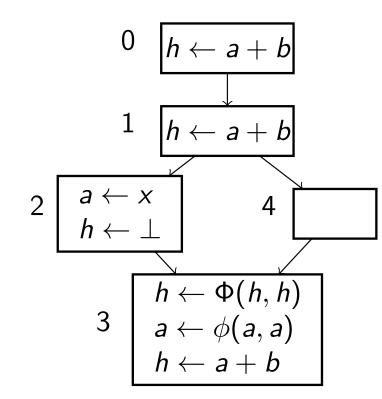
- Recall that Partial Redundancy Elimination, or **PRE**, can eliminate both **full** and **partial** redundancies.
- Full redundancies: when the expression is available from all predecessor basic blocks.
- Partial redundancies: when the expression is only available from some but not all predecessor basic blocks.
- Partial redundancy elimination also covers loops, i.e., PRE can move code out from loops.

- PRE was invented by Morel and Renvoise in 1979.
- Then Fred Chow in his PhD thesis at Stanford from 1983 (with John Hennessy as supervisor) improved it.
- In 1992 Knoop et al. published a version of PRE which is optimal in the sense of minimizing register pressure. They called their algorithm Lazy Code Motion.
- In 1999 Kennedy and Chow and others at SGI published the SSA formulation of Lazy Code Motion and called it **SSAPRE**.
- We will first study a simpler version of it and then note that there exists an efficient variant of SSAPRE which is much faster.



- Both hash-based and global value numbering can optimize the full redundancy in vertex 1.
- None of them can optimize the partial redundancy in vertex 3.

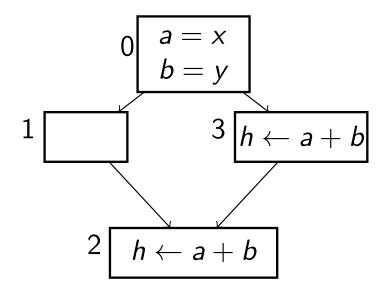
### The Key Idea of SSAPRE



- We create Φ-functions for the hypothetical variable h.
- After SSAPRE, Φ-functions become normal φ-functions and they are really the same (different notation to distinguish between them only).
- By inserting the expression a + b at Φ-operands with the value ⊥ ("bottom"), the partial redundancy in vertex 3 becomes a full redundancy and can be eliminated.

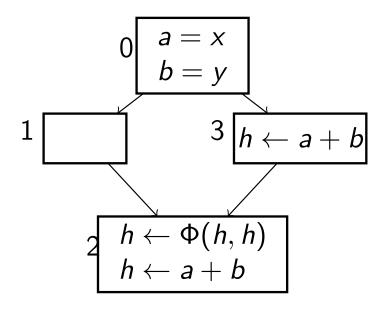
- Insert Φ-functions.
- Perform SSA-renaming for the variable *h* and **all other variables** (again).
- Compute **downsafety**, i.e. where the expression is anticipated.
- Compute can be avail, i.e. where the expression can be available, either because the expression is there or it can replace a ⊥-operand.
- Compute **later**, i.e. if can be lazy and insert the expression further down in the control flow graph.
- Perform **finalize1**, i.e. modify the code.
- Perform **finalize2**, i.e. clean up various things.

#### Insertion of $\Phi$ -functions



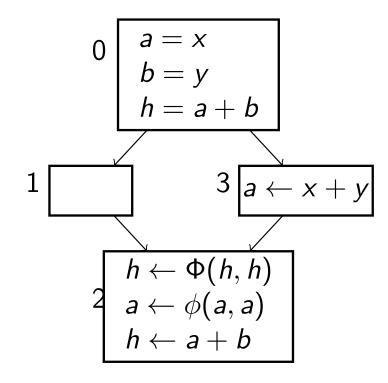
- Recall that in SSAPRE every expression assigns to a hypothetical variable *h*.
- Where should we then insert Φ-functions for *h*?
  - In the iterated dominance frontiers of all evaluations of the expression, i.e. assignment to h.
  - 2 In the iterated dominance frontiers of all assignments to operands in the expression since they mean  $h \leftarrow \bot$

