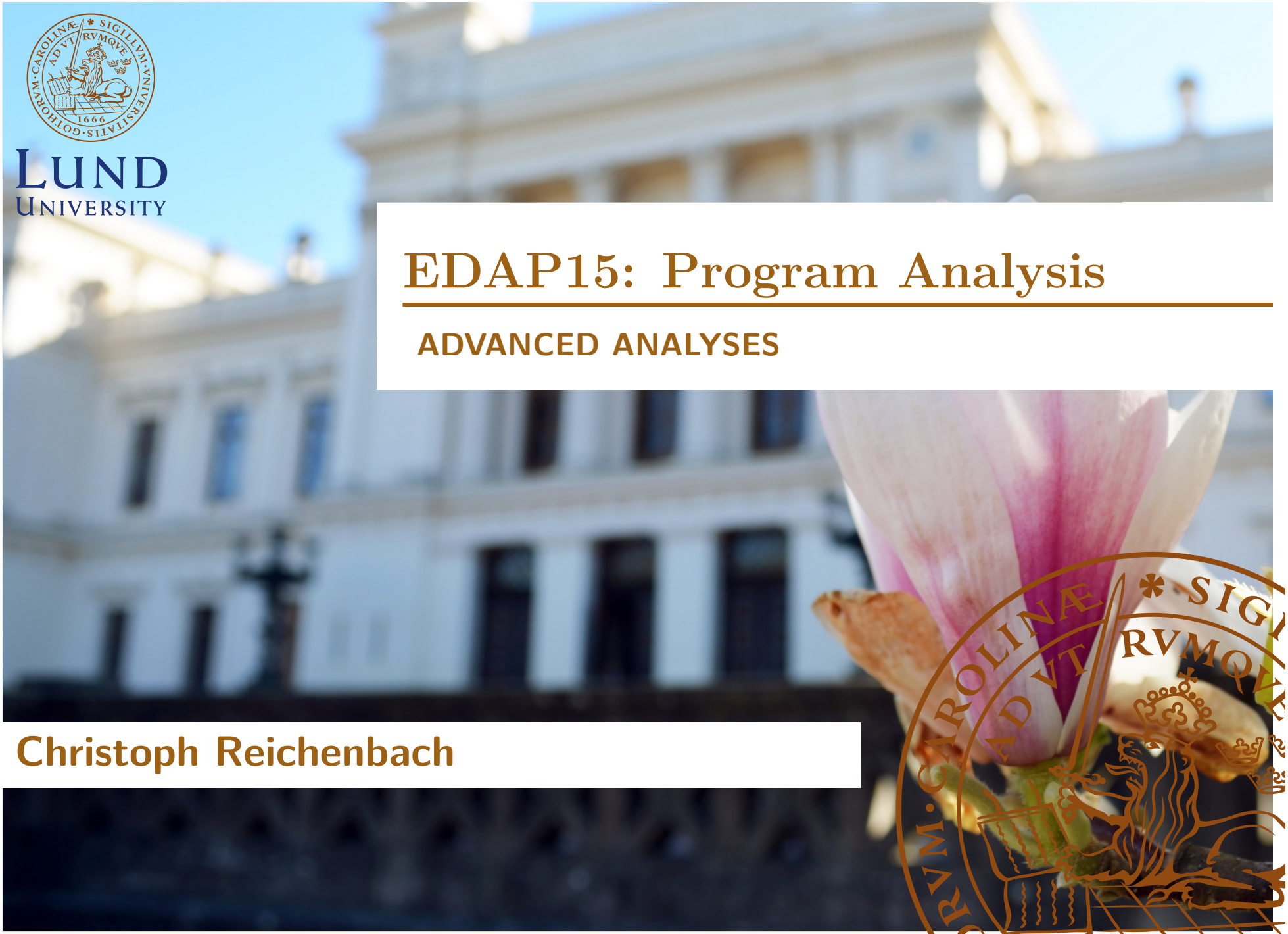

LUND UNIVERSITY

EDAP15: Program Analysis

ADVANCED ANALYSES

Christoph Reichenbach

Andersen's Points-To Analysis

- Asymptotic performance is $O(n^3)$
- More precise than Steensgaard's analysis
- *Subset-based* (a.k.a. *inclusion-based*)
- $\implies$ Flow-sensitive but *directed*
- Popular as basis for current points-to analyses

L. Andersen, "Program Analysis and Specialization for the C Programming Language", PhD. thesis, DIKU report 94/19, 1994

Collecting Constraints

- Collect constraints, resolve as needed
- For each statement in program, we record:
  - If **Referencing** (`x := new`$_{\ell_i}$ `A()`):

$$\ell_i \in pts(\mathtt{x}) \qquad (x \rightarrow \ell_i)$$

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Subset Constraints (1/2)

- Solving $pts(x) \supseteq pts(y)$

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y := new_ℓ0();
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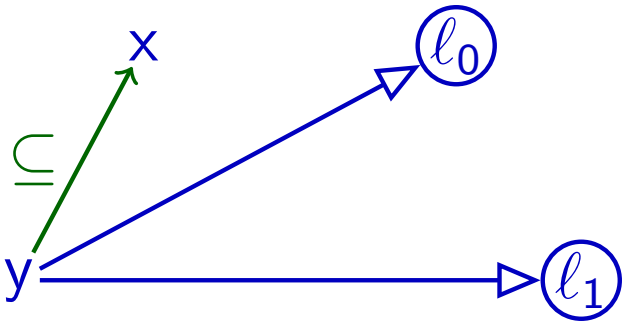

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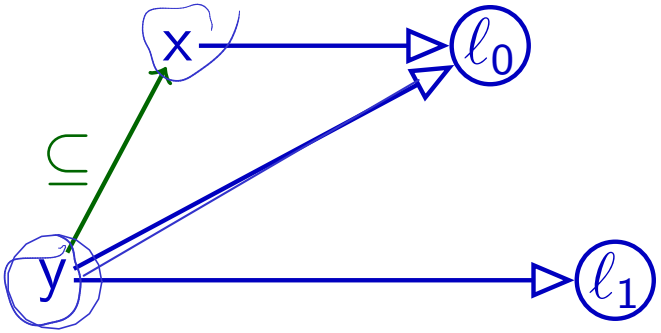

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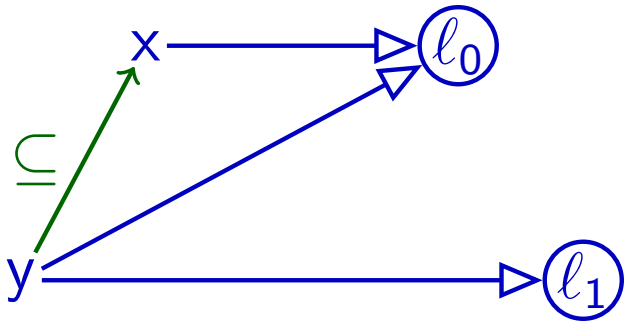

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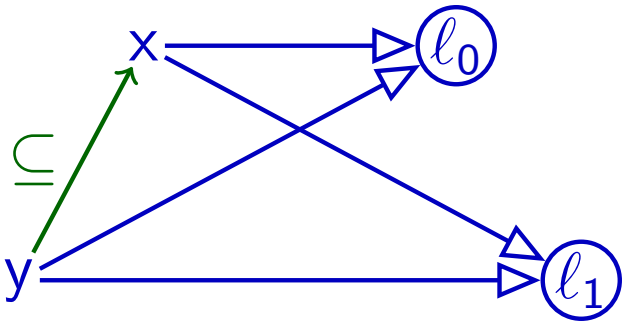

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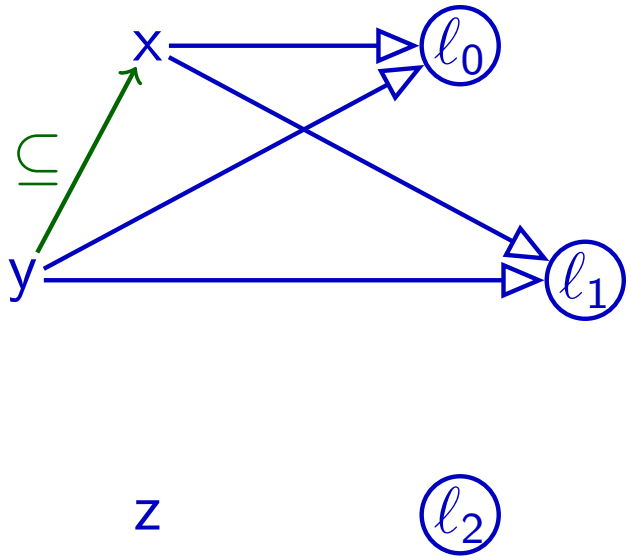

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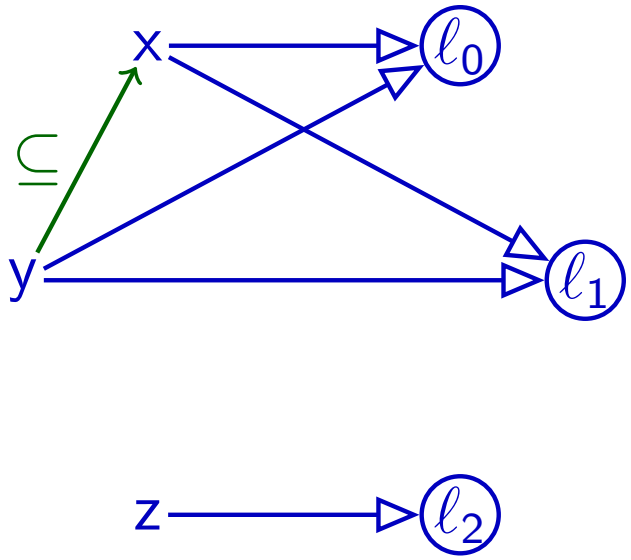

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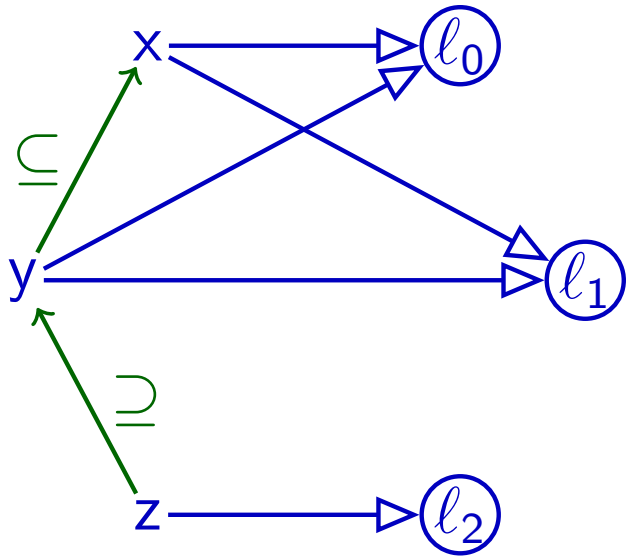

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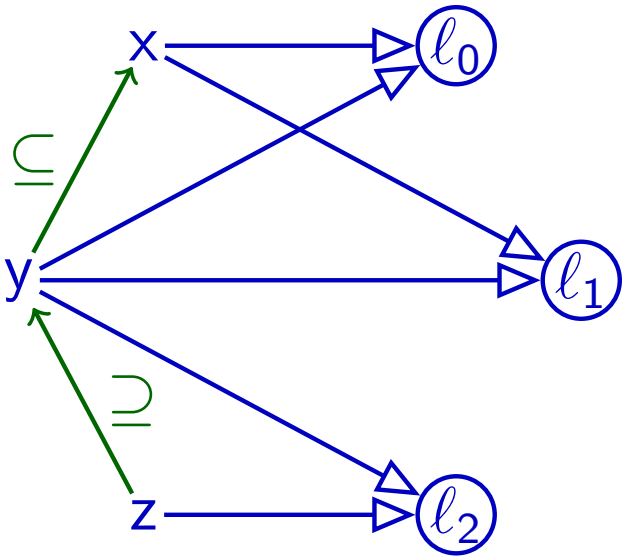

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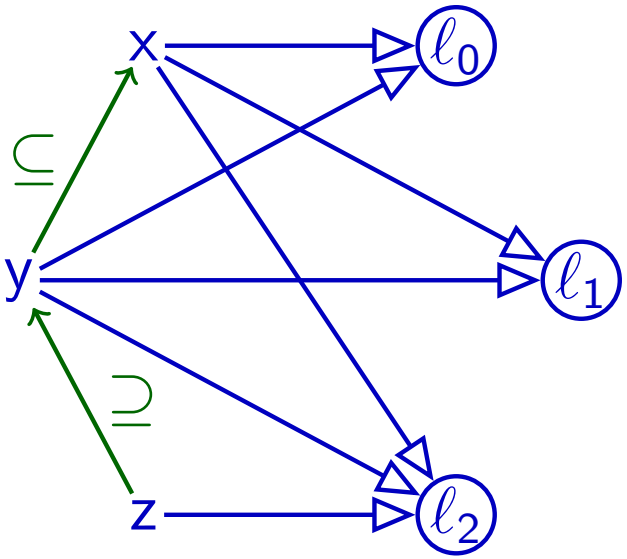

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Solving one ($\supseteq$) can depend on all ($\leftarrow$) and ($\supseteq$) in program

Subset Constraints (2/2)

- Solving $pts(x) \supseteq pts(y.\square)$

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y := new_ℓ0();
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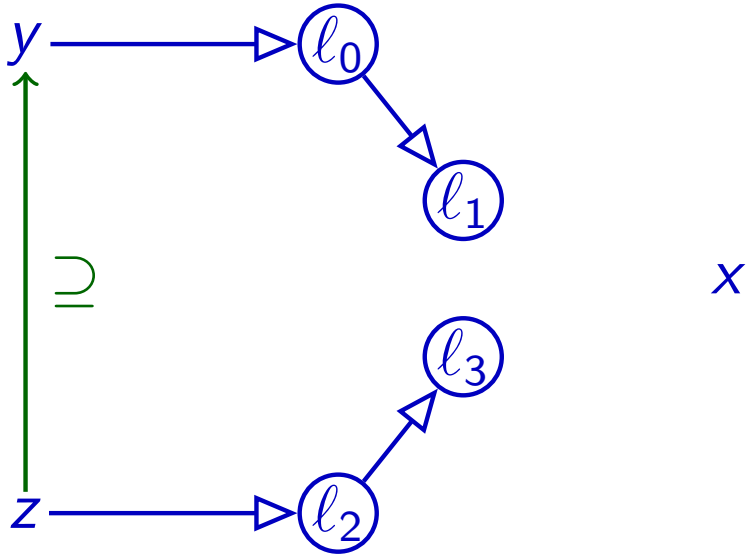

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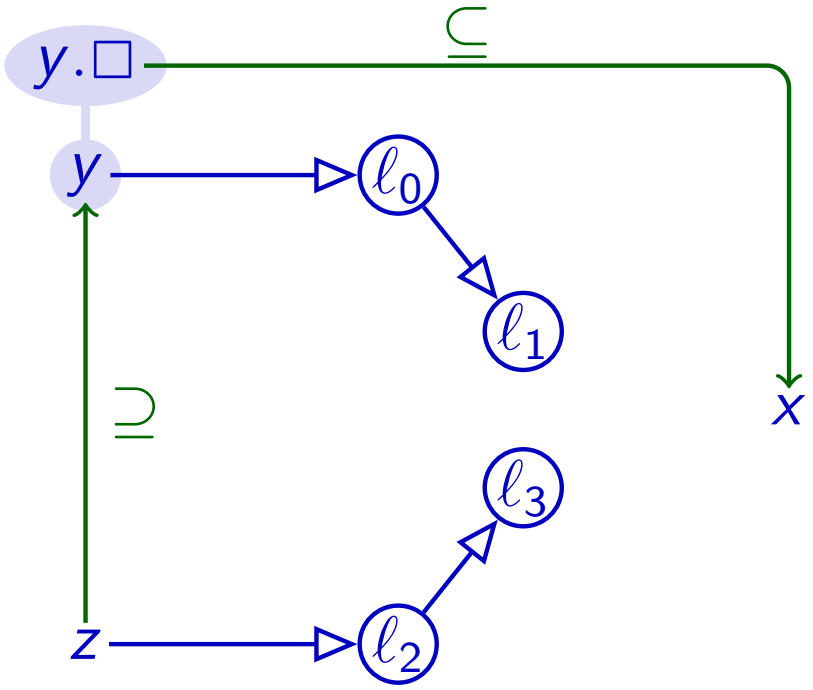

