Contents of Lecture 6

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Copy Propagation During Translation to SSA Form

$$\begin{array}{|l|}\hline a_0 \leftarrow x + y \\ b_0 \leftarrow a_0 \\ c_0 \leftarrow z + 44 \\ d_0 \leftarrow b_0 + c_0 \\ \hline\end{array}$$

$$\begin{array}{|l|}\hline a_0 \leftarrow x + y \\ c_0 \leftarrow z + 44 \\ d_0 \leftarrow a_0 + c_0 \\ \hline\end{array}$$

- Instead of pushing b_0 on b's stack...
- we can push a_0 on b's stack
- Part of Lab 2.

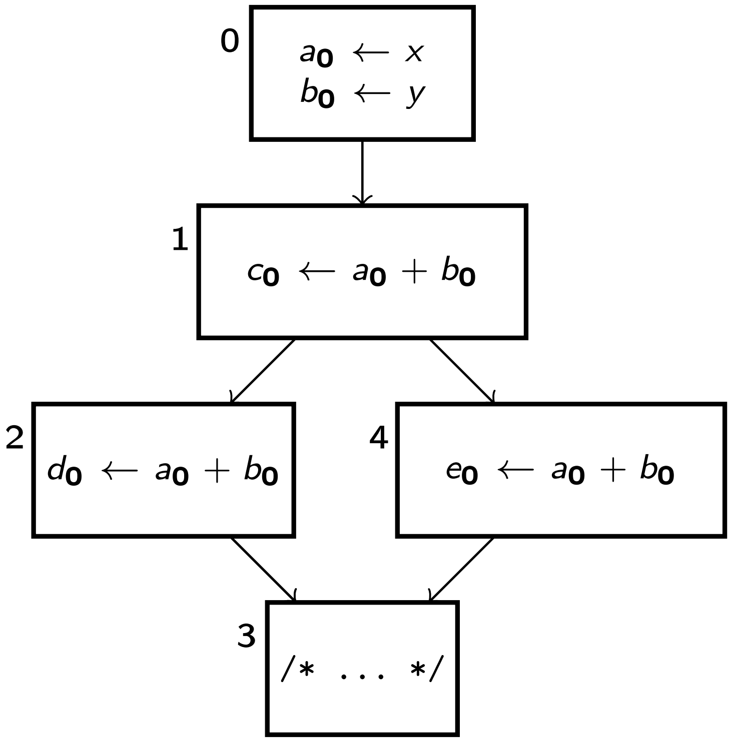
Redundancy Elimination

- An expression $a + b$ is **redundant** if it is evaluated multiple times with identical values of the operands.
- Eliminating redundant expressions is a very important optimization goal.
- There are different approaches to redundancy elimination, including
  1. Hash-Based Value Numbering
  2. Global Value Numbering
  3. Common Subexpression Elimination
  4. Code Motion out of Loops
  5. Partial Redundancy Elimination
- We will study 1, 2, and 5 in detail.

Value Numbering

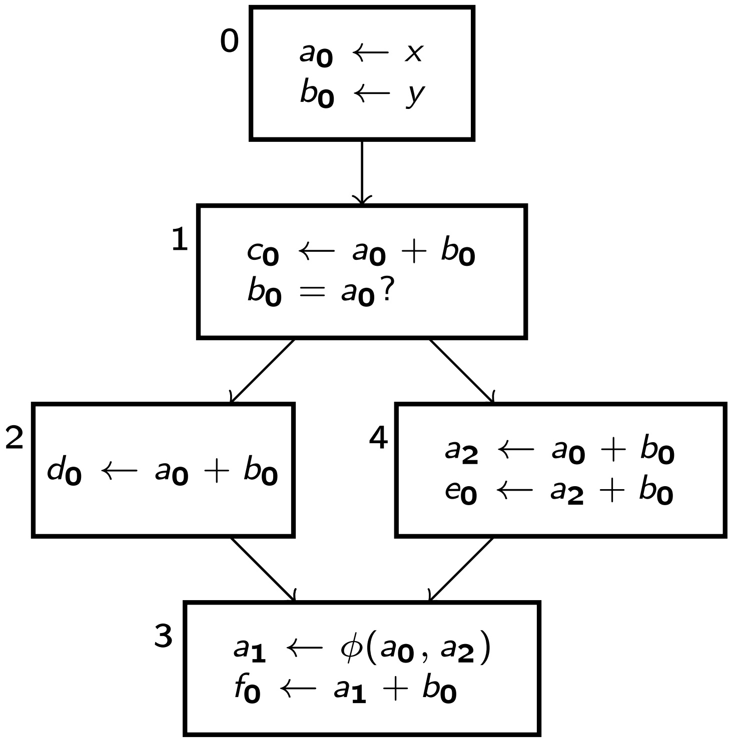
- The name is due to each expression, e.g. $t_i \leftarrow a + b$, is given a number, essentially a hash-table index.
- In subsequent occurrences of $t_j \leftarrow a + b$ it is checked whether the statement can be changed to $t_j \leftarrow t_i$.
- This is a very old optimization technique with one version that is performed during translation to SSA Form and other versions when the code already is on SSA Form.
- There are obviously older versions used before SSA Form but we will not look at them.

