Non-classical search algorithms
by Stuart Russell

modified by Jacek Malec for LTH lectures
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Chapter 4 of AIMA

Outline

◇ Hill-climbing

◇ Simulated annealing (briefly)

◇ Genetic algorithms (briefly)

◇ Local search in continuous spaces (briefly)

◇ Searching with nondeterministic actions (briefly)

◇ Searching with partial observations (briefly)

◇ Online search and unknown environments (briefly)

Iterative improvement algorithms

In many optimization problems, **path** is irrelevant;
the goal state itself is the solution

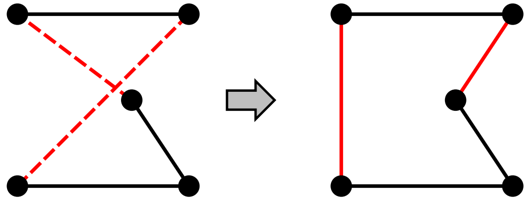
Then state space = set of "complete" configurations;
find **optimal** configuration, e.g., TSP
or, find configuration satisfying constraints, e.g., timetable

In such cases, can use iterative improvement algorithms;
keep a single "current" state, try to improve it

Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

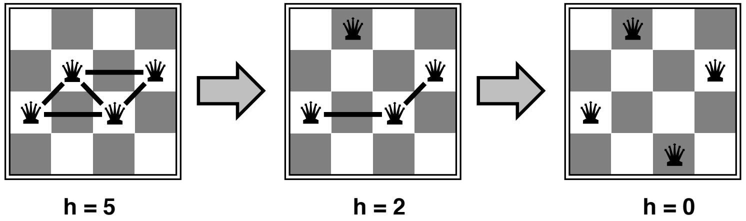


Variants of this approach get within 1% of optimal very quickly with thousands of cities

Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Move a queen to reduce number of conflicts



Almost always solves n-queens problems almost instantaneously for very large n, e.g., $n = 1 million$

Hill-climbing (or gradient ascent/descent)

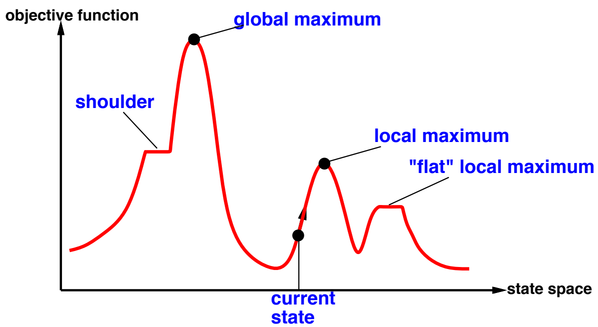
"Like climbing Everest in thick fog with amnesia"

```
function Hill-Climbing(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← Make-Node(Initial-State[problem])
   loop do
       neighbor ← a highest-valued successor of current
       if Value[neighbor] ≤ Value[current] then return State[current]
       current ← neighbor
   end
```


Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves escape from shoulders loop on flat maxima

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
but gradually decrease their size and frequency

```
function Simulated-Annealing(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← Make-Node(Initial-State[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← Value[next] – Value[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

Properties of simulated annealing

At fixed "temperature" T, state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\Longrightarrow$ always reach best state x^*
because $e^{\frac{E(x^*)}{kT}}/e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small T

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!
Searches that find good states recruit other searches to join them

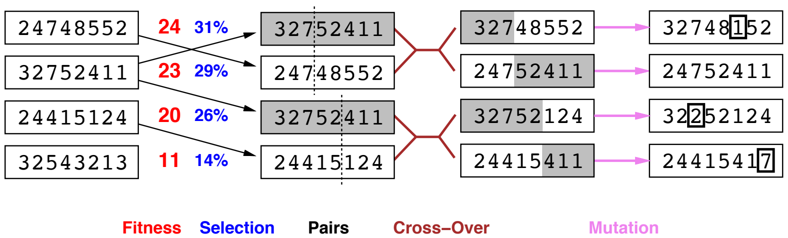
Problem: quite often, all k states end up on same local hill

Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic algorithms

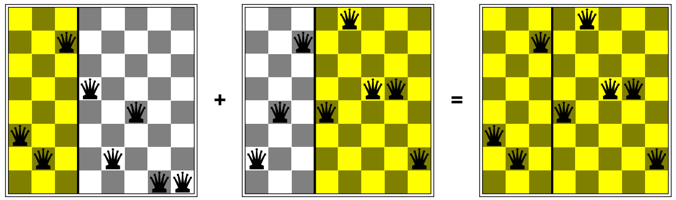
= stochastic local beam search + generate successors from **pairs** of states



Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)

Crossover helps **iff substrings are meaningful components**



GAs $\neq$ evolution: e.g., real genes encode replication machinery!

Continuous state spaces

Suppose we want to site three robot battery loading stations in the hospital:
- 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
  sum of squared distances from each location to nearest loading station

Discretization methods turn continuous space into discrete space,
e.g., empirical gradient considers $\pm\delta$ change in each coordinate

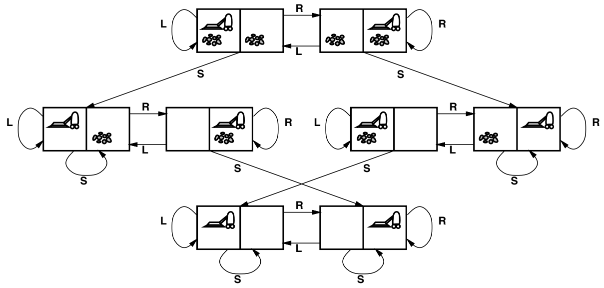
Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one location).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x}) \nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$

Searching with nondeterministic actions



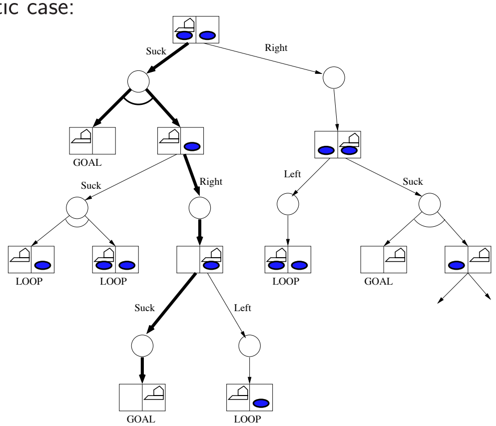
Erratic vacuum world: modified **Suck**;

Slippery vacuum world: modified **Right** and **Left**.

Searching with nondeterministic actions

And-or search trees

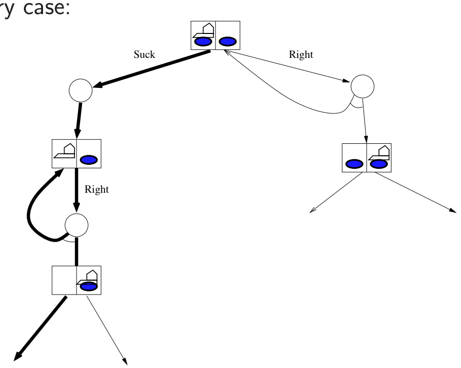
For the erratic case:



Searching with nondeterministic actions

And-or search trees

For the slippery case:



Searching with nondeterministic actions