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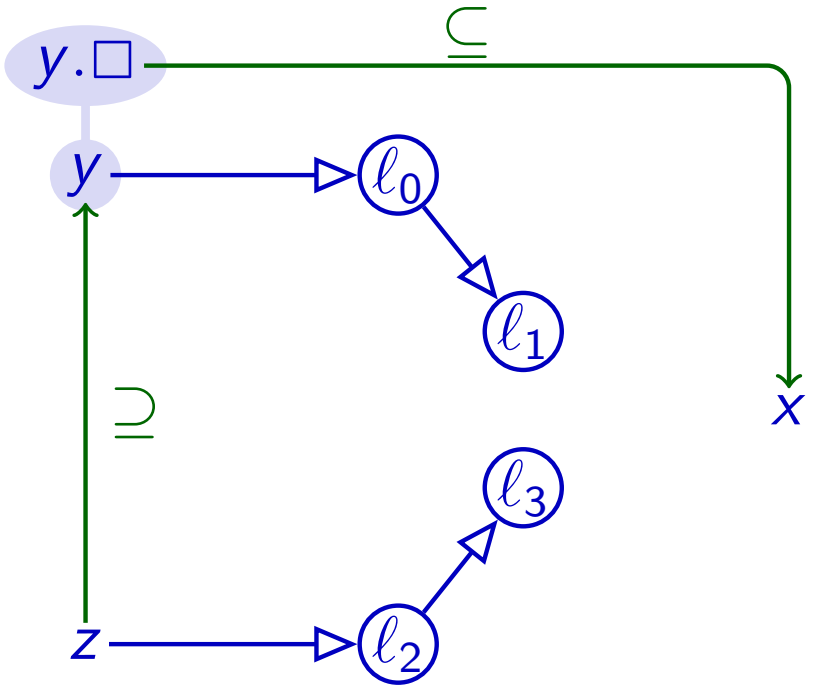

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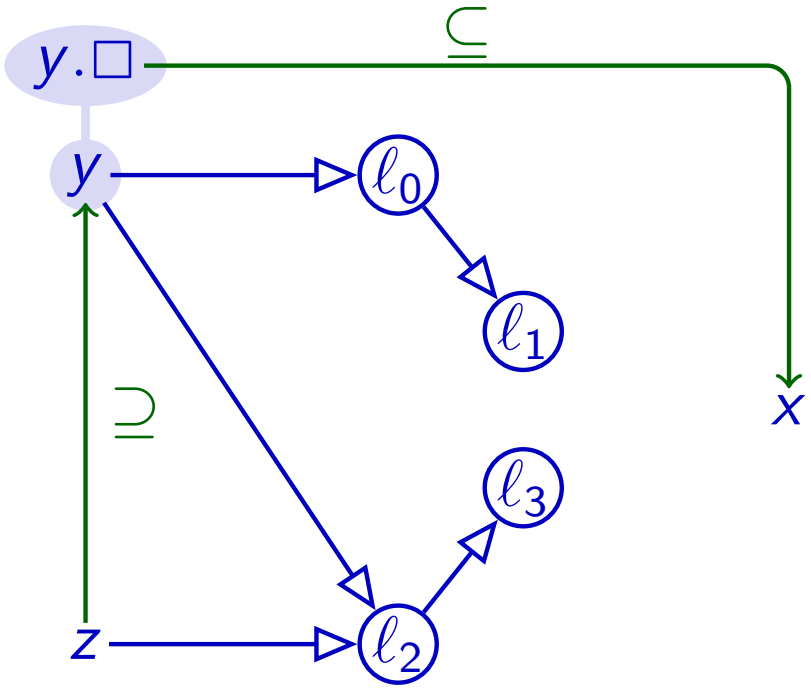

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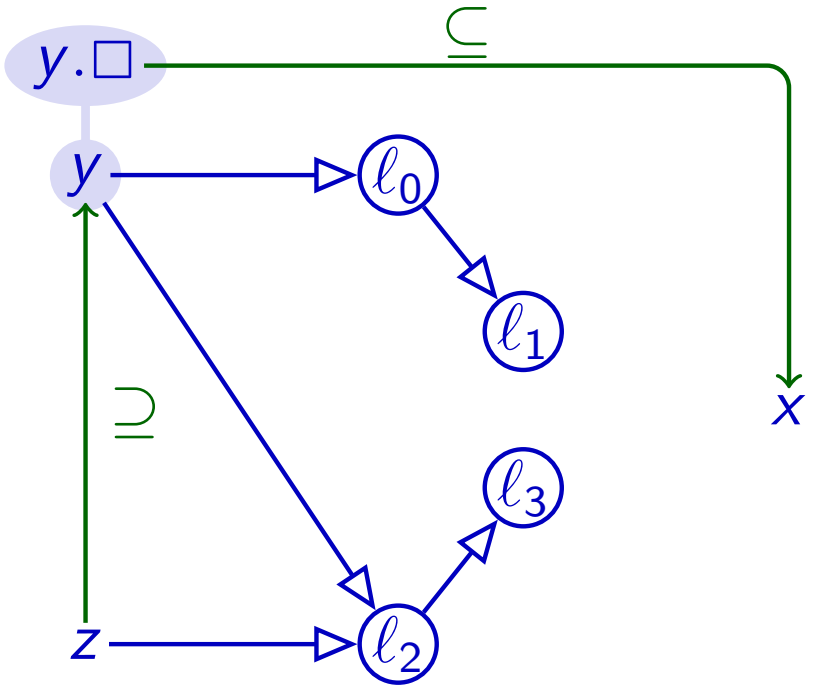

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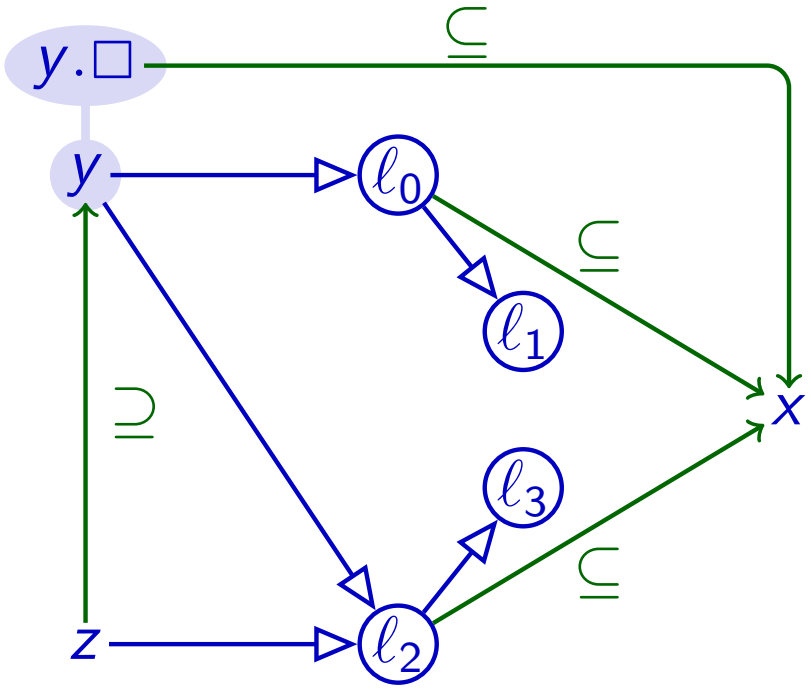

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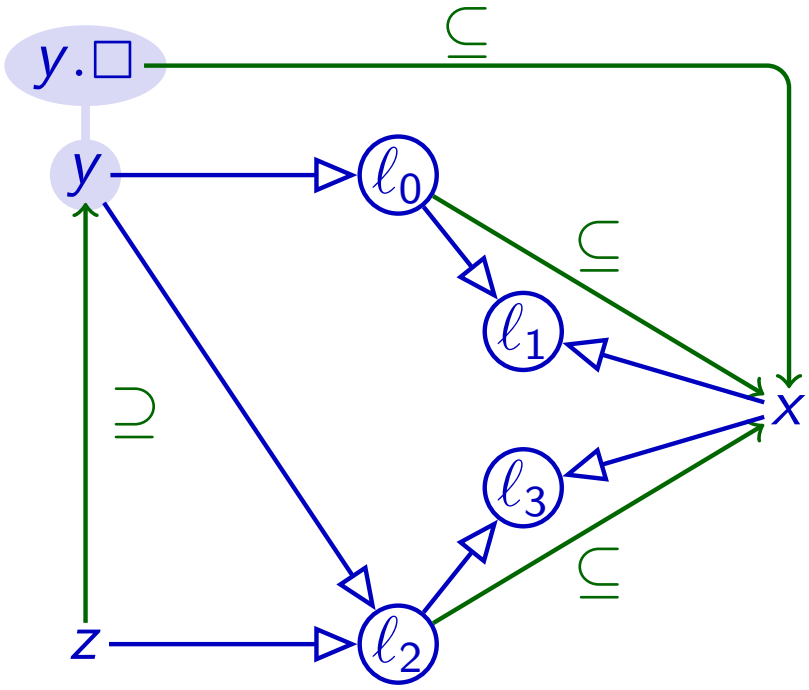

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1. Fact extraction:
   - Initial points-to sets: $\ell \in pts(x)$, meaning $\ell \triangleleft\!\!- x$
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       $\Longrightarrow x \leftarrow v$

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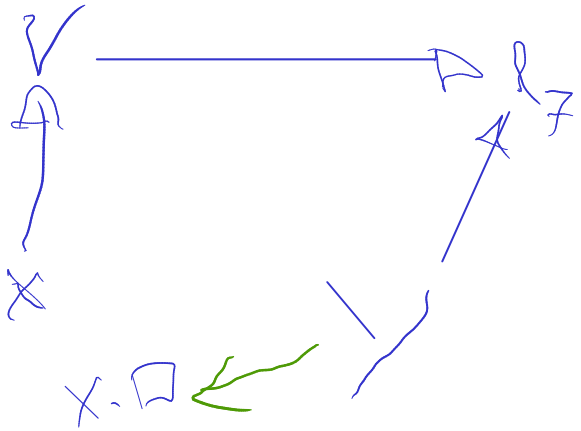

Example

	x := new$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
	x := y	y$\rightarrow$x
$\Rightarrow$	x := y.$\square$	y.$\square\rightarrow$x
	x.$\square$:= y	y$\rightarrow$x.$\square$

- **Actual**:

Teal

```
var a := new_ℓ1();
var b := new_ℓ2();
a := new_ℓ3();
var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
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- **Andersen**:

Example

`x := new`$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
`x := y`	$\mathtt{y} \rightarrow \mathtt{x}$
`x := y.□`	$\mathtt{y.\square} \rightarrow \mathtt{x}$
`x.□ := y`	$\mathtt{y} \rightarrow \mathtt{x.\square}$

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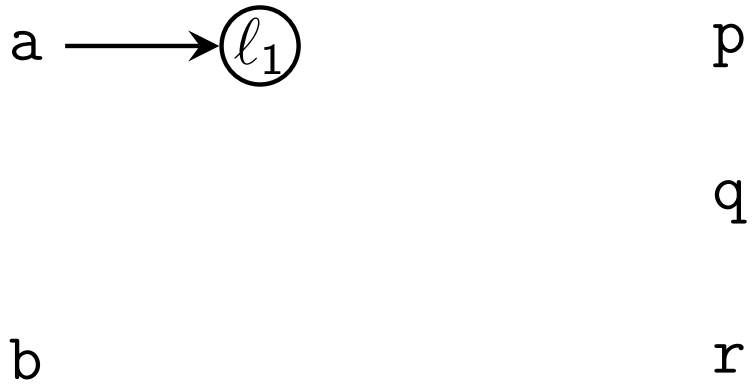

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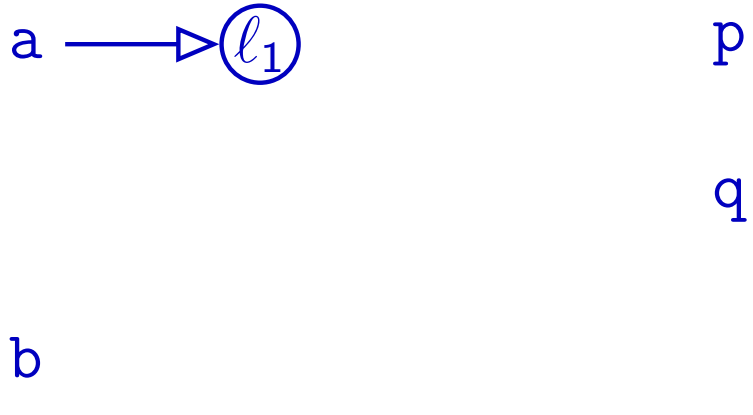

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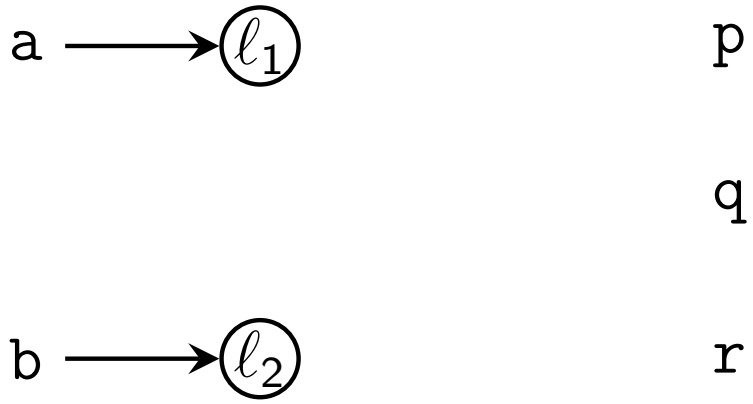

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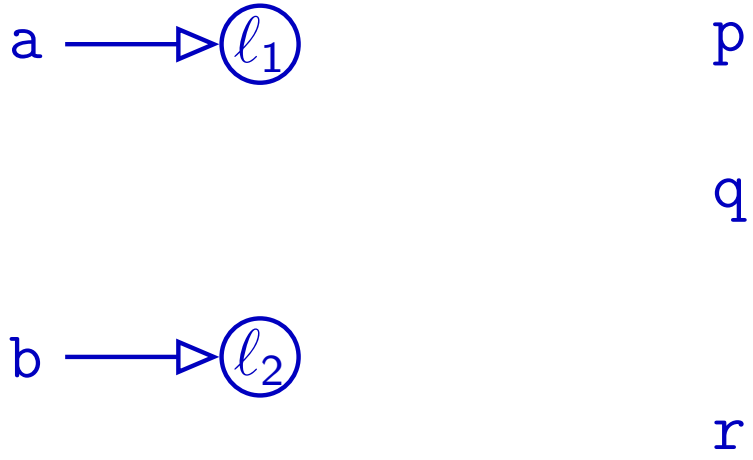

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$\Rightarrow$ `x := new`$_{\ell_z}$	$\ell_z \in pts(\texttt{x})\}$
`x := y`	`y`$\rightarrow$`x`
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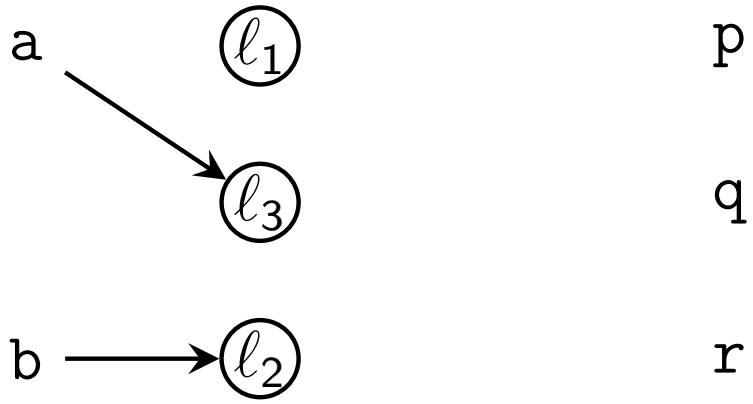

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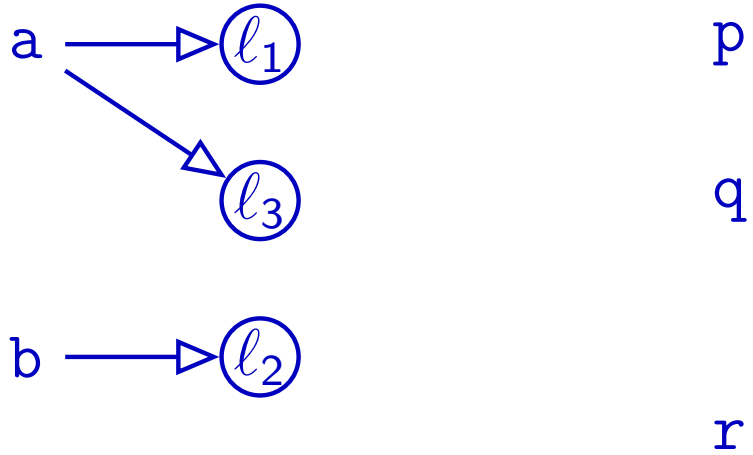

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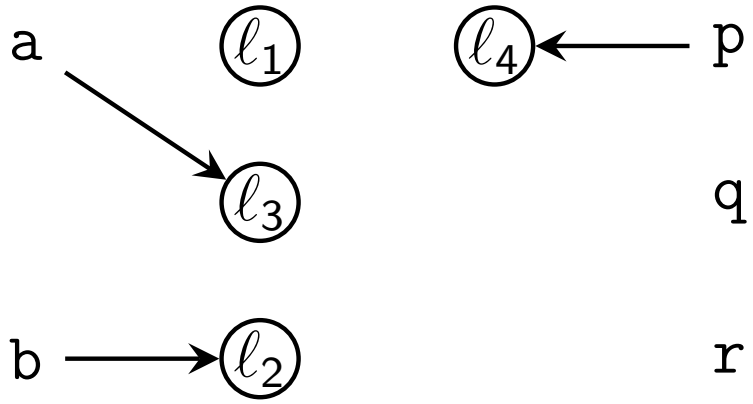
p := q;
var r := q.n;
```