Example 1



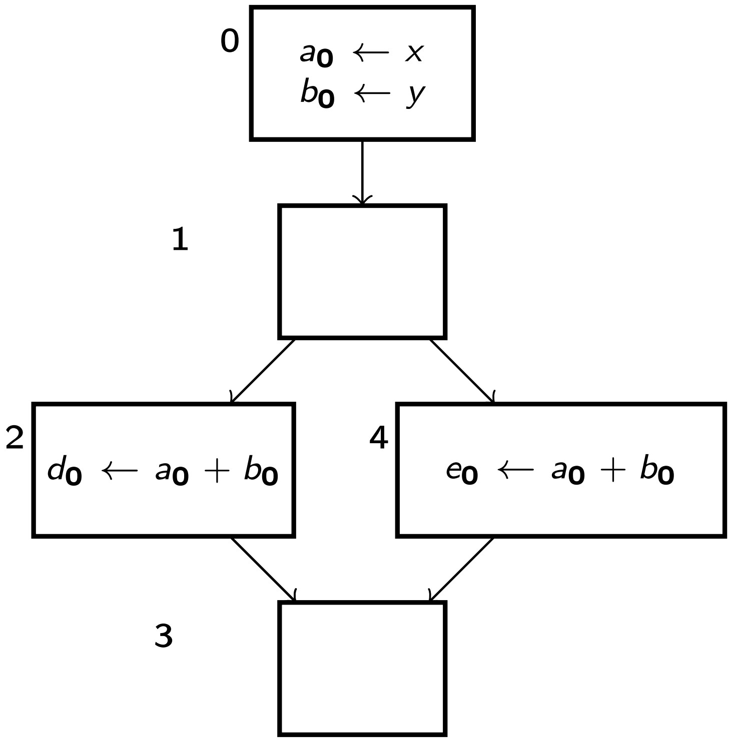
- In vertex 1 the expression $a_0 + b_0$ is first computed.
- The redundant occurrences of $a_0 + b_0$ can easily be removed.
- On SSA Form we simply check that the variable versions are the same in the current and previous occurrence.

Example 2



- The second occurrence in vertex 4 and the only in 3 cannot mistakenly be regarded as useful due to mismatching variable versions.

Example 3



- Obviously there are no redundant expressions here.
- We could perhaps save memory by computing $a_0 + b_0$ in vertex 1 but that is not a goal for redundancy elimination.
- Which data structure should we use for performing value numbering during translation to SSA Form?

Modifying the Rename Function 1(3)

```
procedure rename(w)
    oldLHS ← empty list
    enter new scope in hash table
    for each statement t in w do
        for each variable V ∈ RHS(t)
            replace use of V by use of V_i where i = top(S(V))

        V = LHS(t)
        if (V = null)
            continue
        add V to oldLHS
```

Modifying the Rename Function 2(3)

simplify t using e.g. $V_i - V_i = 0$
$h \leftarrow$ lookup $RHS(t)$ in hash table
if (h was found)
    push left-hand side of h onto $S(V)$
else {
    $i \leftarrow C(V)$
    replace V by V_i
    push i onto $S(V)$
    $C(V) \leftarrow C(V) + 1$
    install t in hash table
}

