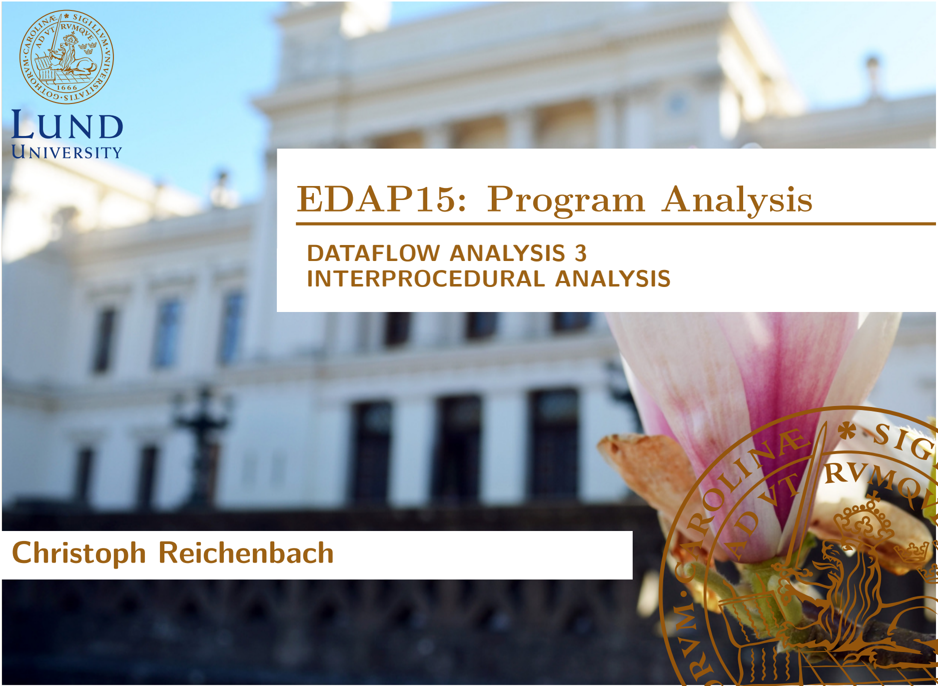


EDAP15: Program Analysis

DATAFLOW ANALYSIS 3
INTERPROCEDURAL ANALYSIS

Christoph Reichenbach

Inter- vs. Intra-Procedural Analysis

- **Intra**procedural: Within one procedure
  - Data flow analysis so far
- **Inter**procedural: Across multiple procedures
  - Type Analysis, especially. with polymorphic type inference

Limitations of Intra-Procedural Analysis

Teal-0

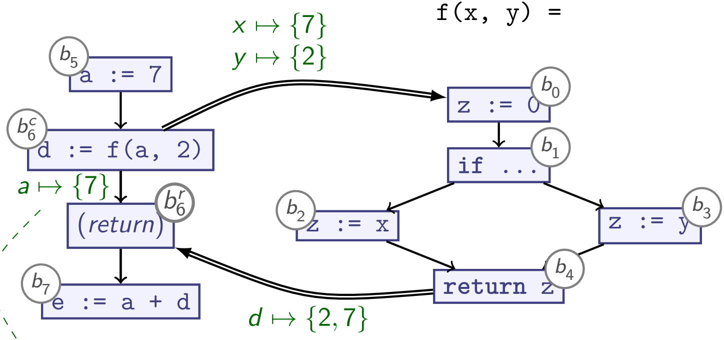
```
a := 7;
d := f(a, 2);
e := a + d;
```

Teal-0

```
fun f(x, y) = {
  z := 0;
  if x > y {
    z := x;
  } else {
    z := y;
  }
  return z;
}
```

How can we compute Reachable Definitions here?