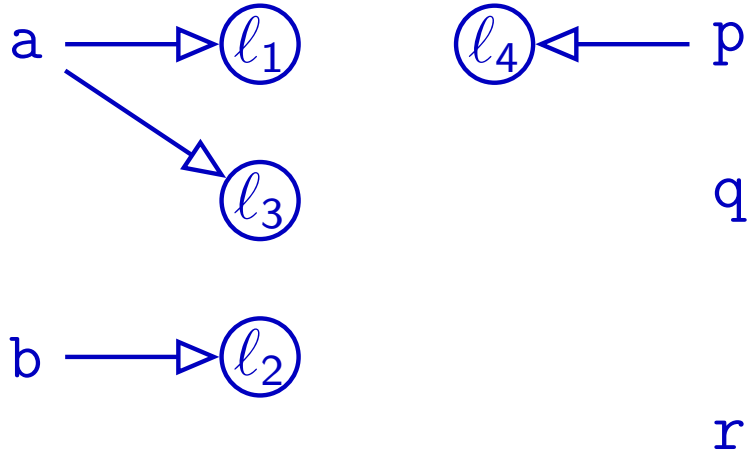
Example

⇒ `x := new`$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
`x := y`	$\mathtt{y} \rightarrow \mathtt{x}$
`x := y.□`	$\mathtt{y.\square} \rightarrow \mathtt{x}$
`x.□ := y`	$\mathtt{y} \rightarrow \mathtt{x.\square}$

- **Actual**:



- **Andersen**:



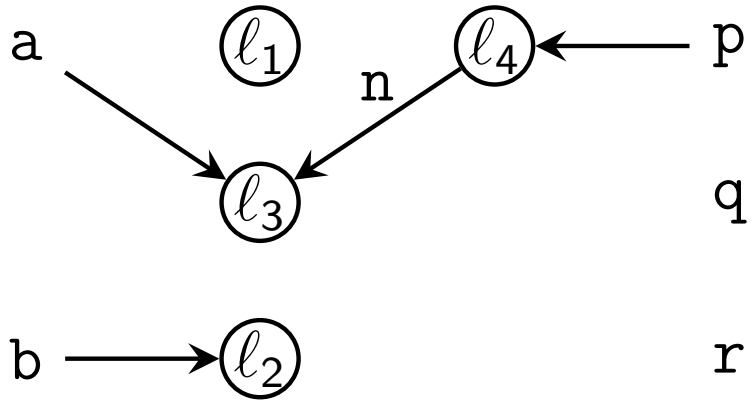
Teal

```
var a := new_{ℓ1}();
var b := new_{ℓ2}();
a := new_{ℓ3}();
var p := new_{ℓ4}(); //⇐
p.n := a;
var q := new_{ℓ6}();
q.n := b;
p := q;
var r := q.n;
```

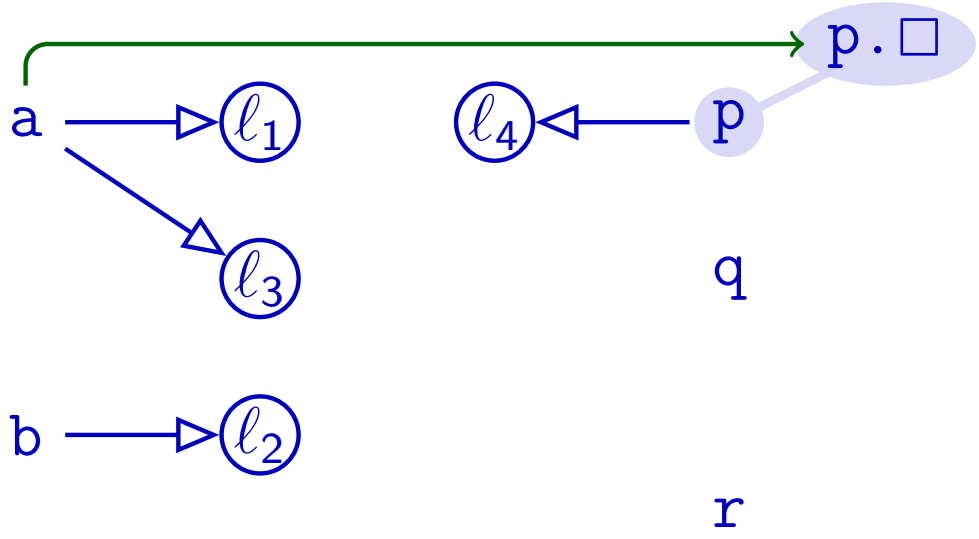
Example

x := new$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
x := y	y→x
x := y.□	y.□→x
⇒ x.□ := y	y→x.□

- **Actual**:



- **Andersen**:



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Example

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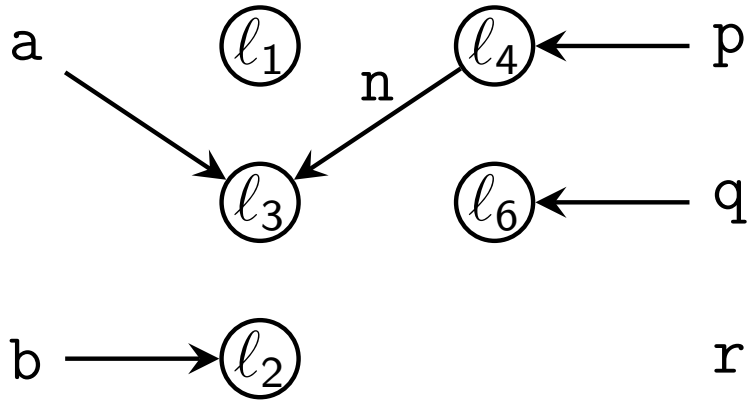

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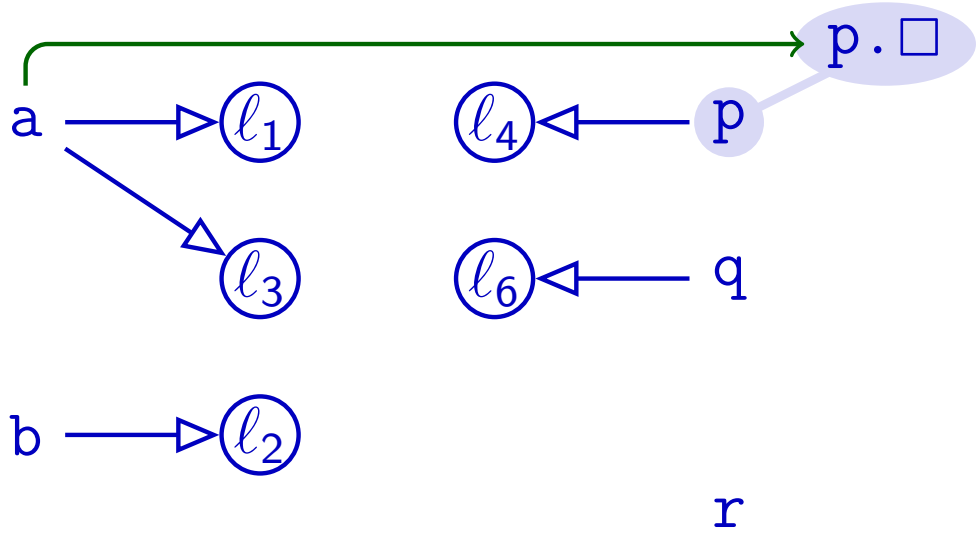

Teal

```
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```

Example

x := new$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
x := y	y→x
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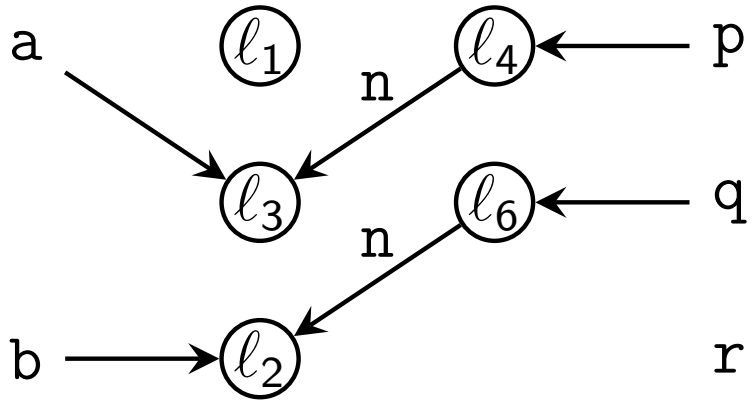
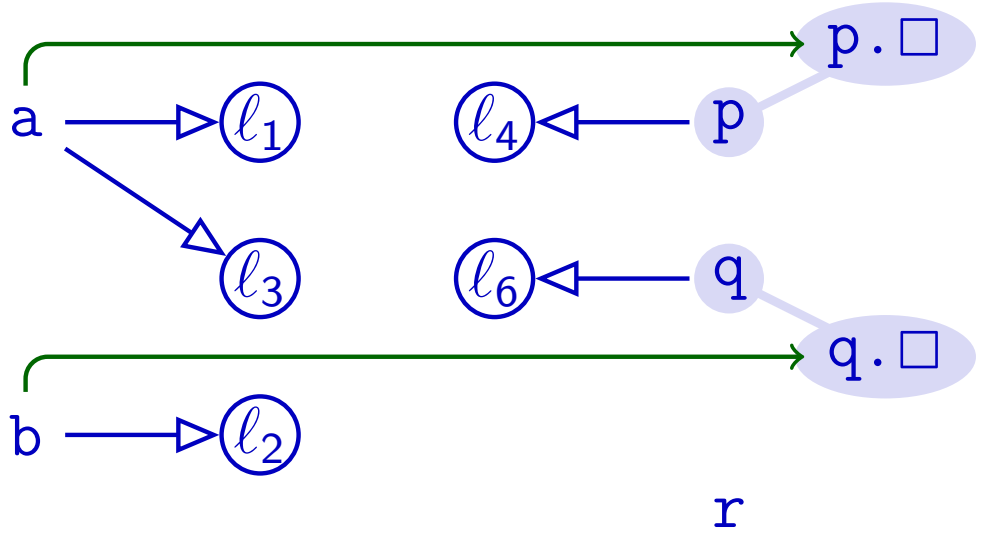
⇒ x.□ := y	y→x.□

- **Actual**:



- **Andersen**:



Teal

```
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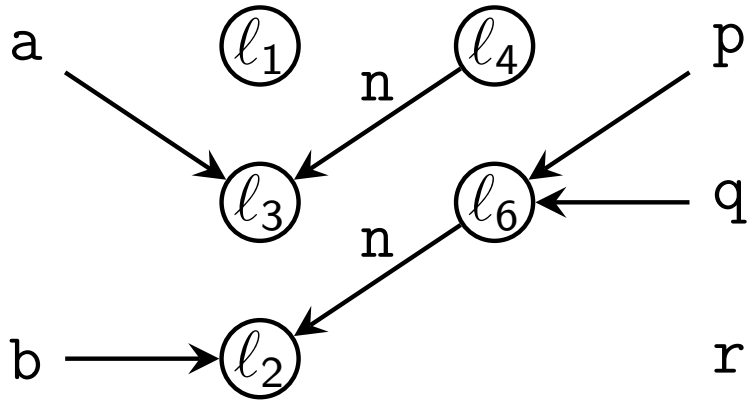
var b := new_ℓ2();
a := new_ℓ3();
var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
q.n := b;          //⇐
p := q;
var r := q.n;
```

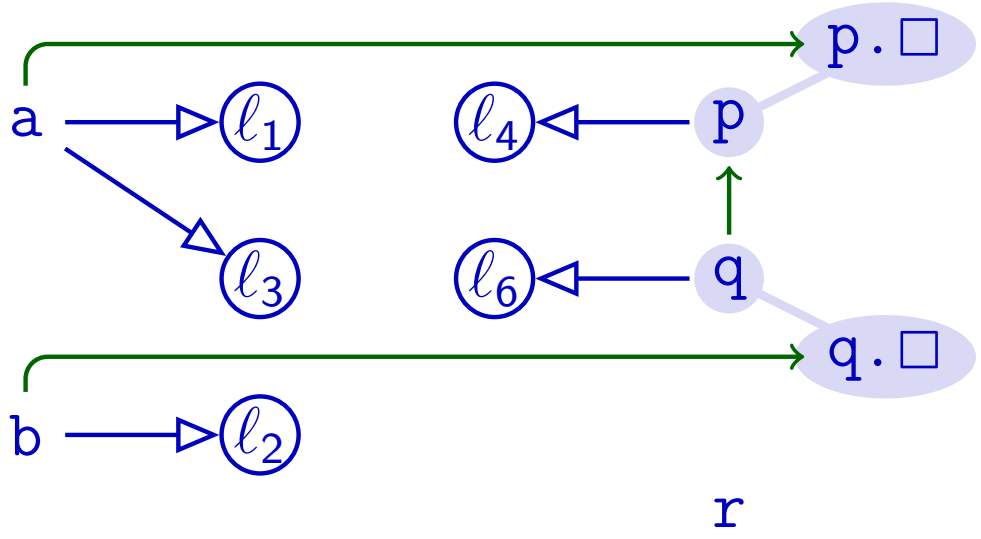
Example

	x := new$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
$\Rightarrow$	x := y	y$\rightarrow$x
	x := y.$\square$	y.$\square\rightarrow$x
	x.$\square$:= y	y$\rightarrow$x.$\square$

- **Actual**:



- **Andersen**:



Teal