Modifying the Rename Function 3(3)

for each $v \in succ(w)$ **do**
    $j \leftarrow which\_pred(w, v)$
    for each ϕ-function in v **do**
        replace the j-th operand in $RHS(\phi)$ by V_i where $i = top(S(V))$
for each $v \in children(w)$ **do**
    $rename(v)$
for each variable V in $oldLHS$ **do**
        $pop(V)$
exit scope in hash table

Global Value Numbering (GVN)

- Global Value Numbering was one of the first optimizations presented on SSA Form, and was invented by IBM Research.
- SSA Form was explained in that paper as well to introduce this novelty to the reader.
- IBM actually uses an unpublished version of this algorithm which is better.
- Mårten Kongstad, D99, implemented this algorithm in his Master's Thesis as a pass in GCC and observed performance improvements of up to 6.1%.

Key Ideas of GVN

- Recall that in constant propagation only the start vertex is initially assumed to be executable.
- In GVN the initial assumption is that all instructions with the same operation will produce the same value.
- I.e. all adds produce the same value, etc.
- This most likely is not the case, of course.
- Then, for example, the add instructions are inspected to check whether the compiler can determine that two such instructions do not produce the same value.
- When the algorithm terminates, it has proved which instructions produce the same value.

Equivalent Instructions

- The set I of all instructions in a control flow graph is partitioned into blocks B_j, i.e. $\bigcup B_j = I$.
- Initially a block B_j consists of all instructions with the same operator and type.
- Each instruction i has a number of operands.
- Two instructions are regarded as equivalent if they belong to the same block B_j and their respective operands come from the same blocks.
- A variable x with unknown value is put in a singleton block B_x.
- ϕ-functions can be equal only if they belong to the same basic block.

An Example of Equivalent Instructions

```
a = x + y;
b = x - z;
c = x + z;
d = a - b;
e = a + d;
f = a + b;
g = b + d;
```

- We denote an instruction with the variable defined by it, and $\mathit{left}(a)$ and $\mathit{right}(a)$ denote the instruction which define the left and right operand of instruction a, respectively.
- $B_x = \{x\}$, $B_y = \{y\}$, and $B_z = \{z\}$
- $B_+ = \{a, c, e, f, g\}$
- $B_- = \{b, d\}$
- Let us check some instructions:
  - $\mathit{left}(e) \in B_+$ and $\mathit{left}(f) \in B_+$ and $\mathit{right}(e) \in B_-$ and $\mathit{right}(f) \in B_-$ so we still think $e \equiv f$.
  - $\mathit{left}(f) \in B_+$ and $\mathit{left}(g) \in B_-$ so $f \not\equiv g$.
- What should we do when we have discovered that two instructions from the same block cannot be equivalent?

Splitting Blocks

```
a = x + y;
b = x - z;
c = x + z;
d = a - b;
e = a + d;
f = a + b;
g = b + d;
```

- We just discovered that $f \neq g$.
- Therefore we split B_+ into B'_+ and B''_+.
- $B_x = \{x\}$, $B_y = \{y\}$, and $B_z = \{z\}$
- $B'_+ = \{a, c, e, f\}$
- $B''_+ = \{g\}$
- $B_- = \{b, d\}$
- Thus, we split blocks when we discover that two members cannot be equivalent due to their respective operands come from different blocks.
- How should we practically perform the splitting?

The Basic Algorithm

```
procedure N²-partition
    let π₀ = {B₀, B₁, B₂, ..., Bₚ} be the initial partition
    i ← 0
    change ← true
    while (change) do
        change ← false
        k ← 0
        for each Bⱼ ∈ πᵢ do
            take one node v from Bⱼ
            create a new block Bₖ in πᵢ₊₁
            put v in Bₖ in πᵢ₊₁
            for each node w ∈ Bⱼ do
                if (match(v, w))
                    add w to Bₖ in πᵢ₊₁
                else {
                    if (Bₖ₊₁ has not already been created)
                        create a new block Bₖ₊₁ in πᵢ₊₁
                        change ← true
                    add w to Bₖ₊₁ in πᵢ₊₁
                }
end
```

Redundancy Elimination

- All instructions in a block B_k in the final partition π produce the same value.
- Suppose a and b are members of B_k.
- Using dominance at the instruction level, if $a \gg b$ then b is redundant and can be replaced with a.