# Iterated Dominance Frontiers of Evaluations of a + b



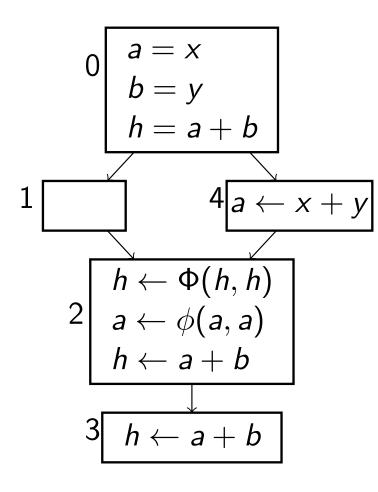
- We have already computed the dominance frontiers of each vertex.
- We thus simply have to collect the vertices which contain such an evaluation.

### Iterated Dominance Frontiers of $h \leftarrow \bot$



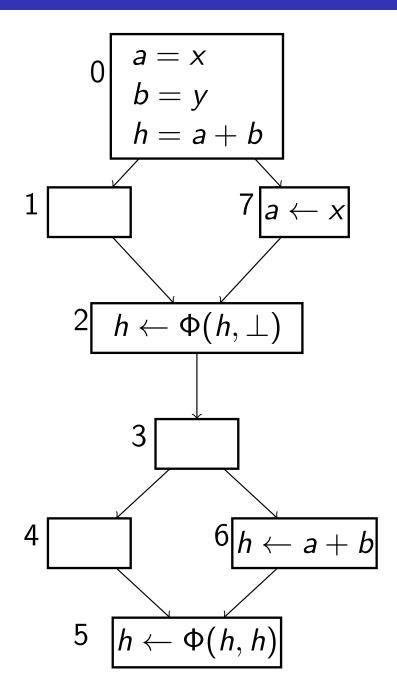
- Although we can collect all vertices with assignments to a or b, and find the iterated dominance frontiers of these, there is a simpler way.
- Every vertex for which we will insert a Φ-function due to an h ← ⊥ must contain a φ-function to any of the variables in the expression, i.e. φ(a) or φ(b).
- So we simply look for φ(a) and φ(b), and insert Φ(h) in the same vertex.
- Recall that  $\phi$ -functions are parallel copy statements.

#### Anticipated Expressions



- An expression is anticipated at a point p in the control flow graph if it is certain it will be evaluated with all operands having the same value on all paths from p.
- At the end of vertex 0, a + b is not anticipated since a might be assigned a new value in vertex 4.
- At the end of vertices 1 and 4 the expression is anticipated due to the evaluation in vertex 2 which certainly will be evaluated.
- The word "evaluated" here means "executed".

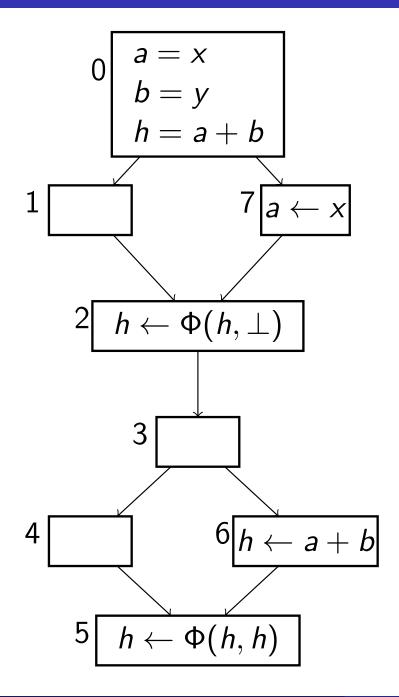
# The Main Rule of the Game of PRE



- No matter what, PRE may never transform a function so it will execute additional instructions due to PRE.
- Should the  $\perp$  in vertex 2 be replaced with  $h \leftarrow a + b$ ?
- No, it's not safe to insert the expression since the expression is not anticipated by the Φ-function.
- The path (0, 7, 2, 3, 4, 5) would execute a + b at the end of vertex 7 (for the Φ-operand) without any purpose.
- Actually, a Φ-operand is regarded as belonging to the predecessor vertex.