```
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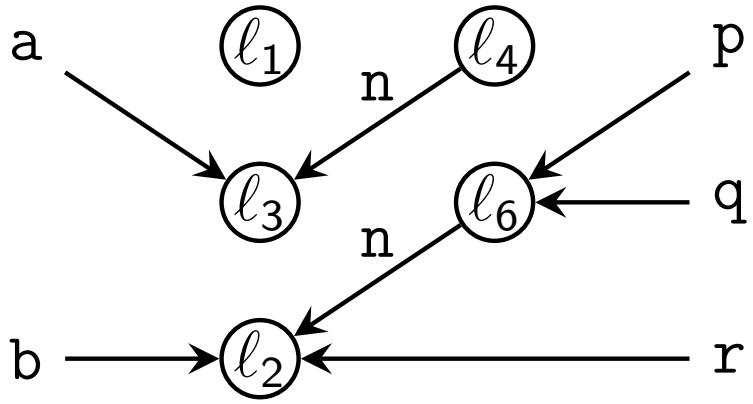
var b := new_ℓ2();
a := new_ℓ3();
var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
q.n := b;
p := q;            //⇐
var r := q.n;
```

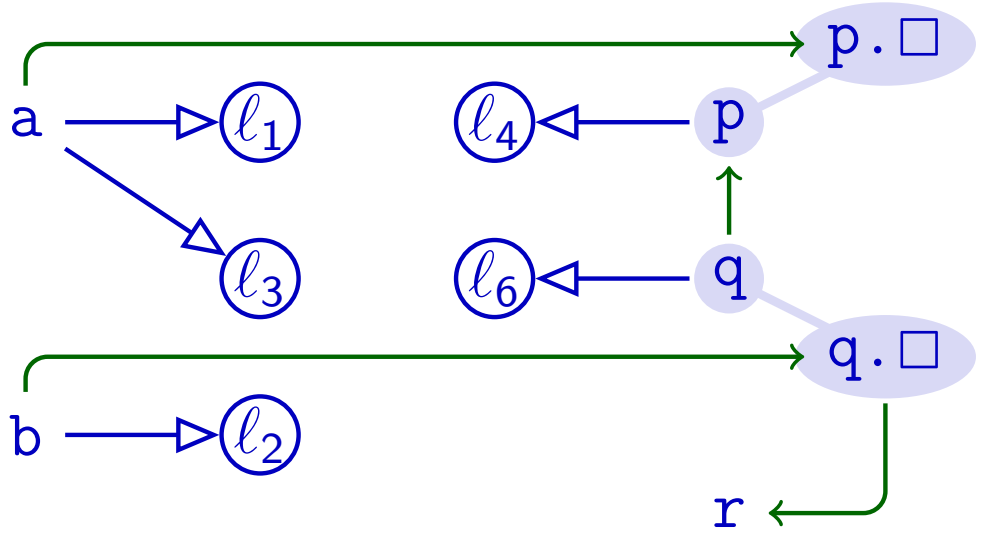
Example

x := new$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
x := y	y→x
x := y.□	y.□→x
x.□ := y	y→x.□

- **Actual**:



- **Andersen**:



Teal

```
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var b := new_ℓ2();
a := new_ℓ3();
var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
q.n := b;
p := q;
var r := q.n;      //⇐
```

Example

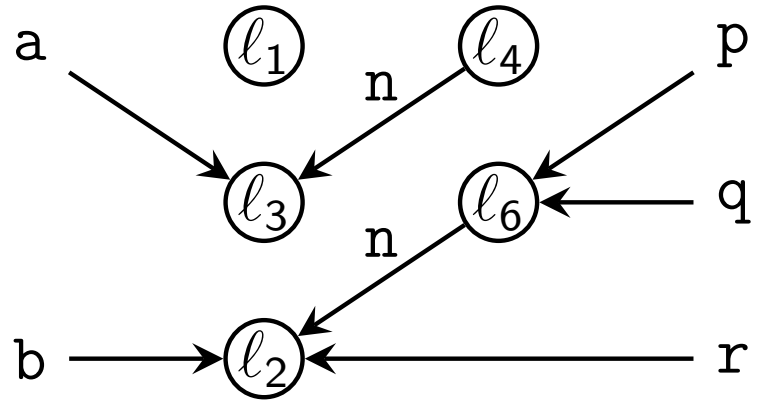
`x := new`$_{\ell_z}$	$\ell_z \in pts(\mathtt{x})\}$
`x := y`	$\mathtt{y} \rightarrow \mathtt{x}$
`x := y.`$\square$	$\mathtt{y.}\square \rightarrow \mathtt{x}$
`x.`$\square$ `:= y`	$\mathtt{y} \rightarrow \mathtt{x.}\square$

$\ell \leftarrow y$ and $x \leftarrow y \implies \ell \leftarrow x$

$v \leftarrow y$ and $x \leftarrow y.\square \implies x \leftarrow v$

$v \leftarrow x$ and $x.\square \leftarrow y \implies v \leftarrow y$

- **Actual**:



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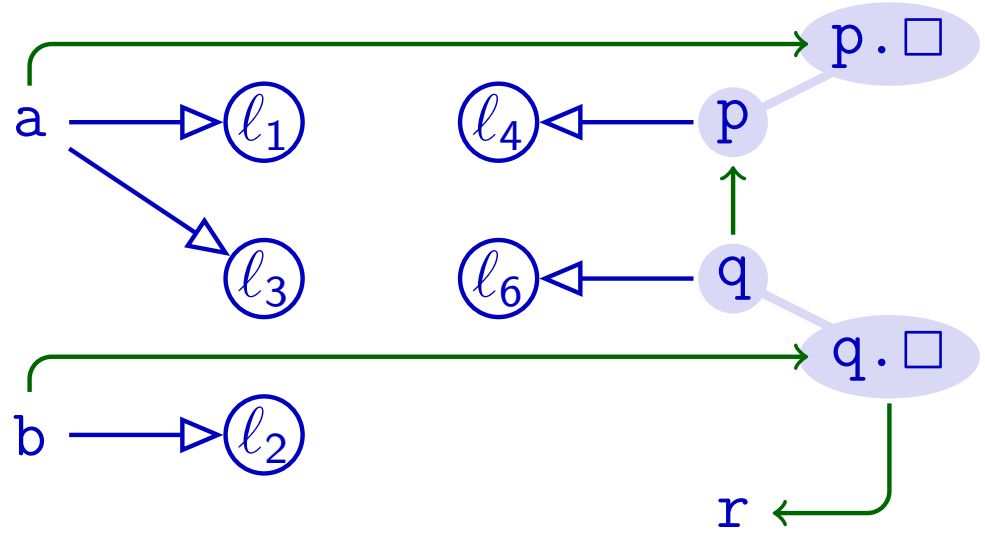

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```
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q.n := b;
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```

Andersen's algorithm must propagate along inclusion graph

Example

$\ell \leftarrow y$ and $x \leftarrow y \implies \ell \leftarrow x$

$v \leftarrow y$ and $x \leftarrow y.\square \implies x \leftarrow v$

$v \leftarrow x$ and $x.\square \leftarrow y \implies v \leftarrow y$

ℓ_6 q q□ b ℓ_6 b

Teal

```
var a := new_ℓ1();
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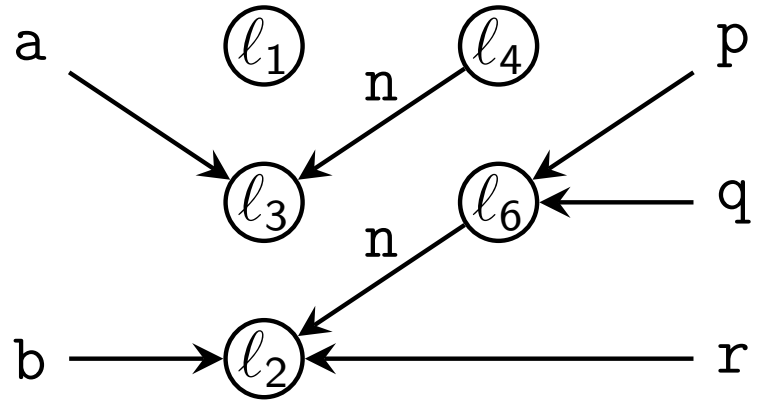
- **Actual**:



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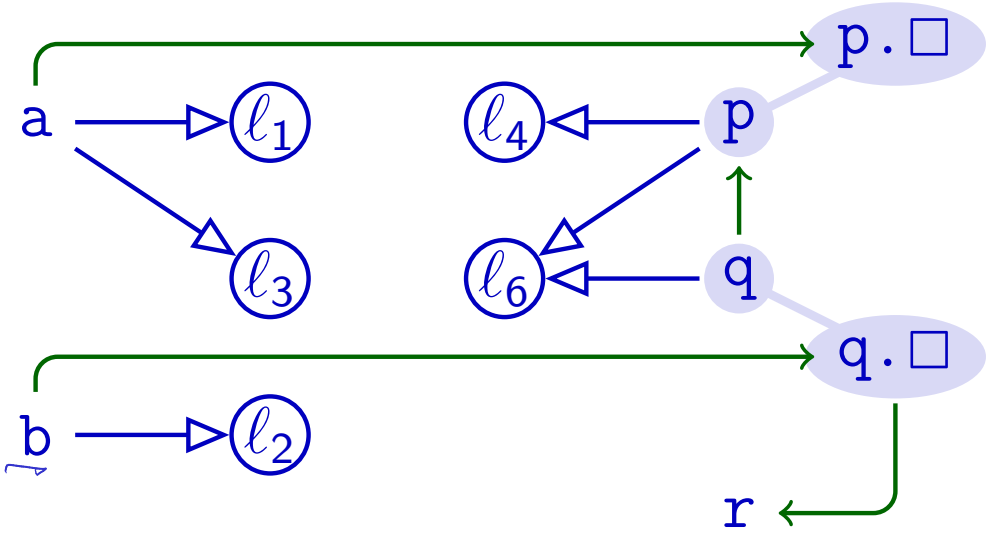

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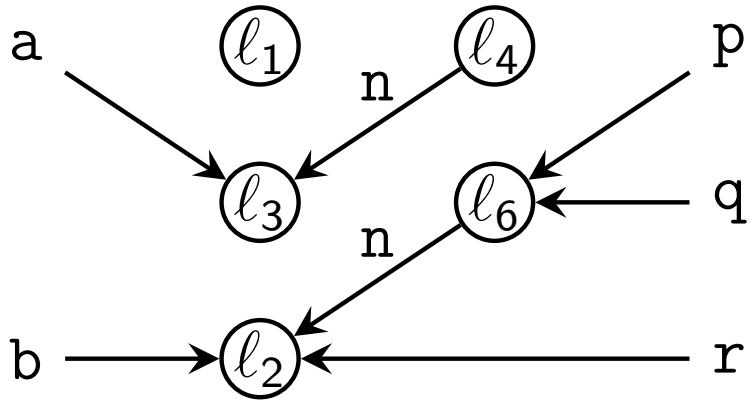

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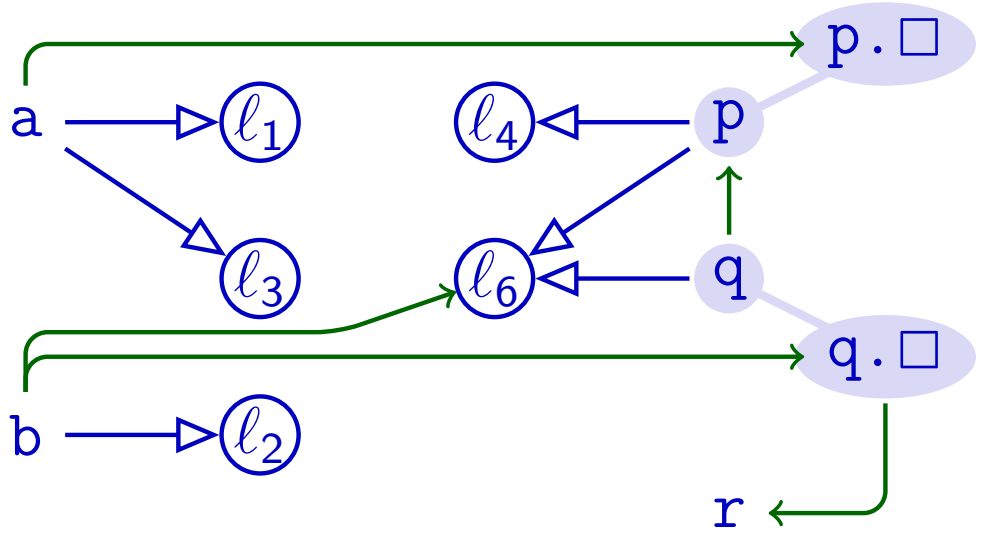

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var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
q.n := b;
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$\ell \leftarrow y$ and $x \leftarrow y \implies \ell \leftarrow x$
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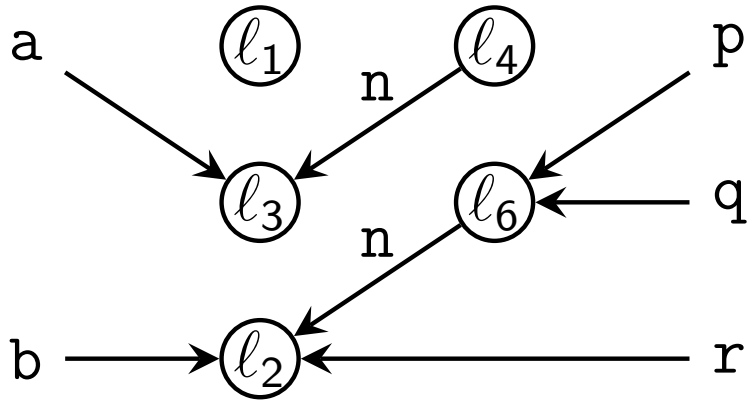
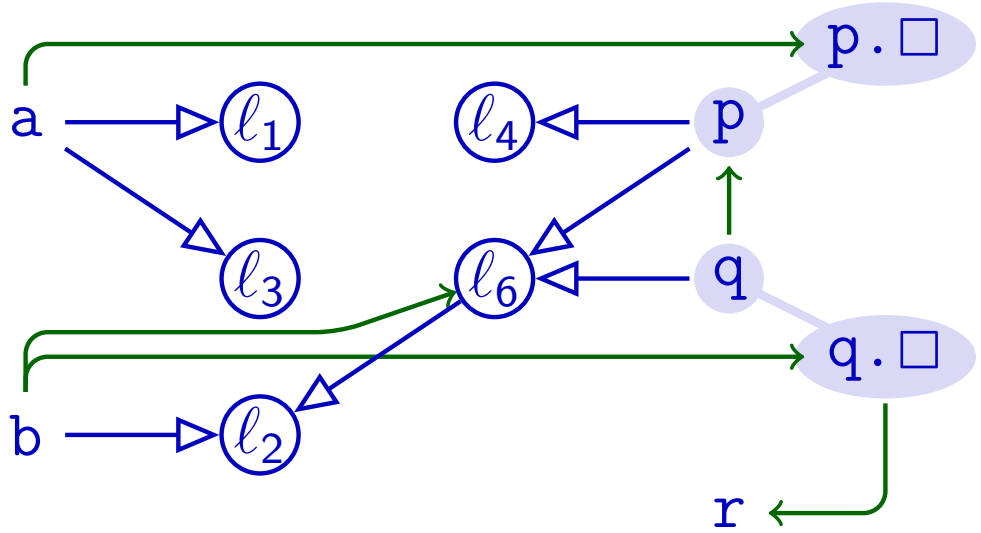


- **Andersen**:



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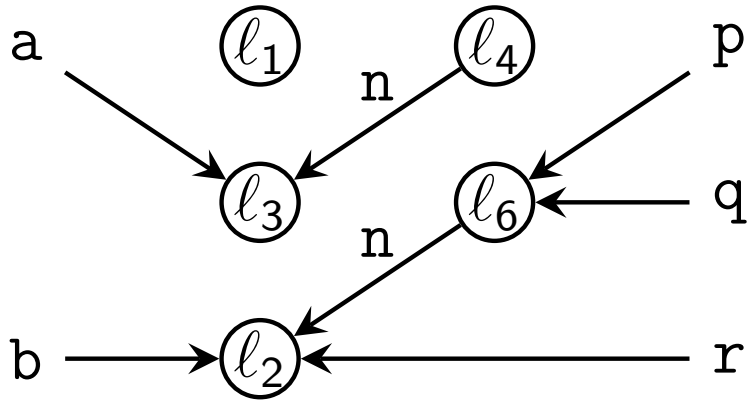

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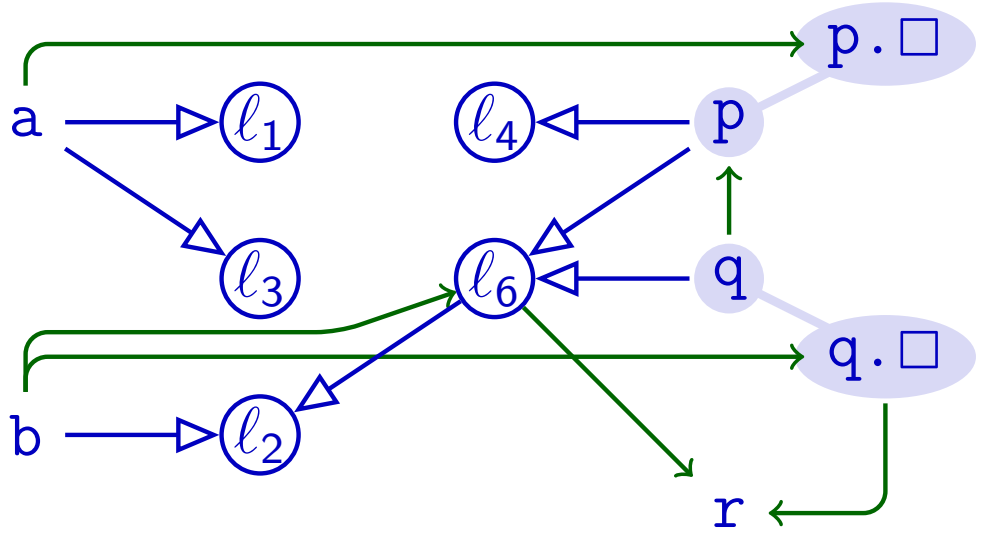

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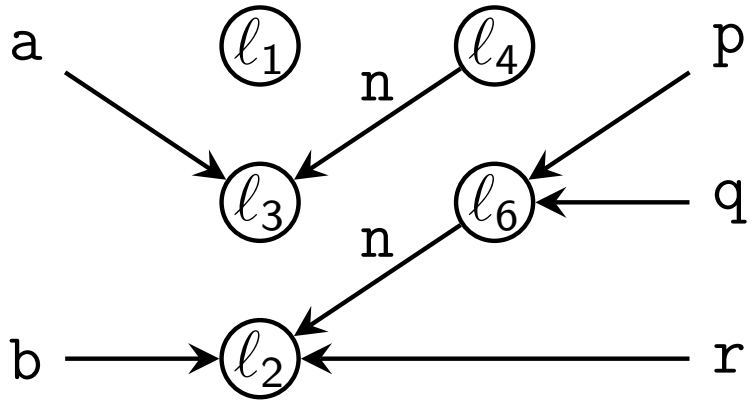

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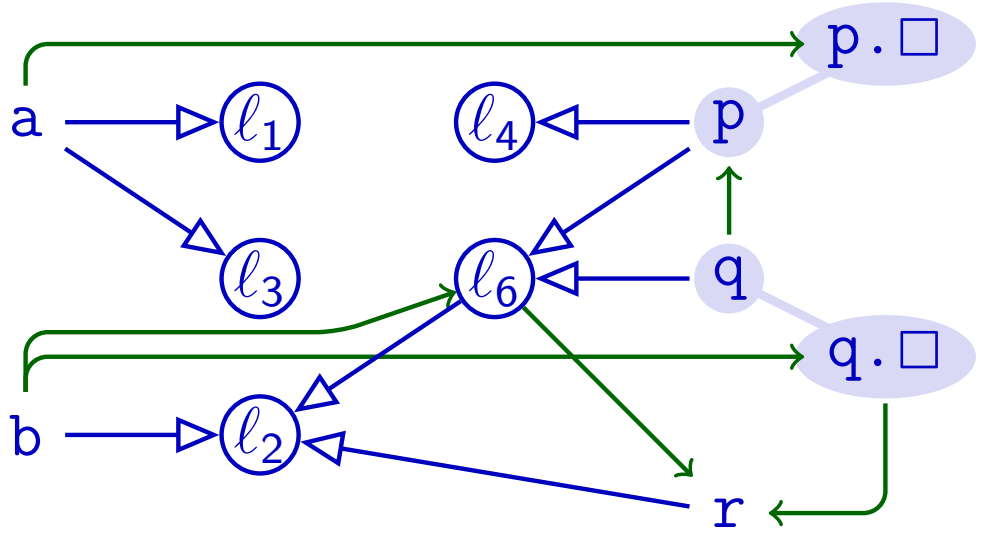

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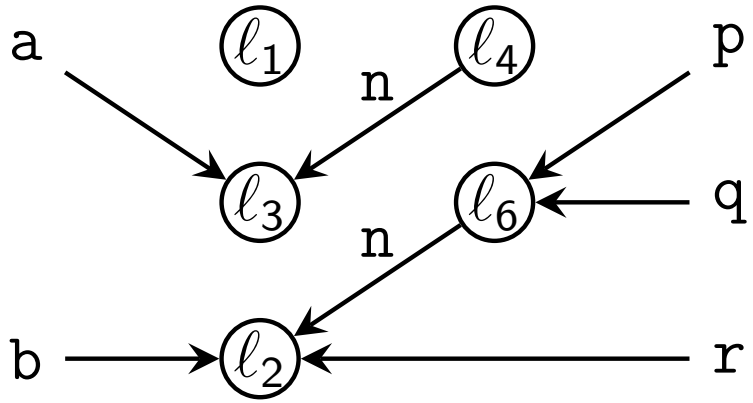

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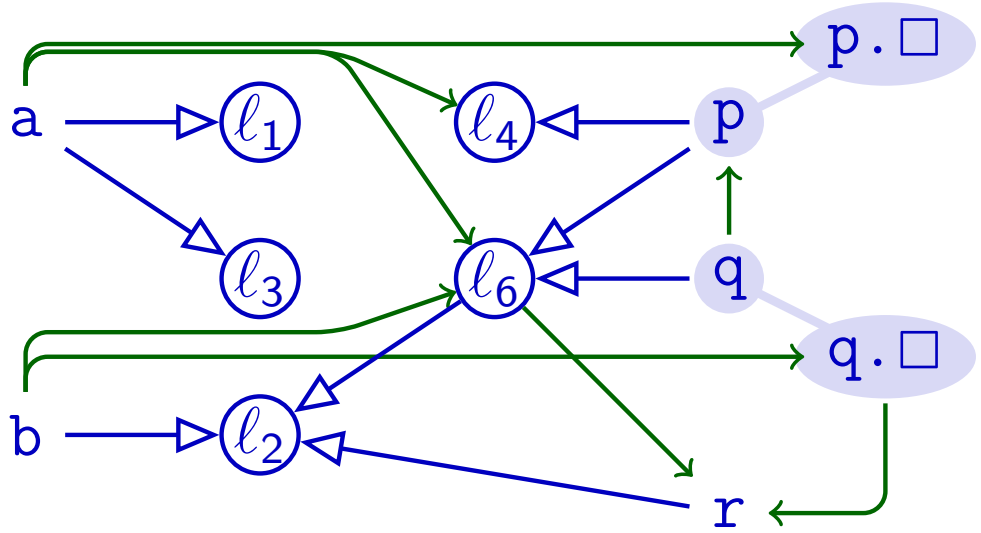

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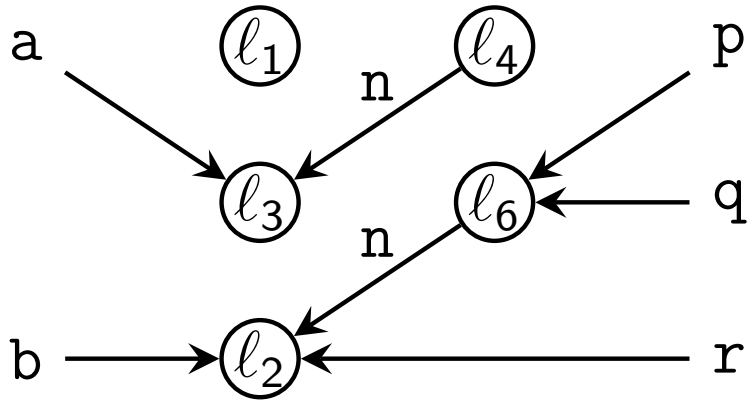

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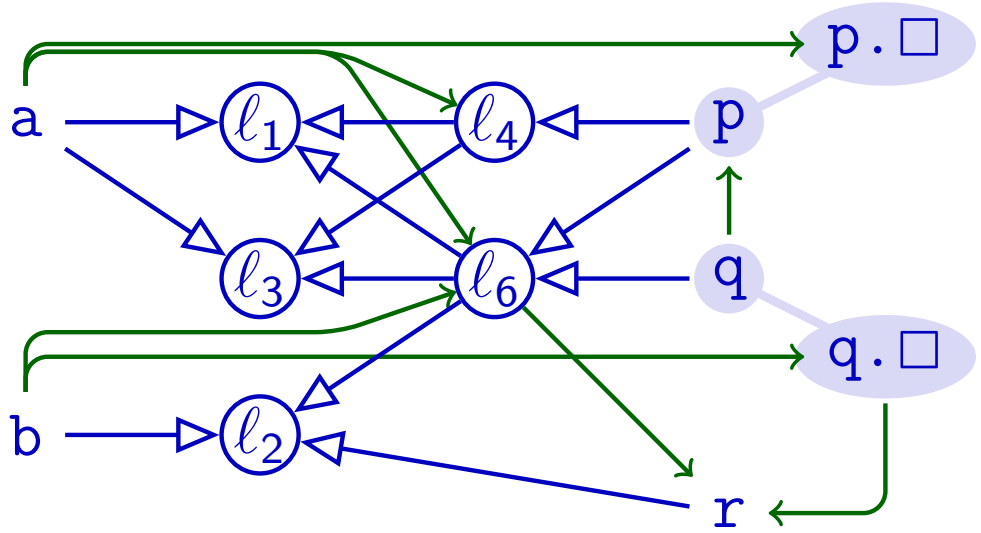

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```
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var p := new_ℓ4();
p.n := a;
var q := new_ℓ6();
q.n := b;
p := q;
var r := q.n;
```

Andersen's algorithm must propagate along inclusion graph

Implementation

- Graph structure
- Two types of edges
- Connection between x and $x.\square$
- Worklist:
  - Track all new edges (initially all)
  - Process one edge at a time:
  - Search for matching non-new edges to complete one of our three rules
  - Produce new edges, if match
  - If edge is new: add to worklist

$$\ell \leftarrow y \text{ and } x \leftarrow y \implies \ell \leftarrow x$$
$$v \leftarrow y \text{ and } x \leftarrow y.\square \implies x \leftarrow v$$
$$v \leftarrow x \text{ and } x.\square \leftarrow y \implies v \leftarrow y$$

Complexity

- Complexity of graph closure: $O(n^3)$
- Traditional assumption about Andersen's analysis
- Recent work observes[1]: Close to $O(n^2)$ if:

1. Few statements dereference each variable
2. Control flow graphs not too complex
3. *Both conditions are common in practical programs*

Summary

- Andersen's analysis:
  - Subset-based
  - Builds inclusion graph for propagating memory locations along subset constraints
  - $O(n^3)$ worst-case behaviour
  - Closer to $O(n^2)$ in practice
  - More precise than Steensgaard's analysis
  - Less scalable than Steensgaard's analysis

Challenges Towards OO Support

- (+) Flow-sensitivity
- (+) Points-to information
- Dynamic Dispatch

Challenges Towards OO Support

- (+) Flow-sensitivity
- (+) Points-to information
- Dynamic Dispatch
- Advanced features:
  - Pointer arithmetic
  - Dynamic Class Loading
  - "Native Calls" (into C/assembly/Syscalls)
  - Reflection

int f

The Call Graph

```
int main(int argc,
         char *argv) {
    if (argc > 1) {
        f(argv[0]);
    }
    g();
    return 0;
}
```

```
void f(char *s) {
    for (char *p = s; *p; p++) {
        *p = up(*p);
    }
    puts(s);
}
```

```
char up(char c) {
    if (c >= 'a' && c <= 'z') {
        return c - ('a' - 'A');
    }
    return c;
}
```

```
void g(void) {
    puts("Hello, World!");
}
```

The Call Graph

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int main(int argc,
         char *argv) {
    if (argc > 1) {
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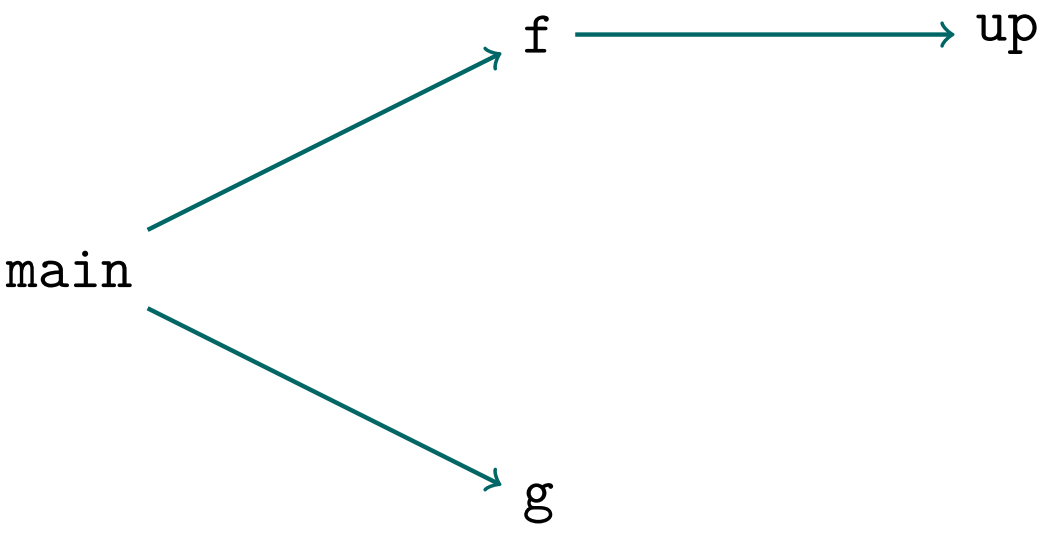
char up(char c) {
    if (c >= 'a' && c <= 'z') {
        return c - ('a' - 'A');
    }
    return c;
}
```

```
void g(void) {
    puts("Hello, World!");
}
```

The Call Graph

- $G_{\text{call}} = \langle P, E_{\text{call}} \rangle$
- Connects procedures from P via call edges from E_{call}
- 'Which procedure can call which other procedure?'
- Often refined to:
  'Which *call site* can call which procedure?'
- Used by program analysis to find procedure call targets



Finding Calls and Targets

```
class Main {
  public void
  main(String[] args) {
    A[] as = { new A(), new B() };
    for (A a : as) {
      A a2 = a.f();
      print(a.g());
      print(a2.g());
    }
  }
}
```

```
class A {
  public A
  f() { return new C(); }

  public String
  g() { return "A"; }
}
```

```
class D extends A {
  @Override
  public String
  g() { return "D"; }
}
```

```
class C extends A {
  @Override
  public String
  g() { return "A"; }
}
```

```
class B extends A {
  @Override
  public String
  g() { return "B"; }
}
```