A Naïve Inter-Procedural Analysis



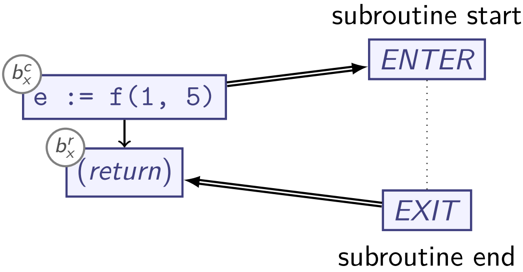
omitting 'obvious' transfer functions

$\mathbf{in}_{b_6^r} = \mathbf{out}_{b_6} \sqcup \mathbf{out}_{b_4} = \mathbf{out}_{b_6^r}$

- $\mathbf{out}_{b_7}$: $e \mapsto \{9, 14\}$

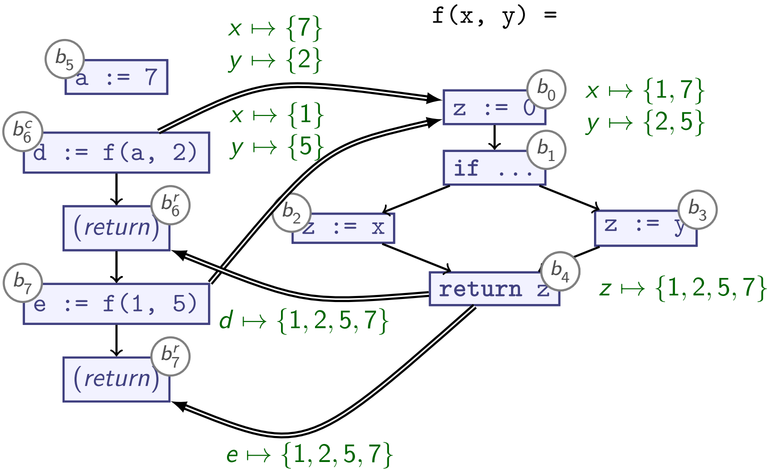
Works rather straightforwardly!

Inter-Procedural Data Flow Analysis



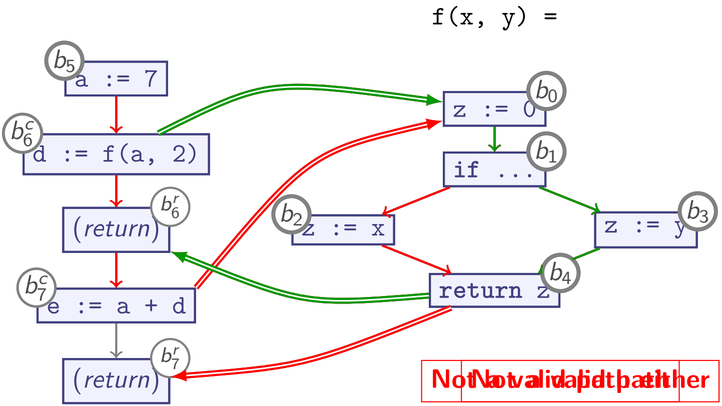
- Split call sites b_x into *call* (b_x^c) and *return* (b_x^r) nodes
- Intra-procedural edge $b_x^c \longrightarrow b_x^r$ carries environment/store
- Inter-procedural edge ($\Longrightarrow$):
  - Caller $\Longrightarrow$ subroutine, substitutes parameters (for pass-by-value)
  - Caller $\Longleftarrow$ return, substitutes result (for pass-by-result)
  - Otherwise as intra-procedural data flow edge

A Naïve Inter-Procedural Analysis



Imprecision!

Context Sensitivity: Valid Paths



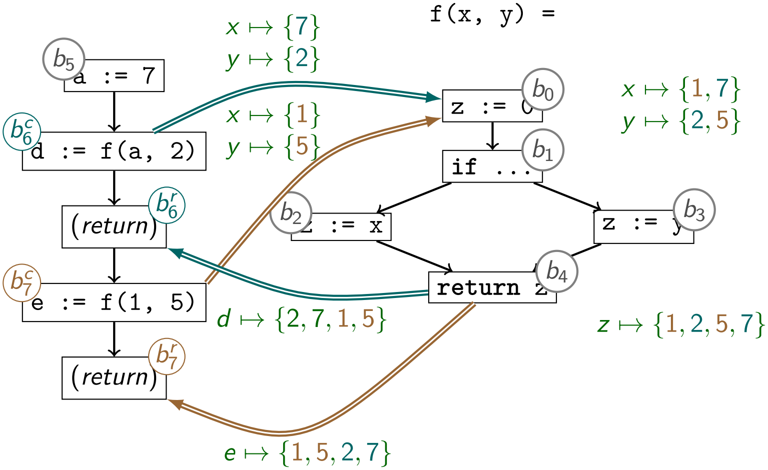
- $[b_5, b_6^c, b_0, b_1, b_3, b_4, b_6^r]$