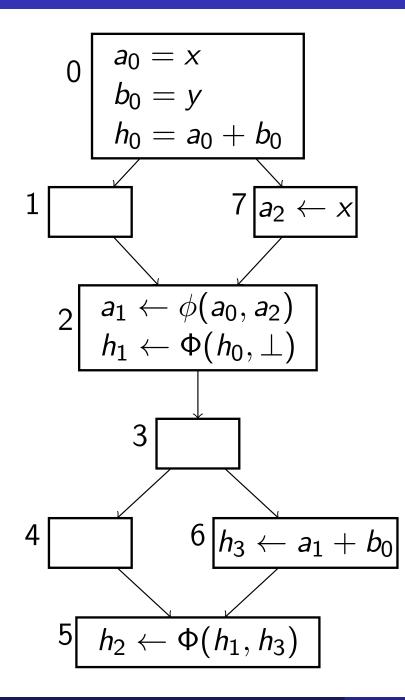
- There are three main types of so called **occurrences** of an expression:
  - **(1)** A real occurrence, i.e. the expression a + b,
  - A Φ-function occurrence, and
  - **3** A Φ-operand occurrence.
- Note that  $\Phi$ -operands are placed in the predecessor basic block.

#### Attributes of $\Phi$ -functions



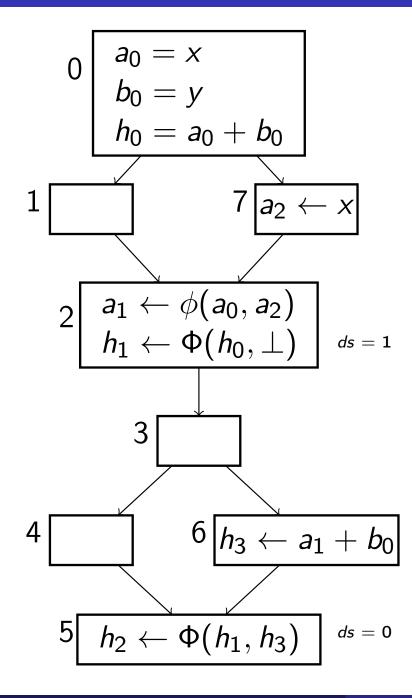
- Each Φ-function has a number of boolean attributes:
  - downsafe or ds
  - can\_be\_available or cba
  - later
  - will be available or wba
- If a Φ-function is downsafe, it's OK to replace a ⊥ operand with the expression.
- We will soon see how downsafe is computed.
- A Φ-operand has the boolean attribute
   has real use which is true if the value comes from a real occurrence.

# Renaming



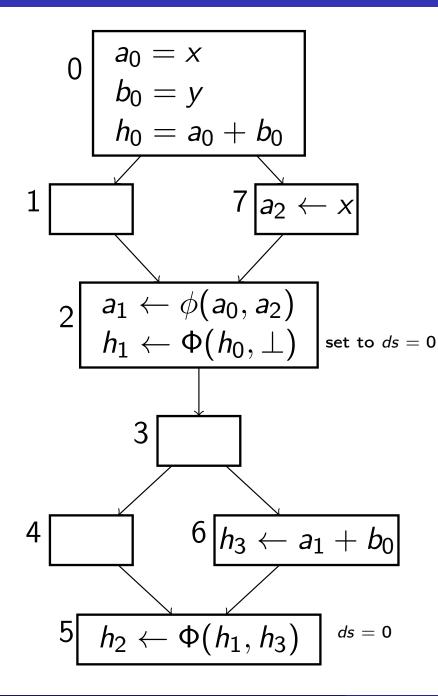
- The dominator tree is traversed.
- At a Φ-function occurrence, a new version of h is always created.
- At a Φ-operand occurrence it is noted if the value comes from a real occurrence, in which case
   has real use is set to true.
- At a real occurrence, a new version of h is created if any operand has a new version (compared to the stack of h), and then if the top of the stack of h is a Φ-function, that Φ-function is marked as ds = 0
- Both real and  $\Phi$ -function occurrences are pushed on the rename stack of h.

