Type Systems Course

Derivation (Chapter 3)

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Syntax & Evaluation Rules

Example from TAPL (p.34):

$$\frac{}{\text{if } \boxed{\text{true}} \text{ then } t_2 \text{ else } t_3 \rightarrow t_2}\ (\text{E-IFTRUE})$$

t ::= true
| false
| if t then t else t

$$\frac{}{\text{if } \boxed{\text{false}} \text{ then } t_2 \text{ else } t_3 \rightarrow t_3}\ (\text{E-IFFALSE})$$

$$\frac{t \rightarrow t'}{\text{if } \boxed{t} \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } \boxed{t'} \text{ then } t_2 \text{ else } t_3}\ (\text{E-IF})$$

Constructing Derivation Trees

Let's start with something simple ...

$$\frac{}{\text{if } \boxed{\text{true}} \text{ then false else true} \rightarrow \text{false}}\,(\text{E-IFTRUE})$$

$$\frac{}{\text{if } \boxed{\text{false}} \text{ then false else true} \rightarrow \text{true}}\,(\text{E-IFFALSE})$$

Note that this is the same as:

$$(\text{if } \boxed{\text{true}} \text{ then false else true}, \text{false}) \in \rightarrow$$

$$(\text{if } \boxed{\text{false}} \text{ then false else true}, \text{true}) \in \rightarrow$$

Constructing Derivation Trees

Derivation trees become more interesting when there is at least one premise, e.g.,

$$\dfrac{\dfrac{}{\text{if true then true else true} \rightarrow \text{true}}\,(\text{E-IFTRUE})}{\text{if } \text{if true then true else true} \text{ then false else true} \rightarrow \text{if } \text{true} \text{ then false else true}}\,(\text{E-IF})$$

Again, there is a different representation, i.e., given

$$(\text{if true then true else true}, \text{true}) \in \rightarrow$$

we have

$$(\text{if } \boxed{\text{if true then true else true}} \text{ then false else true},$$
$$\text{if } \boxed{\text{true}} \text{ then false else true}) \in \rightarrow$$

Constructing Derivation Trees

The example from TAPL (p.36):

$$s \stackrel{\text{def}}{=} \text{if true then false else false}$$

$$t \stackrel{\text{def}}{=} \text{if } s \text{ then true else true}$$

$$u \stackrel{\text{def}}{=} \text{if false then true else true}$$

$$\dfrac{\dfrac{\dfrac{}{\text{if true then false else false} \rightarrow \text{false}}\,(\text{E-IFTRUE})}{\text{if } s \text{ then true else true} \rightarrow \text{if false then true else true}}\,(\text{E-IF})}{\text{if } t \text{ then false else true} \rightarrow \text{if } u \text{ then false else false}}\,(\text{E-IF})$$

For this rule schema, keep going upwards until you hit E-IFTRUE or E-IFFALSE