State-copying and Recomputation in Parallel Constraint Programming with Global Constraints

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## Problem declaration

- \( X \in \{0..9\} \)
- \( Y \in \{0..9\} \)
- \( X < Y \)
Constraint Programming (CP)

Problem declaration

\[
X, Y \in \{0..9\} \quad X < Y
\]

Store

\[
X, Y \in \{0..9\} \quad X < Y
\]

Solver

Create
Constraint Programming (CP)

Problem declaration

\begin{align*}
X &\in \{0..9\} \\
Y &\in \{0..9\} \\
X &< Y
\end{align*}

Store

\begin{align*}
X &\in \{0..9\} \\
Y &\in \{0..9\} \\
X &< Y
\end{align*}

Solver

\[X = 4\]
Constraint Programming (CP)

Problem declaration
\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Create

Store
\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Decision

Consistency
\[ X \in \{4\} \]
\[ Y \in \{5..9\} \]
\[ X < Y \]

Solver
\[ X = 4 \]
Constraint Programming (CP)

Problem declaration

\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Create

Store

\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Decision

Consistency

Solver

\[ X = 4 \]

\[ Y = 5 \]
Constraint Programming (CP)

Problem declaration

\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Create

Store

\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Decision

Solver

\[ X = 4 \]
\[ Y = 5 \]

Consistency

Consistency
Constraint Programming (CP)

Problem declaration

\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Store

\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]

Solver

\[ X = 4 \]

Solution

\[ X = 4 \]
\[ Y = 5 \]
Depth First Search in CP

\[ X \in \{0..9\} \]
\[ Y \in \{0..9\} \]
\[ X < Y \]
Depth First Search in CP

\[ X \in \{0..9\} \quad Y \in \{0..9\} \quad X < Y \]

Consistency checking

\[ X \in \{0..8\} \quad Y \in \{1..9\} \quad X < Y \]
Depth First Search in CP

Consistency checking

$X \in \{0..9\}$
$Y \in \{0..9\}$
$X < Y$

$X \in \{0..8\}$
$Y \in \{1..9\}$
$X < Y$

$X = 4$

$X \in \{4\}$
$Y \in \{1..9\}$
$X < Y$
Depth First Search in CP

Consistency checking

X ∈ {0..9}
Y ∈ {0..9}
X < Y

X ∈ {0..8}
Y ∈ {1..9}
X < Y

X = 4

X ∈ {4}
Y ∈ {1..9}
X < Y

X ∈ {4}
Y ∈ {5..9}
X < Y
Depth First Search in CP

Consistency checking

$X \in \{0..9\}$
$Y \in \{0..9\}$
$X < Y$

$X \in \{0..8\}$
$Y \in \{1..9\}$
$X < Y$

$X = 4$

$X \neq 4$

Consistency

$X \in \{4\}$
$Y \in \{1..9\}$
$X < Y$

$X \in \{0..3, 5..9\}$
$Y \in \{1..9\}$
$X < Y$

$X \in \{4\}$
$Y \in \{5..9\}$
$X < Y$
Depth First Search in CP

Consistency checking

X \in \{0..9\}
Y \in \{0..9\}
X < Y

X \in \{0..8\}
Y \in \{1..9\}
X < Y

X = 4
X \neq 4

Consistency

X \in \{4\}
Y \in \{1..9\}
X < Y

X \in \{0..3, 5..9\}
Y \in \{1..9\}
X < Y

Consistency

X \in \{4\}
Y \in \{5..9\}
X < Y

X \in \{0..3, 5..9\}
Y \in \{1..9\}
X < Y
Depth First Search in CP

The search tree changes shape during the search
# Models of Communication

<table>
<thead>
<tr>
<th></th>
<th>Local machine</th>
<th>Remote machine</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State-copying</strong></td>
<td>Variables</td>
<td>Variables</td>
</tr>
<tr>
<td></td>
<td>Domains</td>
<td>Domains</td>
</tr>
<tr>
<td></td>
<td>Constraints</td>
<td>Constraints</td>
</tr>
<tr>
<td><strong>State-recomputation</strong></td>
<td>Variables</td>
<td>Variables</td>
</tr>
<tr>
<td></td>
<td>Domains</td>
<td>Domains</td>
</tr>
<tr>
<td></td>
<td>Constraints</td>
<td>Constraints</td>
</tr>
</tbody>
</table>
Models of Communication

State-copying

Local machine
- Variables
- Domains
- Constraints

Remote machine
- Variables
- Domains
- Constraints

State-recomputation

Local machine
- Variables
- Domains
- Constraints

Remote machine
- Variables
- Domains
- Constraints
Models of Communication

State-copying

Local machine
- Variables
- Domains
- Constraints

Remote machine
- Variables
- Domains
- Constraints

State-recomputation

Local machine
- Variables
- Domains
- Constraints

Remote machine
- Variables
- Domains
- Constraints

Replace references
Models of Communication

State-copying

- Local machine: Variables, Domains, Constraints
- Remote machine: Variables, Domains, Constraints
  - Replace references

State-recomputation

- Local machine: Variables, Domains, Constraints
- Remote machine: Assignments performed, Variables, Domains, Constraints
Models of Communication

State-copying

- Local machine
  - Variables
  - Domains
  - Constraints

- Remote machine
  - Variables
  - Domains
  - Constraints

State-recomputation

- Local machine
  - Variables
  - Domains
  - Constraints

- Remote machine
  - Variables
  - Domains
  - Constraints
  - Assignments performed
  - Perform assignments, compute consistency
  - Replace references
Problem

• Copying sometimes too slow
• Recomputation not always faster
Our Solution: *Dual Com*

Can send work
Our Solution: *Dual Com*

- Can send work
- Is the network busier than the assignment list is long?
Our Solution: *Dual Com*

- Can send work
- Is the network busier than the assignment list is long?
  - Yes
    - Use recomputation
Our Solution: Dual Com

Can send work

Is the network busier than the assignment list is long?

Yes

Use recomputation

No

Use copying
Experiment Setup

- Benchmark Problems: Golomb, n-Queens
- Cluster of AMD Opteron 2.2 GHz CPUs
- 1 MB cache per processor
- Gigabit Ethernet network
- CentOS Linux 4.4
Experiment: Optimal Golomb Ruler

Proving the optimality with $n = 12$

<table>
<thead>
<tr>
<th>Number of Processors</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
</tr>
</tbody>
</table>

- **Copying**
- **Recomputation**
- **Dual Com**
Experiment: n-Queens

Finding all solutions with n = 15

Number of Processors

Copying  Recomputation  Dual Com

Speed-up

Finding all solutions with n = 15

Number of Processors

Copying  Recomputation  Dual Com

Speed-up

Finding all solutions with n = 15

Number of Processors

Copying  Recomputation  Dual Com

Speed-up
Benefit of Global Constraints

\[ X \neq Y \neq Z \rightarrow \text{alldifferent}\{X,Y,Z\} \]

<table>
<thead>
<tr>
<th>Execution Time (s)</th>
<th>Copying</th>
<th>Recomputation</th>
<th>Dual Com</th>
</tr>
</thead>
<tbody>
<tr>
<td>640</td>
<td></td>
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<tr>
<td>512</td>
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<tr>
<td>0</td>
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</tr>
</tbody>
</table>

Results for 32 Processors

- Binary Queens
- Global Queens
Conclusions

- Dual Com faster than copying or recomputation
- Global constraints doubles the performance
- Global constraints increases benefit of Dual Com