Finding Calls and Targets

```
class Main {
  public void
  main(String[] args) {
    A[] as = { new A(), new B() };
    for (A a : as) {
      A a2 = a.f();
      print(a.g());
      print(a2.g());
    }
  }
}
```

```
class A {
  public A
  f() { return new C(); }

  public String
  g() { return "A"; }
}
```

```
class D extends A {
  @Override
  public String
  g() { return "D"; }
}
```

```
class C extends A {
  @Override
  public String
  g() { return "A"; }
}
```

```
class B extends A {
  @Override
  public String
  g() { return "B"; }
}
```

Finding Calls and Targets

[A, B]

```
class Main {
  public void
  main(String[] args) {
    A[] as = { new A(), new B() };
    for (A a : as) {
      A a2 = a.f();
      print(a.g());
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  public String
  g() { return "A"; }
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class D extends A {
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  public String
  g() { return "D"; }
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class C extends A {
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  public String
  g() { return "A"; }
}
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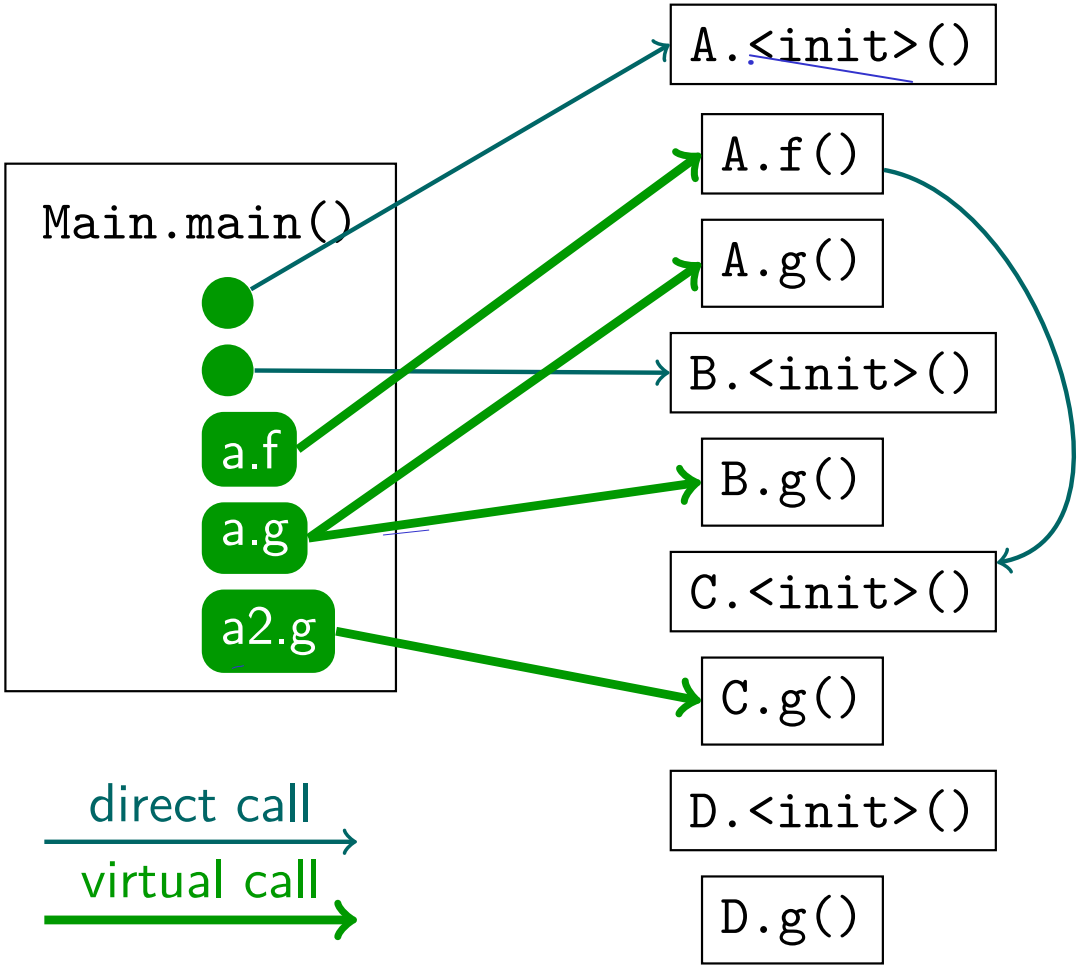
class B extends A {
  @Override
  public String
  g() { return "B"; }
}
```

Dynamic Dispatch: Call Graph

Challenge: Computing the precise call graph:



Summary

- **Call Graphs** capture which procedure calls which other procedure
- For program analysis, further specialised to map:

$$\text{Callsite} \rightarrow \text{Procedure}$$

- Direct calls: straightforward
- Virtual calls (dynamic dispatch):
  - Multiple targets possible for call
  - Not straightforward

Callgraphs with Points-to Data

```
class A {
 public A
 f() {
  return new C();
 }
}
```

```
class B extends A {
 public A
 f() {
  return new A();
 }
}
```

```
class C extends A {
 public A
 f() {
  return new B();
 }
}
```

```
A a = new A();
a = a.f();
a = a.f();
```

- Precision of call graph affects quality of all interprocedural analyses
  - IFDS, IDE
  - Points-to analyses

Callgraphs with Points-to Data

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- Precision of call graph affects quality of all interprocedural analyses
  - IFDS, IDE
  - Points-to analyses
- Idea: Use points-to analysis to determine *dynamic* type of objects
  - More precise virtual call resolution!

Callgraphs with Points-to Data

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- Precision of call graph affects quality of all interprocedural analyses
  - IFDS, IDE
  - Points-to analyses
- Idea: Use points-to analysis to determine *dynamic* type of objects
  - More precise virtual call resolution!
- **Problem**: Mutual dependency between call-graph and points-to analysis!