Context-sensitive interprocedural analyses consider only valid paths

Summary

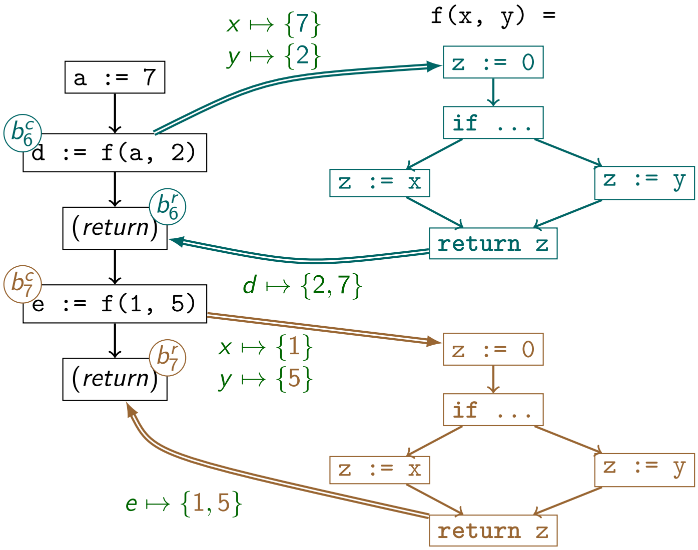
- **Intraprocedural** Data Flow Analysis is highly imprecise with subroutine calls
- **Interprocedural** Data Flow Analysis is more precise:
  - Split call site into call site + return site
  - Add flow edges between call sites, subroutine entry
  - Add flow edges between subroutine return, return site
  - Carry environment from call site to return site
- Interprocedural analysis must typically consider the entire program
  $\Rightarrow$ **whole-program analysis**
- Naïve interprocedural analysis is **context-insensitive**
  - Merge all callers into one

Interprocedural Data Flow Analysis



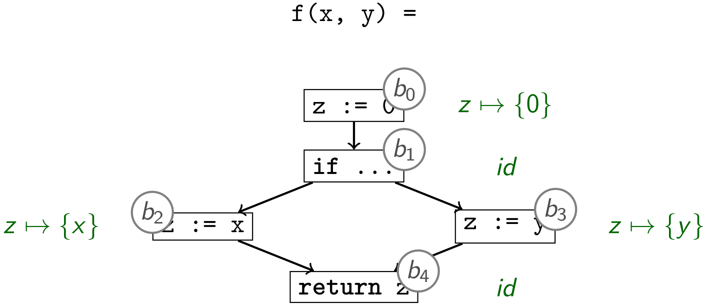
***Context-insensitive*: analysis merges all callers to `f()`**

Inlining



Clone subroutine IRs for each *calling context*

Alternative to Inlining: Summarise Procedure (Here: Reaching Defs.)



- *Compose* transfer functions:
  - $trans_{b_0} \circ trans_{b_1} = [z \mapsto 0]$
  - $trans_{b_0} \circ trans_{b_1} \circ trans_{b_2} = [z \mapsto \{x\}]$
  - $trans_{b_0} \circ trans_{b_1} \circ trans_{b_3} = [z \mapsto \{y\}]$
  - $trans_{b_0} \circ trans_{b_1} \circ (trans_{b_2} \sqcup trans_{b_3}) = [z \mapsto \{x, y\}]$
  - $trans_{b_0} \circ trans_{b_1} \circ (trans_{b_2} \sqcup trans_{b_3}) \circ trans_{b_4} = [z \mapsto \{x, y\}]$