```
function And-Or-Graph-Search(problem) returns a cond. plan, or failure
   Or-Search(problem.Initial-State, problem, [])

function Or-Search(state, problem, path) returns a conditional plan or failure
   if problem.Goal-Test(state) then return the empty plan
   if state is on path then return failure
   for each action in problem.Actions(state) do
      plan ← And-Search(Results(state, action), problem, [state | path])
      if plan ≠ failure then return [action | plan]
   return failure

function And-Search(states, problem, path) returns a conditional plan or failure
   for each s_i in states do
      plan_i ← Or-Search(s_i, problem, path)
      if plan_i = failure then return failure
   return [if s_1 then plan_1 else if s_2 then plan_2 else if ... plan_{n−1} else plan_n ]
```


Searching with partial observations

◇ no-information case:

sensorless problem, or

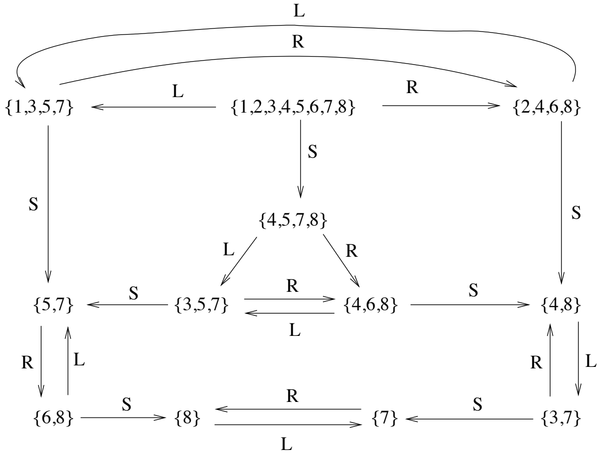
conformant problem

◇ state-space search is made in **belief space**

◇ Problem solving: and-or search!

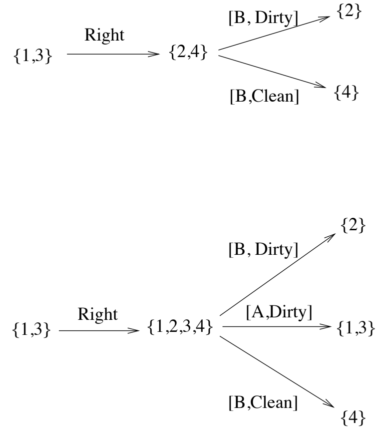
Searching with partial observations

Deterministic case:



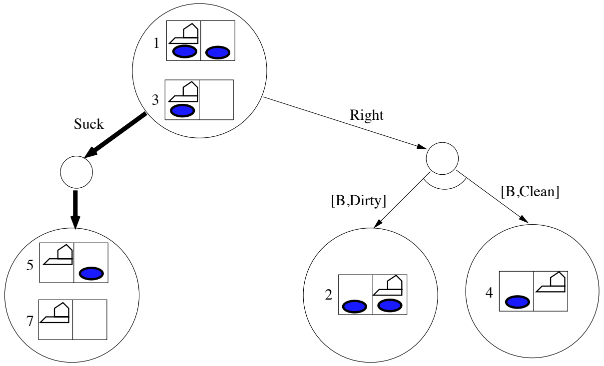
Searching with partial observations

Local sensing, deterministic and slippery cases:



Searching with partial observations

Planning for the local sensing case:



Online search and unknown environments

Interleaving computations and actions:

♢ act

♢ observe the results

♢ find out (compute) next action

Useful in dynamic domains.

Online search usually exploits locality of depth-first-like methods.

♢ random walk

♢ modified hill-climbing

♢ Learning Real-Time A* (LRTA*)

optimism under uncertainty
(unexplored areas assumed to lead to goal with least possible cost)