Finding Calls and Targets

```
class Main {
  public void
  main(String[] args) {
    A[] as = { new A(), new B() };
    for (A a : as) {
      A a2 = a.f();
      print(a.g());
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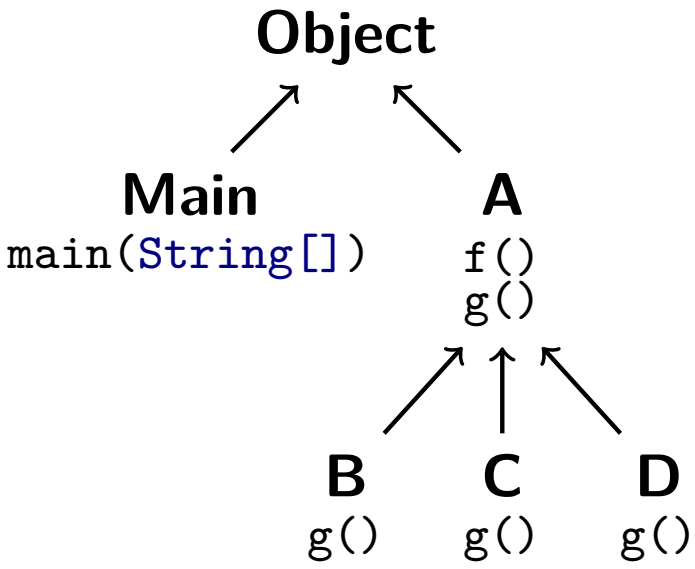
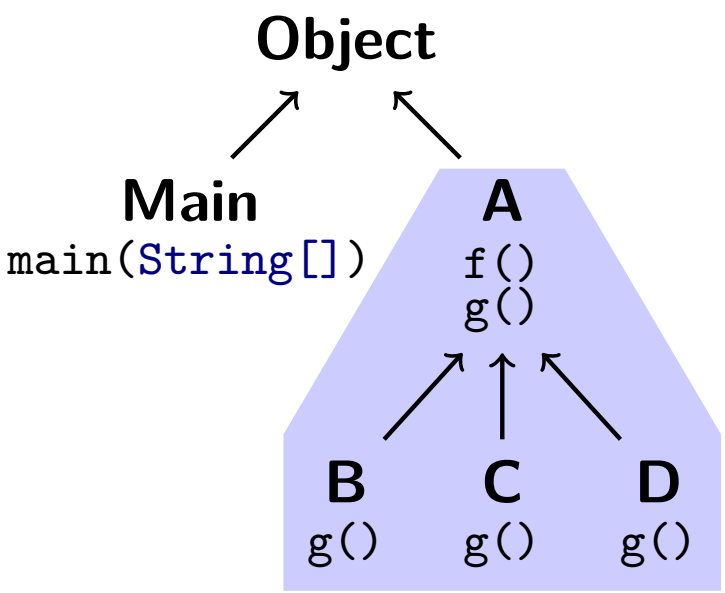
Class Hierarchy Analysis



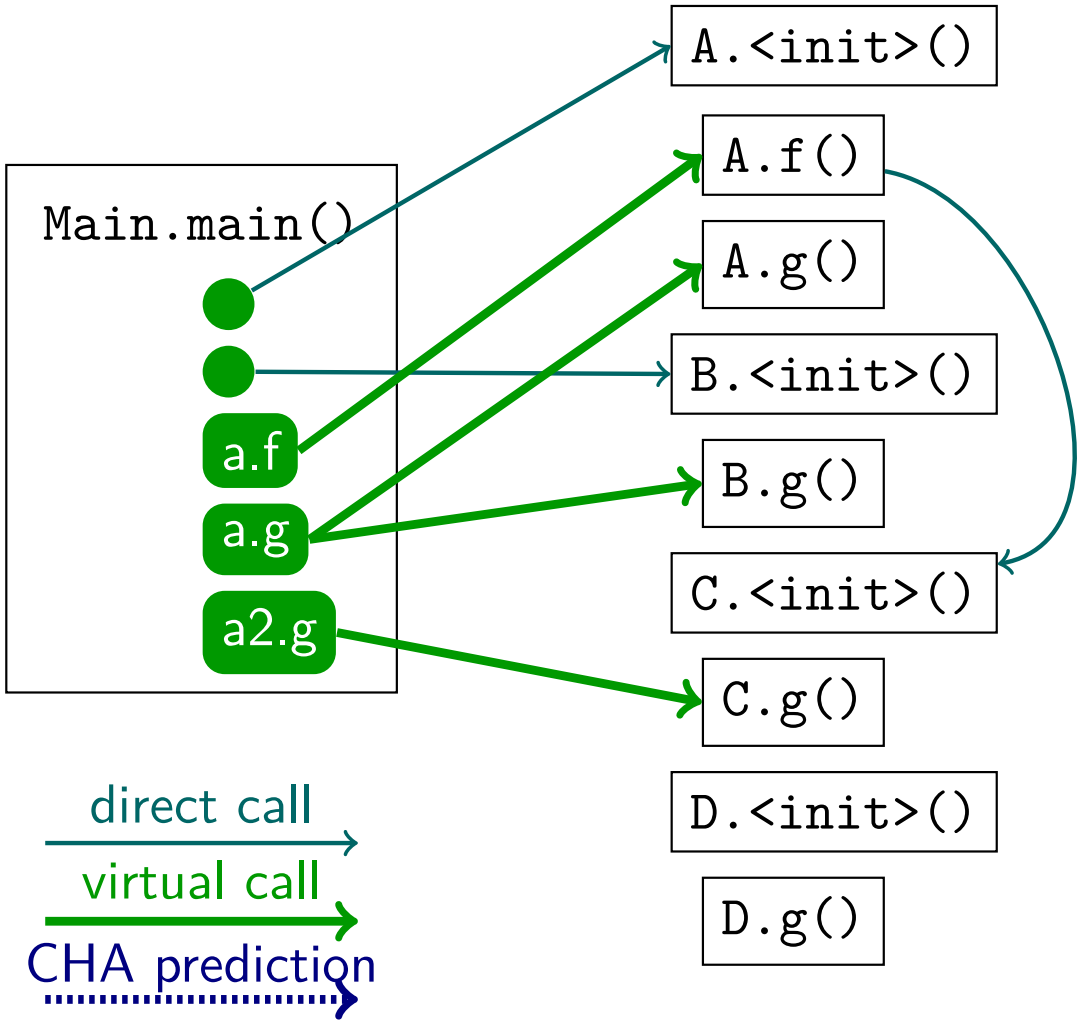
- Use declared type to determine possible targets

Class Hierarchy Analysis



- Use declared type to determine possible targets
- Must consider all possible subtypes
- In our example: assume a.f can call any of: `A.f()`, `B.f()`, `C.f()`, `D.f()`

Class Hierarchy Analysis: Example



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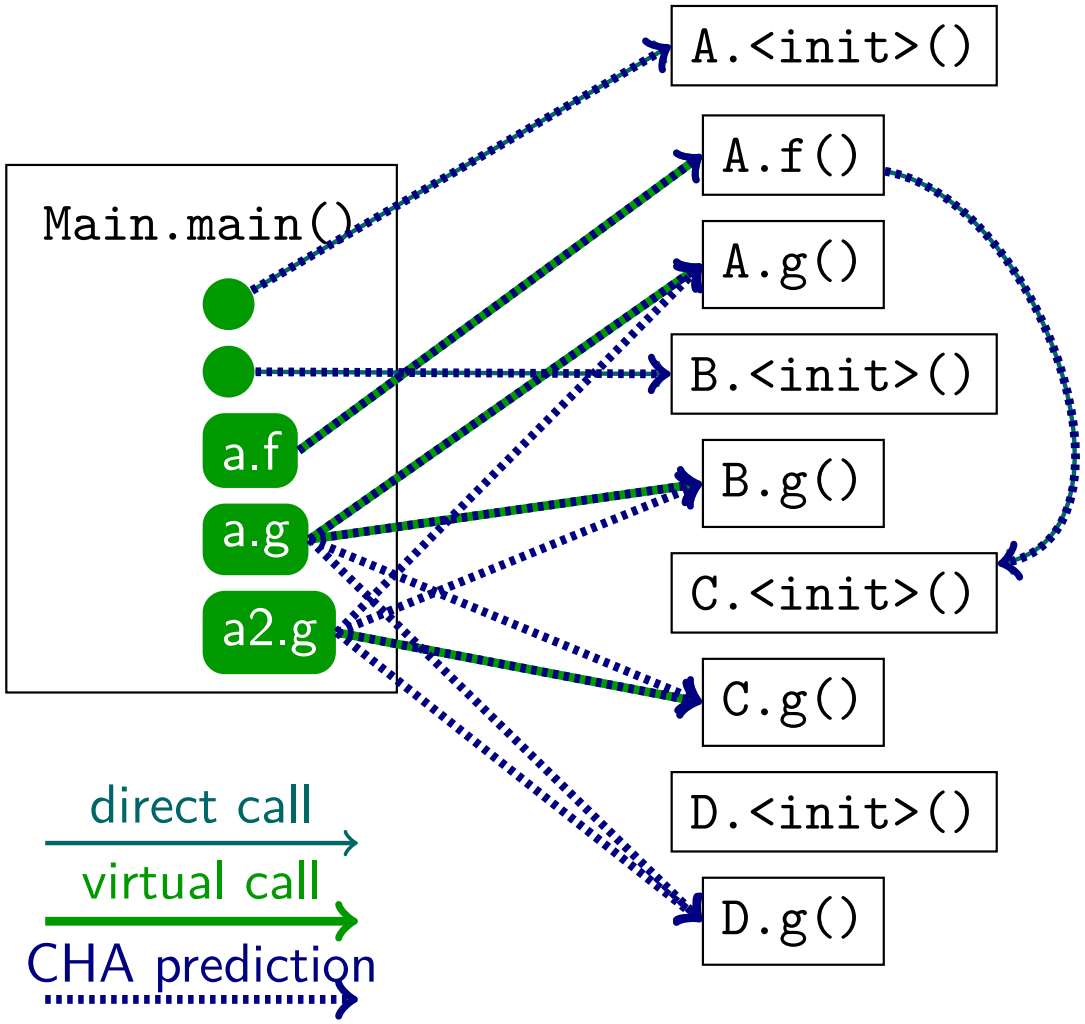

Summary

- **Call Hierarchy Analysis** resolves virtual calls $a.f()$ by:
  - Examining static types T of receivers $(a : T)$
  - Finding all subtypes $S <: T$
  - Creating call edges to all $S.f$, if $S.f$ exists
- **Sound**
  - Assuming strongly and statically typed language with subtyping
- Not very **precise**

Rapid Type Analysis

- Intuition:
  - Only consider reachable code
  - Ignore unused classes
  - Ignore classes instantiated only by unused code

Finding Calls and Targets

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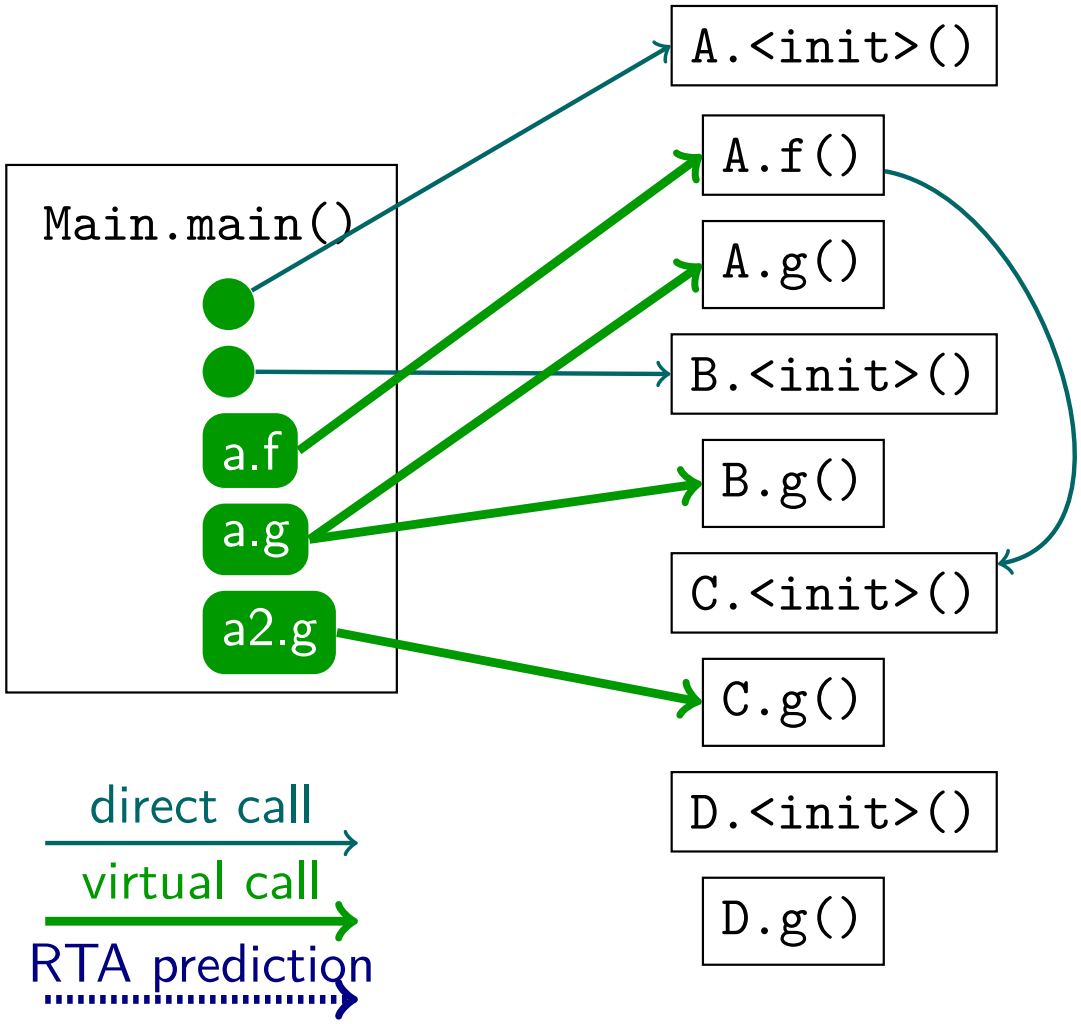
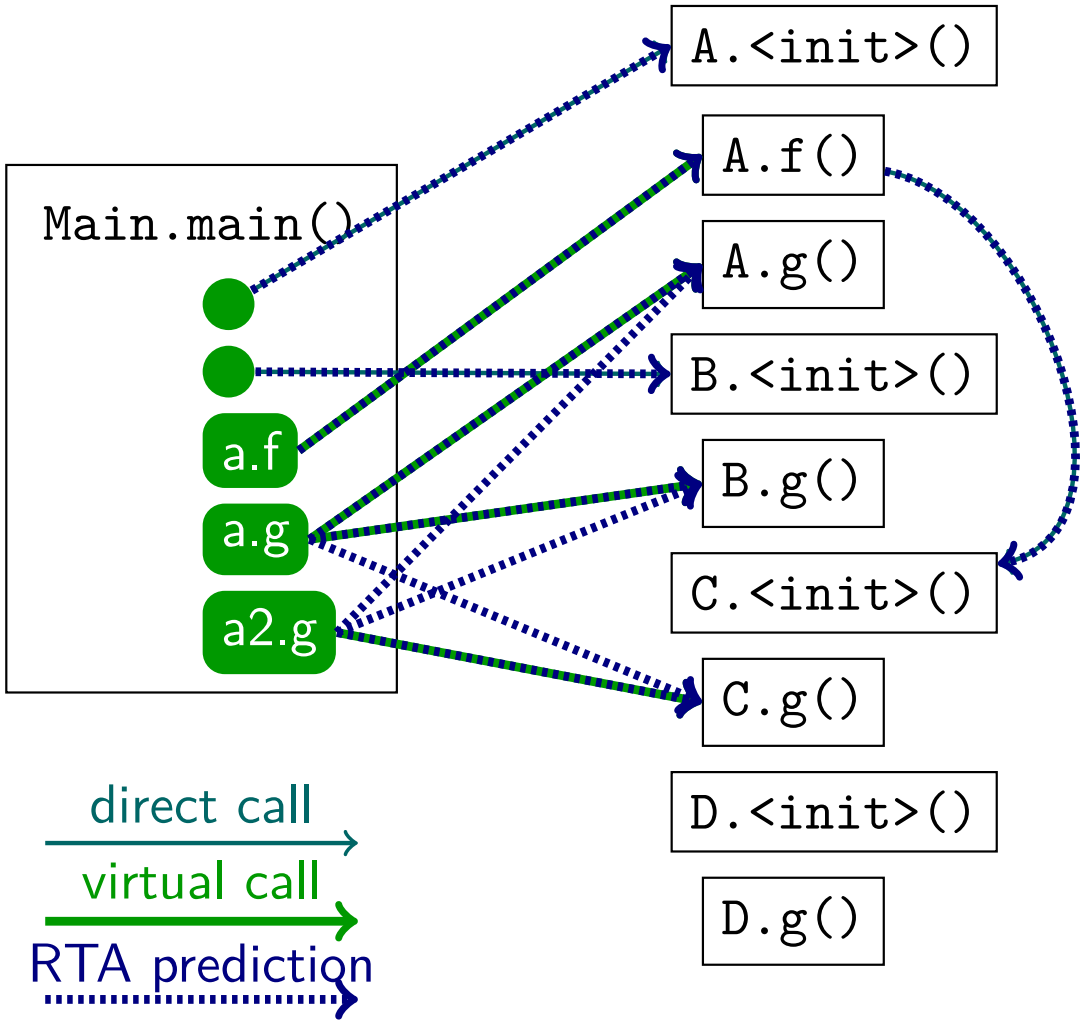
Rapid Type Analysis: Example



Rapid Type Analysis: Example



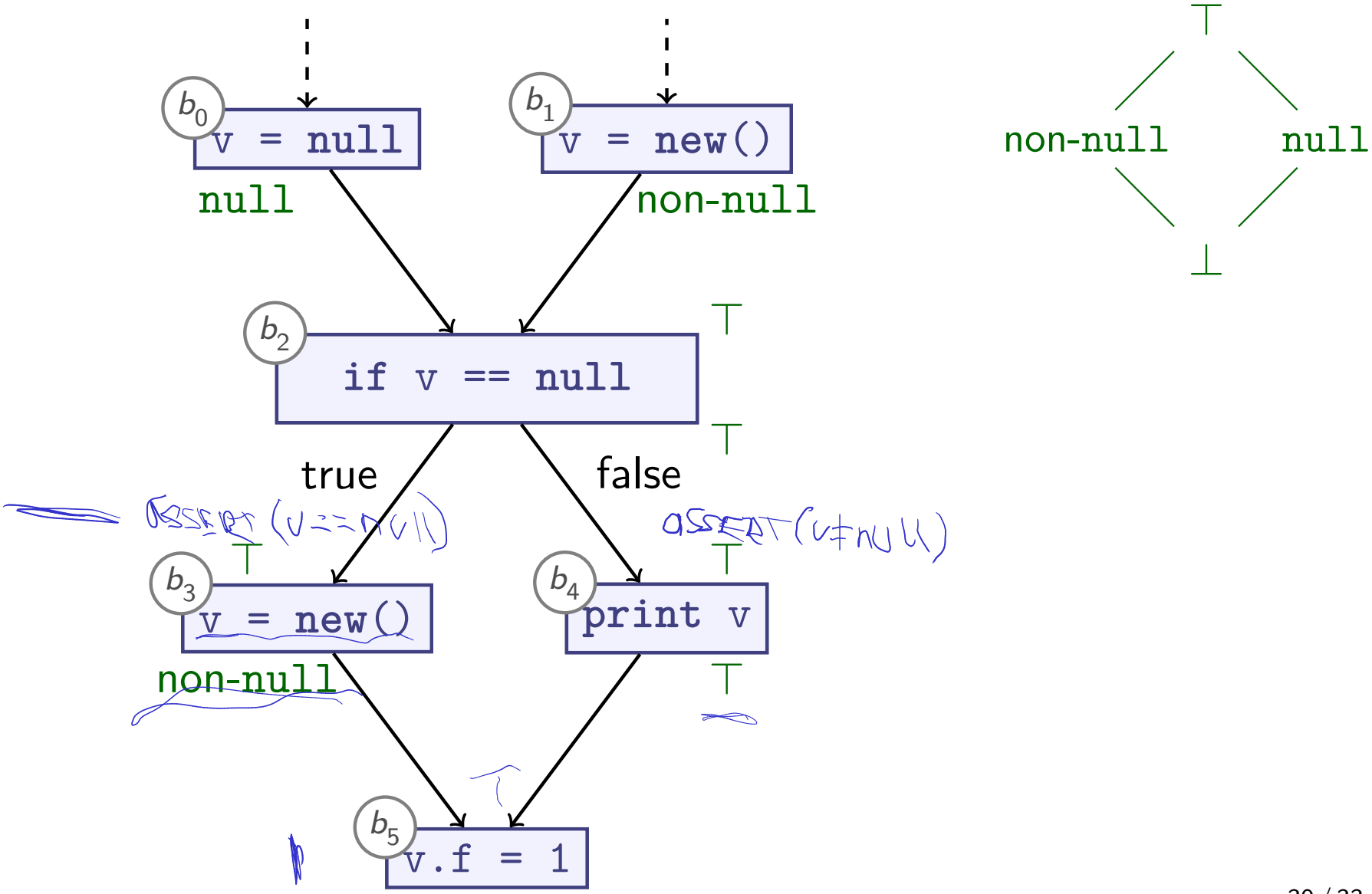
Rapid Type Analysis Algorithm Sketch