Procedure Summaries vs Recursion

`f` calls `g` calls `h` calls `f`

- Reqiures additional analysis to identify who calls whom
- Compute summaries of mutually recursive functions together
- Recursive call edges analogous to loops

Procedure Summaries

- Composing transfer functions yields a combined transfer function for `f()`:

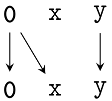
$$trans_f = [\textbf{return} \mapsto \{x, y\}]$$

- Use $trans_f$ as transfer function for `f()`, discard `f`'s body
- **Advantages:**
  - Can yield compact subroutine descriptions
  - Can speed up call site analysis dramatically
- **Disadvantages:**
  - More complex to implement
  - Recursion is challenging
- **Limitations:**
  - Requires suitable representation for summary
  - Requires mechanism for abstracting and applying summary
  - Worst cases:
    - $trans_f$ is symbolic expression as complex as `f` itself

Representation Relations

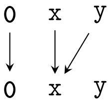
Example procedure summary representation:

```
x := null;
y := y;
```

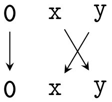


'May be null' analysis

```
if x != y {
  x := y;
}
y := 1;
```




```
{ t := x
  x := y
  y := t }
```



- $c \longrightarrow d$:
  if $P(c) \in \mathbf{in}_b$ then $P(d) \in \mathbf{out}_b$
- Representation Relations relate $\mathbf{in}_b$ and $\mathbf{out}_b$ variables $\mathcal{V}$
- $R \subseteq (\mathcal{V} \cup \{\mathbf{0}\}) \times (\mathcal{V} \cup \{\mathbf{0}\})$
- if $\langle \mathbf{0}, X \rangle \in R$:
  X always 'may be null' in $\mathbf{out}_b$
- if $\langle Y, X \rangle \in R$:
  If Y 'may be null' in $\mathbf{in}_b$:
  $\Rightarrow X$ 'may be null' in $\mathbf{out}_b$

Summary

- **Context-sensitive** analysis distinguishes 'calling context' when analysing subroutine
  - 'Who called me'?
  - Can go deeper: 'And who called them?'
- **Inlining** is one strategy for **context-sensitive** analysis
- Copy subroutine bodies for each caller
- Alternative: **Procedure summaries** built from composed transfer functions
- Can speed up context-sensitive analysis of popular functions, compared to inlining
- Needs some suitably abstract analysis *for the given program*
  - Example: IFDS-style **Representation Relations**
- Recursion is nontrivial:
  - Analyse function calls (*call graph*)
  - Analyse strongly connected components together

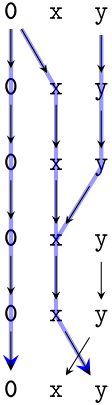
Composing Representation Relations

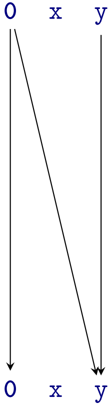
Recall Representation Relations (*may be null* analysis):

```
x := null;
y := y;
```

```
if x != y {
  x := y;
}
y := 1;
```

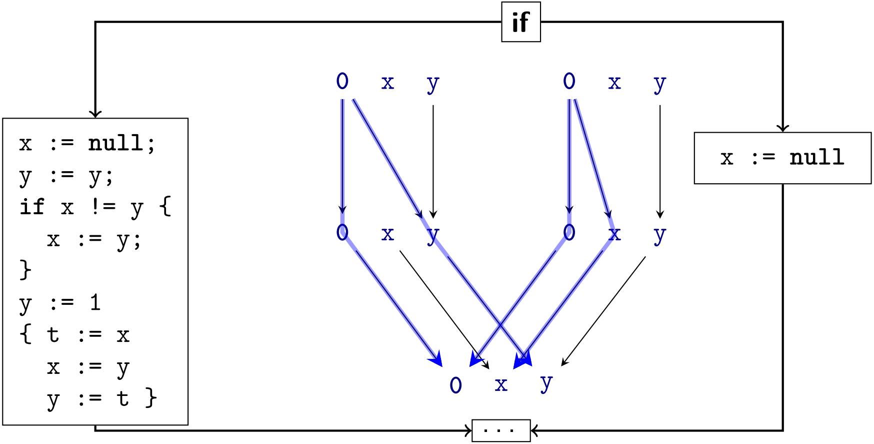
```
{ t := x;
  x := y;
  y := t; }
```





Composed representation relations are again representation relations

Merging Control-Flow Paths



Logical "Or"