#### More initialization of downsafe



- In addition to the previous slide, downsafe is also set as follows.
- If there is a path from a Φ-function to the exit vertex that Φ-function is not downsafe unless the expression was evaluated.
- When renaming comes to the exit vertex, it checks the top of the stack of *h*.
- If the top is a  $\Phi$ -function, it is marked with ds = 0.

### Computing Downsafety



- After the initialization of downsafety during rename, the downsafety is computed for all Φ-functions.
- What should be done?
- A Φ-function with ds = 0 should tell other Φ-functions that also they are not downsafe!
- A Φ-function with ds = 0 and with a Φ-operand that is defined by a Φ-function and for which
   has\_real\_use = 0, should reset its downsafety and continue the recursion.
- In this example both  $\Phi$ -functions have ds = 0.

```
procedure reset downsafe(x)
    if (has real use(x) or def(x) is not a \Phi)
         return
    f \leftarrow def(x)
    if (not down safe(f))
        return
    down safe(f) \leftarrow false
    for each operand \omega of f do
         reset downsafe(\omega)
procedure downsafety
    for each f \in \mathcal{F} do
        if (not down safe(f))
             for each operand \omega of f do
                  reset downsafe (\omega)
```

```
procedure compute _{can}be_{avail}
for each f \in \mathcal{F} in the program do
_{can}be_{avail}(f) \leftarrow true
for each f \in \mathcal{F} in the program do
if (not down_safe(f)
and _{can}be_{avail}(f)
and \exists an operand of f that is \bot)
reset_can_be_avail(f)
```

end

```
procedure reset_can_be_avail(g)

can_be_avail(g) \leftarrow false

for each f \in \mathcal{F} with operand \omega with g = def(\omega) do

if (not has_real_use(\omega)

and not downsafe(f)

and can_be_avail(f))

reset_can_be_avail(f)
```

end

```
procedure reset_later(g)

later(g) \leftarrow false

for each f \in \mathcal{F} with operand \omega with g = def(\omega) do

if (later(f))

reset_later(f)
```

end

```
procedure compute_later

for each f \in \overline{\mathcal{F}} do

later(f) \leftarrow can\_be\_avail(f)

for each f \in \mathcal{F} do

if (later(f) and

\exists an operand \omega of f such that def(\omega) \neq \bot and has\_real\_use(\omega))

reset\_later(f)
```

end

procedure will\_be\_avail compute\_can\_be\_avail compute\_later end

### Finalize1

```
procedure finalize1(g)
      let E \leftarrow the current expression
      for each redundancy class x of E do
             avail def [x] = \bot
      for each occurrence \psi of E in preorder DT traversal order do
             x \leftarrow class(\psi)
             if (\psi is a \Phi occurrence) {
                    if (will be avail (\psi))
                           avail def[x] = \psi
             } else if (\psi is a real occurrence) {
                    if (avail def[x] is \perp or avail def[x] does not dominate \psi)
                           reload (\psi) \leftarrow false
                           avail def[x] = \psi
                    } else {
                           reload (\psi) \leftarrow true
                           def(\psi) \leftarrow avail def[x]
                    }
             } else {
                    /* \psi is a \Phi operand occurrence. */
                    let f be the \Phi in the successor vertex of this operand
                    if (will be avail(f)) {
                           if (\psi satisfies insert) {
                                  insert E at the end of the vertex containing \psi
                                  def(\psi) \leftarrow inserted occurrence
                           } else
                                  def(\psi) \leftarrow avail def[x]
                    }
             }
end
```