```
Procedure RTA(mainproc, <:):
begin
  WORKLIST := {mainproc}
  VIRTUALCALLS := ∅
  LIVECLASSES := ∅
  while s ∈ mainproc do
    foreach call c ∈ s do
      if c is direct call to p then
        addToWorklist(p)
        registerCallEdge(c → p)
      else if c = v.m() and v : T then begin
        VIRTUALCALLS := VIRTUALCALLS ∪ {c}
        foreach S <: T do
          addToWorklist(S.m)
          registerCallEdge(c → S.m)
        done
      end else if c = new C() and C ∉ LIVECLASSES then begin
        LIVECLASSES := LIVECLASSES ∪ {C}
        foreach v.m() ∈ VIRTUALCALLS with v : T and C <: T do
          addToWorklist(C.m)
          registerCallEdge(c → C.m)
        done
      end
done done end
```

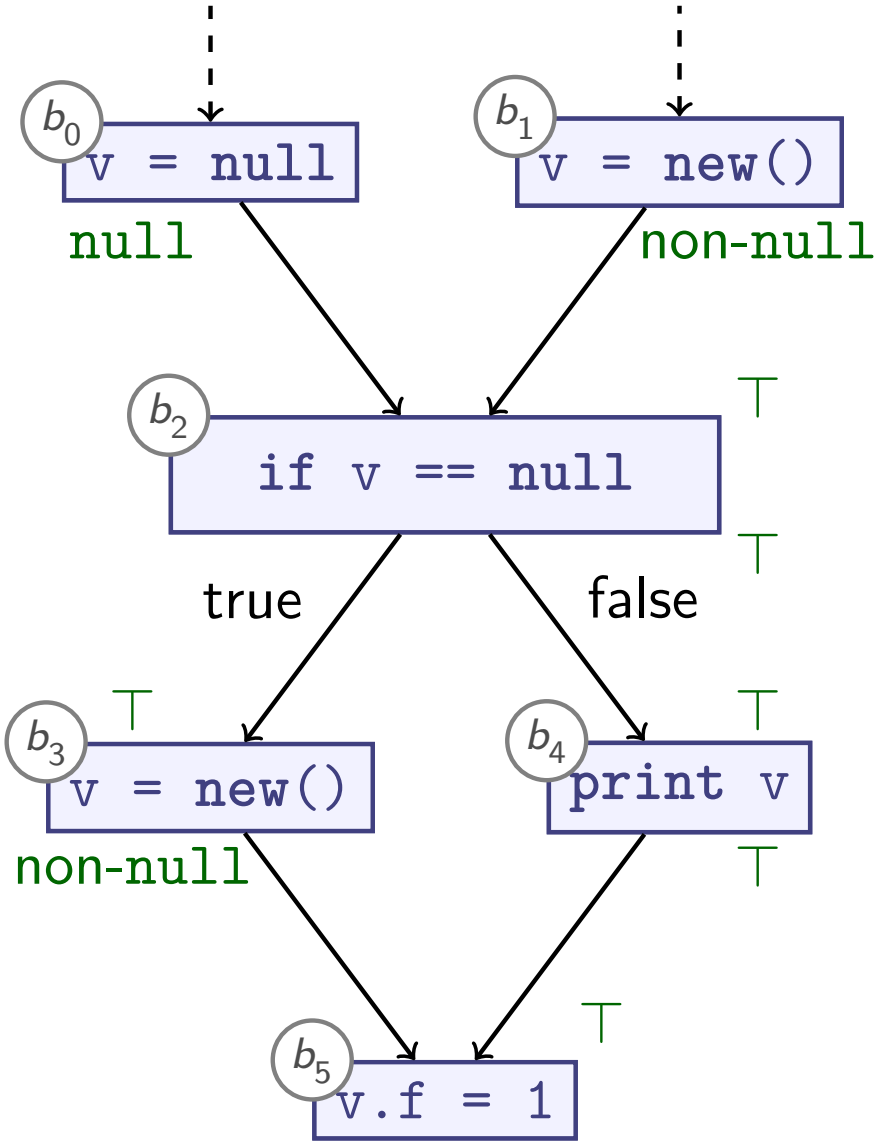
Summary

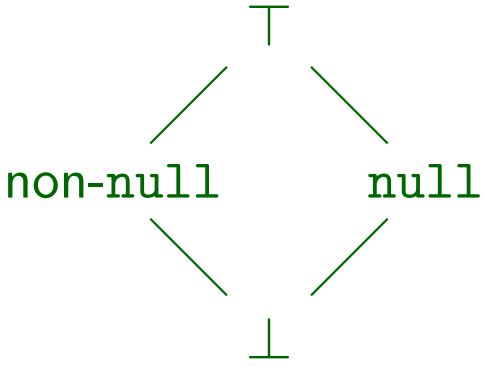
- **Rapid Type Analysis** resolves virtual calls $a.f()$ as follows:
  - Find all classes that can be instantiated in reachable code
  - Expand reachable code:
    - For direct calls to p, add p as reachable
    - For all virtual calls to $v.m()$ with $v : T$:
      $\Rightarrow$ Add $S.m()$ as reachable
  - Iterate until we reach a fixpoint
- **Sound**
  - Assuming strongly and statically typed language with subtyping
- More **precise** than Class Hierarchy Analysis

Path Sensitivity



Path Sensitivity

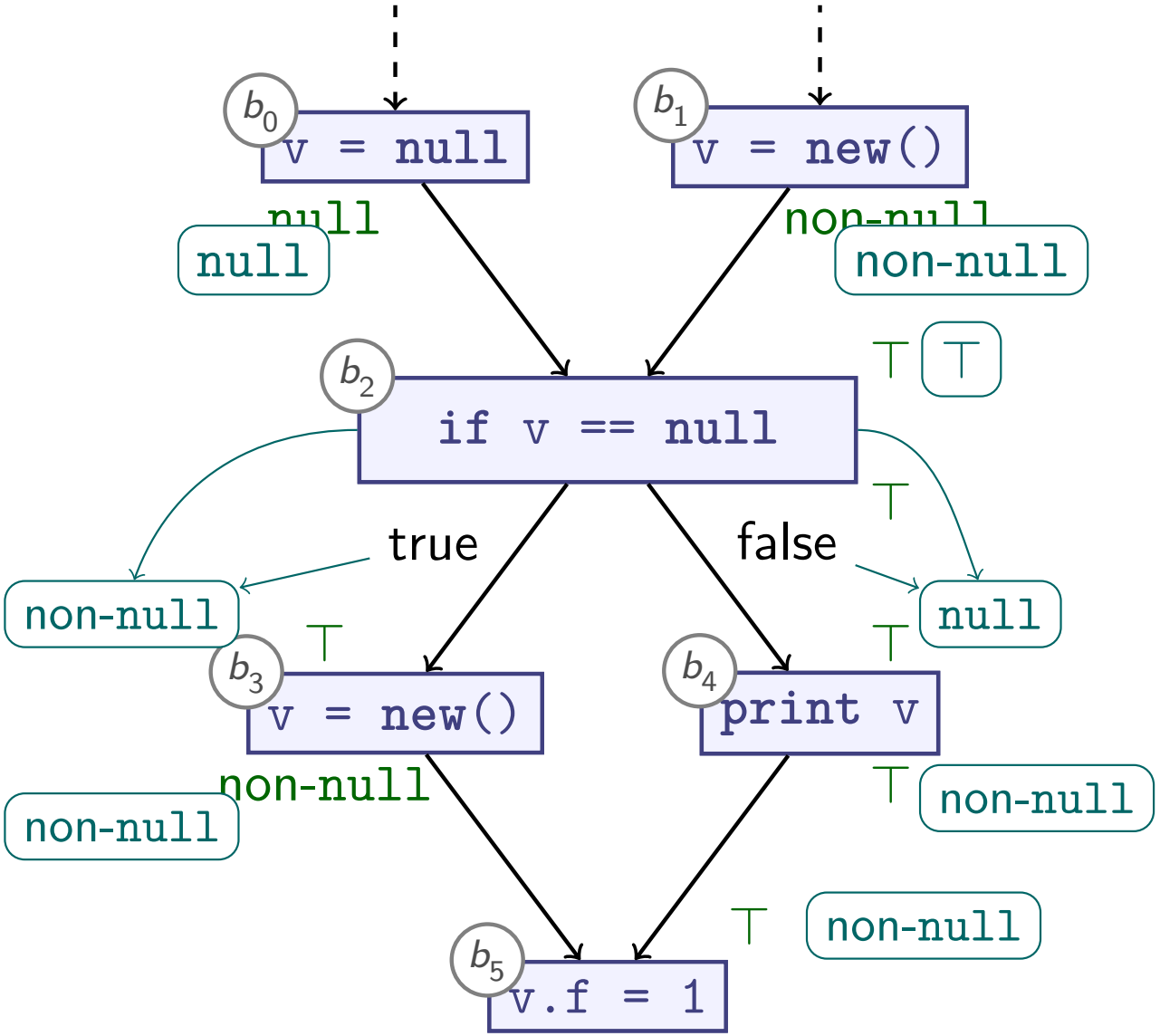


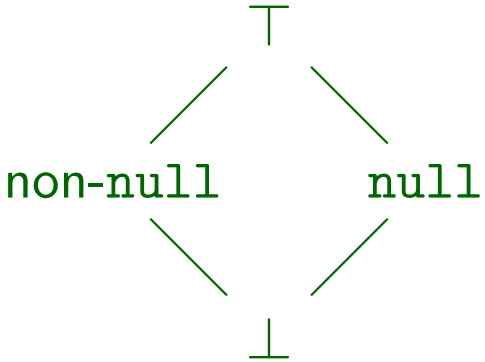


Path-insensitive

~~Path~~ Sensitivity

Control

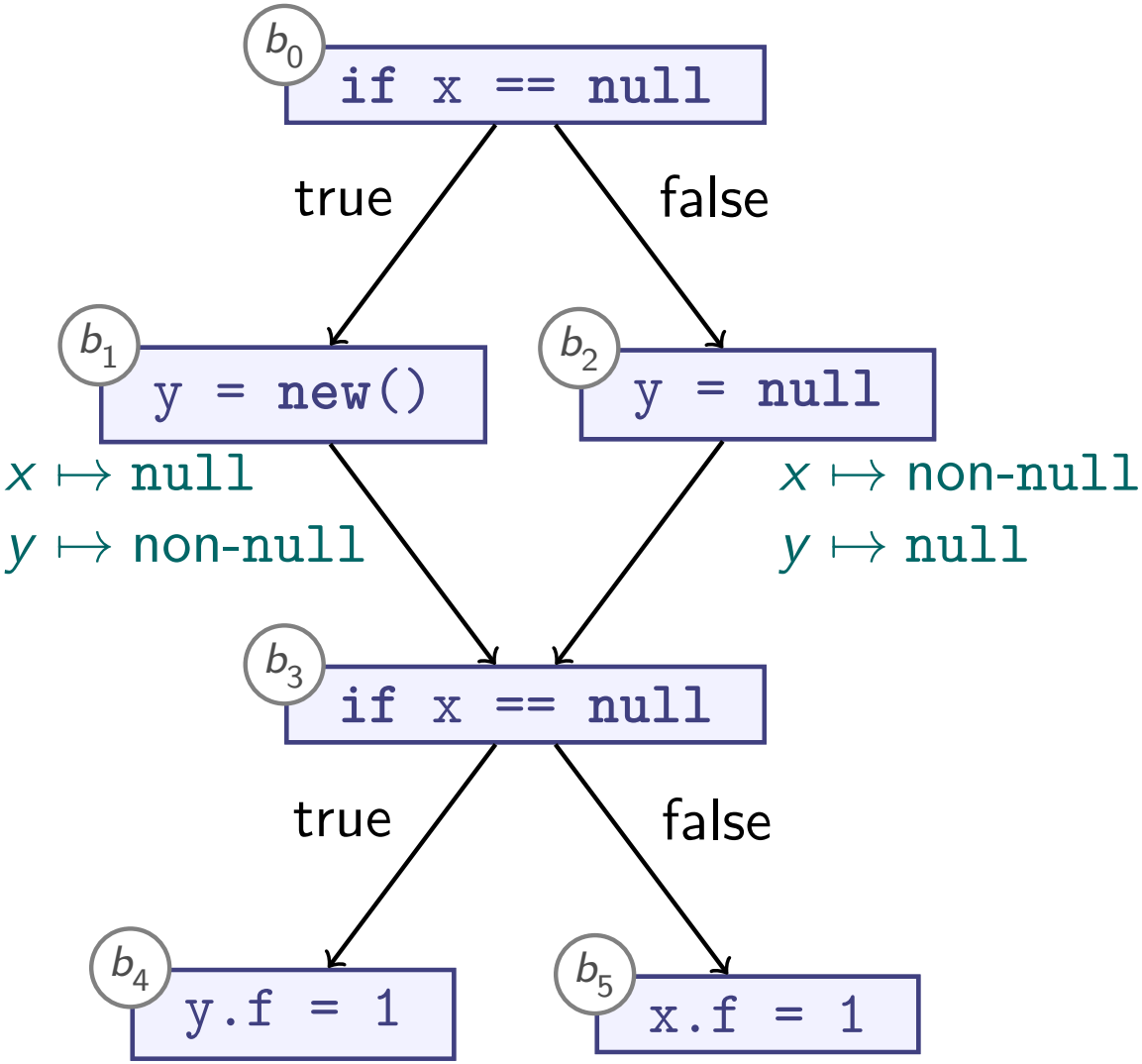




Path-insensitive

Path-sensitive

Multiple Conditionals



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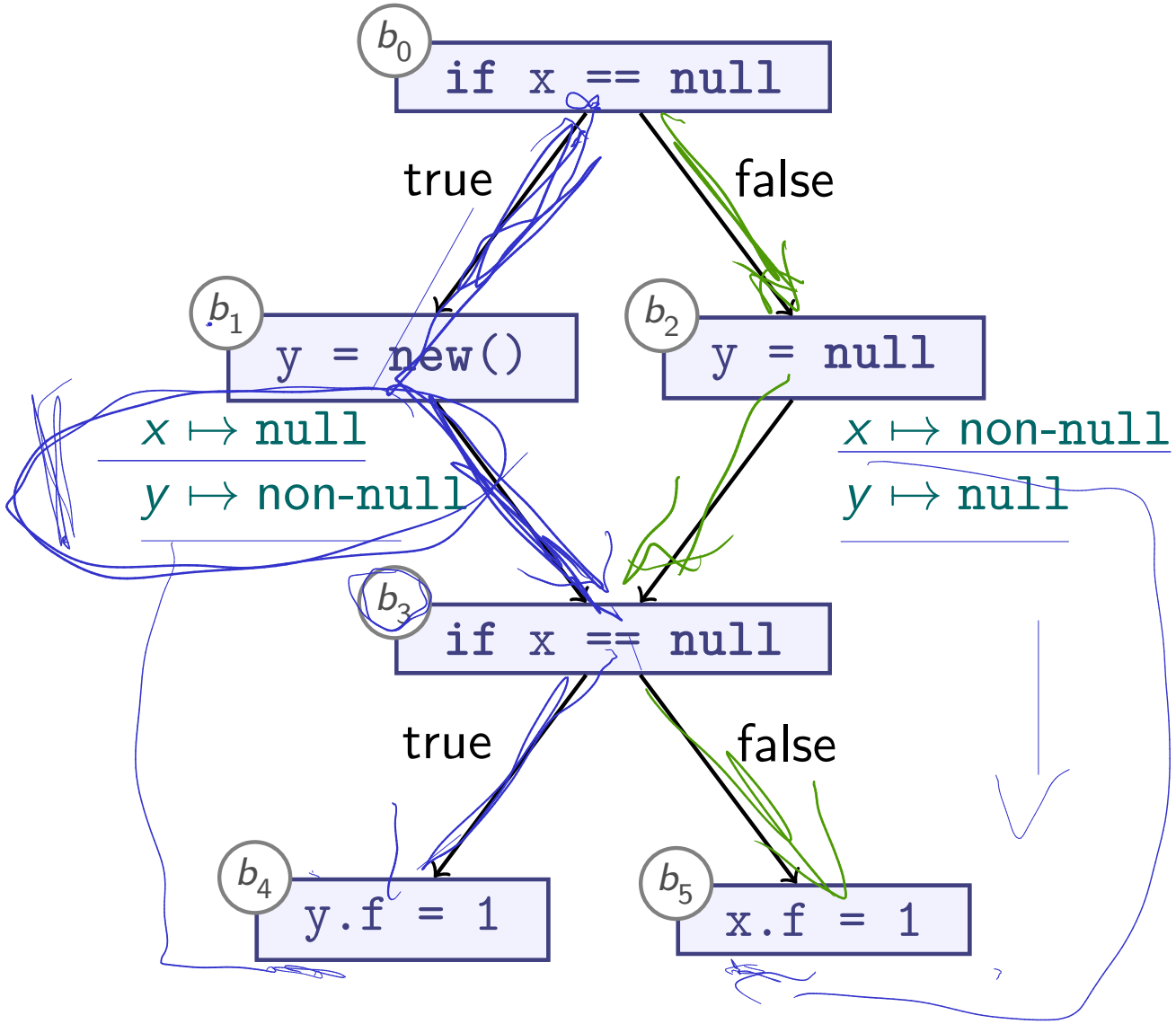
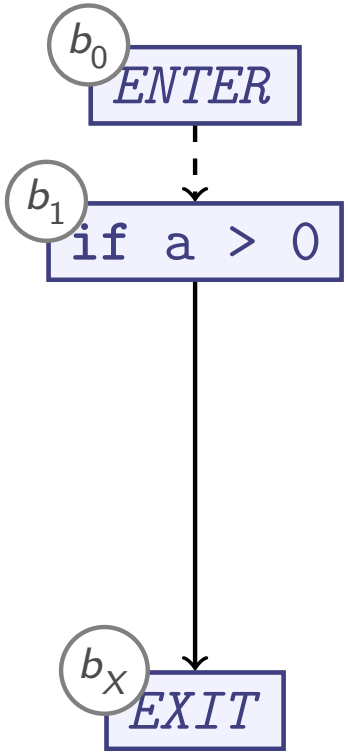


Should we carry path information across merge points?

Paths

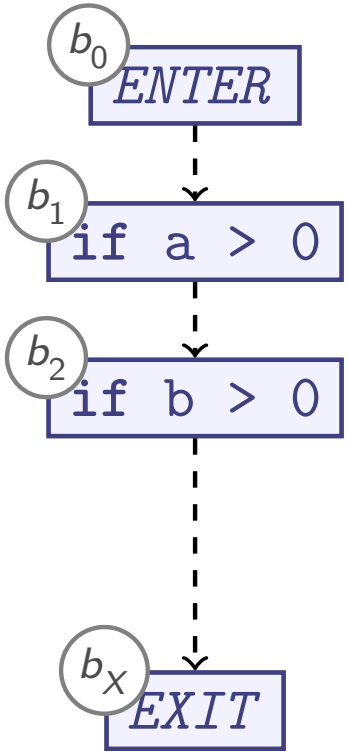
proc f(a, b, c)



2 paths

Paths

proc f(a, b, c)

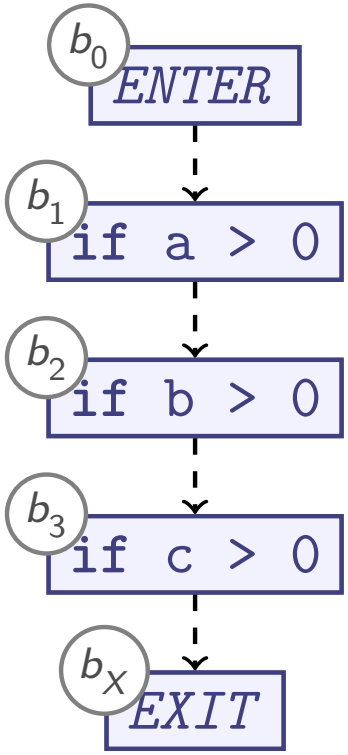


2 paths

4 paths

Paths

proc f(a, b, c)



2 paths

4 paths

8 paths

Number of paths grows exponentially

Summary

- **Path-sensitive** analysis considers conditionals:
  - May propagate different information along different paths
- Only for forward analyses
- Number of paths $O(\#$ of conditionals$)$
  - Avoid exponential blow-up by merging (as before)
  - Path-sensitive procedure summaries might require exponential number of cases
- Exponential analyses/representations usually not practical