Inefficiency of this Algorithm

- Assume we have the statement:
  `b = a[0] + a[1] + a[2] + ... + a[n];`
- All the add instructions will belong to the same block in π_0.
- Then one instruction is removed each iteration which results in an N^2 algorithm.
- This algorithm is too slow in practice and we will next look at a faster algorithm.
- The main problem with the N^2-algorithm is that a block is used to split itself by inspecting all its members.

A Key Idea for a Faster Algorithm

- Instead of taking one block B_k and inspect every instruction in it, and either putting it in B'_k or in B''_k we can take one block and use it to split other blocks. Each block is given a sword.
- To simplify the description let us for the moment only consider unary operators.
- Consider blocks B_i and B_j and whether B_j should be split due to B_i.
- Assume some of the members of B_j take their operand from B_i while some others don't.
- Then B_j must be split, and those with operands from B_i should be put in B'_j and the rest in B''_j.
- $INV(B_i)$ is the set of instructions which take their operand from B_i.
- If $INV(B_i) \cap B_j \neq \emptyset$ and $B_j \not\subseteq INV(B_i)$ then B_j must be split.

More Details About Splitting

- Consider a block B_j with instructions.
- Let $x \in B_j$ and assume x only operand was defined in B_i.
- We write this as $f(x) \in B_i$. $INV(B_i) = \{ \ v \mid f(v) \ \in \ B_i \ \}$.
- We might split B_j due to B_i into B'_j and B''_j.
- $B'_j = \{ \ v \mid v \in B_j \wedge f(v) \in B_i \ \}$ and $B''_j = \{ \ v \mid v \in B_j \wedge f(v) \notin B_i \ \}$.
- Recall B_j is split due to B_i if $INV(B_i) \cap B_j \neq \emptyset$ and $B_j \not\subseteq INV(B_i)$
- If $INV(B_i) \cap B_j = \emptyset$ then B_j is completely unrelated to B_i, and B_j should therefore not be split due to B_i.
- If $B_j \subseteq INV(B_i)$ then all instructions in B_j take their operand from B_i, and B_j should therefore not be split due to B_i.

A Worklist

- Every initial block B_j is put on a worklist W.
- B_i is taken from W and $INV(B_i)$ is computed.
- Then all other blocks are inspected and split if: $INV(B_i) \cap B_j \neq \emptyset$ and $B_j \not\subseteq INV(B_i)$
- A block B_i on the worklist W is equipped with a sword to cut all other blocks into two pieces.
- If a split block B_j also was on W, then both pieces of B_j, i.e. B_j' and B_j'', will remain on W (but not B_j obviously).
- What should we do if B_j is not on the worklist?

Splitting a Block B_j not on the Worklist

- Assume $B_j \notin W$.
- It means B_j already has had its chance to cut other blocks.
- However, when B_j is split into B_j' and B_j'' some block B_k might now have to be split!
- Do we have to put both B_j' and B_j'' into W?
- Assume that $\forall v \in B_k \; f(v) \in B_j$. Then, if B_j is split into B_j' and B_j'', we must either have $f(v) \in B_j'$ or $f(v) \in B_j''$, hence $f(v) \in B_j' \;\Leftrightarrow\; f(v) \notin B_j''$. Therefore, to split B_k we can use either B_j' or B_j'':

$$\{ \; v \mid v \in B_k \land f(v) \in B_j' \; \} = B_k - \{ \; v \mid v \in B_k \land f(v) \in B_j'' \; \}$$
$$\{ \; v \mid v \in B_k \land f(v) \in B_j'' \; \} = B_k - \{ \; v \mid v \in B_k \land f(v) \in B_j' \; \}$$

- By using the smaller of the sets B_j' and B_j'' we can achieve a time complexity of $O(NlogN)$ where N is the number of nodes in the value graph.

The Power of Global Value Numbering

```
int h(int a, int b)
{
    int x, y;

    x = 1;
    y = 1;
    do {
        a = a + b;
        x = x + a;
        y = y + a;
    } while (a > 0);
    return x + y;
}
```

```
int h(int a, int b)
{
    int x, y;

    x = 1;
    do {
        a = a + b;
        x = x + a;
    } while (a > 0);
    return x + x;
}
```