Active Logic

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May 22, 2017

Active logic

- Integer labelled logic
- Introduced by Elgot-Drapkin and Perlis

Short recap of logics

- Propositional logic: $\alpha \wedge \beta \rightarrow \neg\gamma$

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- Propositional logic: $\alpha \wedge \beta \rightarrow \neg\gamma$
- First order / Predicate logic: $\forall x : p(x) \rightarrow q(x)$
- Modal logic: $\Box\alpha \wedge \Diamond\beta \rightarrow \Diamond\gamma$

Active logic

- Labelled formulae

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- Rules of inference

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- Labelled formulae
- Rules of inference

$$\frac{\overbrace{i:}^{\textit{label}} \quad \overbrace{\alpha, \alpha \to \beta}^{\textit{premises}}}{\underbrace{i+1:}_{\textit{label}} \quad \underbrace{\beta}_{\textit{conclusion}}}$$

Active logic

Basic rules of inference:

$$\frac{i : \ldots}{i+1 : Now(i+1)}$$

$$\frac{i : A, A \rightarrow B}{i+1 : B}$$

$$\frac{i : A}{i+1 : A}$$

Uses for active logic

- Reasoning within time
- Reasoning about time
- Controlling expansion

Reasoning about time

The *Three Wise Men Problem*

- All three can see each others hats
- At least one blue hat

Reasoning about time

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Solution:

Reasoning about time

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Reasoning about time

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Solution:

- No two white hats: the one who saw them would know
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Therefore: three blue hats

The Theorem Prover

- Developed by Victor Nilsson in 2010
- Written in Prolog

My work

- Modularizing rules

My work

- Modularizing rules
- Extending theorem prover

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- Modularizing rules
- Extending theorem prover
- Extending belief structure

My work

- Modularizing rules
- Extending theorem prover
- Extending belief structure
- Creating rules for multiple knowledge bases

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$$\frac{\mathcal{K}_\alpha^i A, \mathcal{K}_\alpha^i A \rightarrow B}{\mathcal{K}_\alpha^{i+1} B}$$

Questions?