Dataflow via Graph Reachability

$$n = \langle b, v \rangle$$

- Assume binary latice $(\{\top, \bot\}, \sqsubseteq, \sqcap, \sqcup)$
  - $a \sqcup b = \top$ iff $a = \bot$ and $b = \bot$, otherwise $a \sqcup b = \top$
  - Typical for 'May be X' analysis ('may be `null`')

- We can encode Dataflow problem as *Graph-Reachability*
- Graph nodes $n = \langle b, v \rangle$
  - b: CFG node
  - v: Variable or $\mathbf{0}$
    - Variable: Property of interest connected to variable
    - $\mathbf{0}$: Property of interest connected to executing this statement/block

Dataflow via Graph Reachability

$$n = \langle b, v \rangle$$

- Assume binary latice $(\{\top, \bot\}, \sqsubseteq, \sqcap, \sqcup)$
  - $a \sqcup b = \top$ iff $a = \bot$ and $b = \bot$, otherwise $a \sqcup b = \top$
  - Typical for 'May be X' analysis ('may be `null`')
  - Equivalently for 'Must' analysis:
    'must be `null`' = not ('may be `non-null`')
- We can encode Dataflow problem as *Graph-Reachability*
- Graph nodes $n = \langle b, v \rangle$
  - b: CFG node
  - v: Variable or $\mathbf{0}$
    - Variable: Property of interest connected to variable
    - $\mathbf{0}$: Property of interest connected to executing this statement/block

A Dataflow Worklist Algorithm: IFDS

- Context-sensitive interprocedural dataflow algorithm
- Historical name: IFDS
  (**I**nterprocedural **F**inite **D**istributive **S**ubset problems)
- 'Exploded Supergraph': $G^\sharp = (N^\sharp, E^\sharp)$
  - $N^\sharp = N_{\mathsf{CFG}} \times \mathcal{V} \cup \{0\}$
  - Plus parameter/return call edges
- b^s_{main} is the CFG *ENTER* node of the main entry point
- Property-of-interest holds if reachable from $\langle b^s_{\mathsf{main}}, \mathbf{0}\rangle$
- **Key ideas**:
  - Worklist-based
  - Construct Representation Relations on demand
  - Construct 'Exploded Supergraph'
    - CFG of all functions $\times\ \mathcal{V} \cup \{\mathbf{0}\}$

IFDS Datastructures

Instead of $\langle\langle b_0, v_0\rangle, \langle b_3, v_0\rangle\rangle$ we also write:

$$\langle b_0, v_0\rangle \rightarrow \langle b_3, v_0\rangle$$

WorkList edge

$$\langle b_0, v_0\rangle \dashrightarrow \langle b_3, v_0\rangle$$

All WorkList edges are also PathEdge edges

PathEdge edge

Result of our analysis

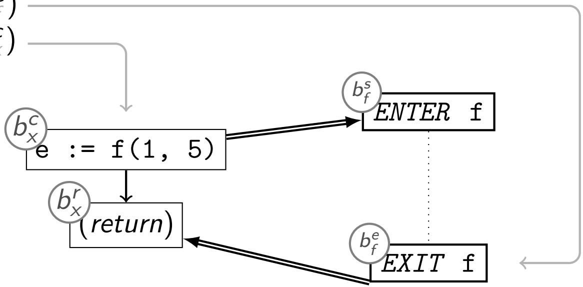
$N^\sharp$-edge

SummaryInst

Generated from summary nodes
Otherwise equivalent to $N^\sharp$-edges

IFDS Strategy

- Algorithm distinguishes between three types of nodes:
  - *Exit* nodes (b^e_f)
  - *Call* nodes (b^c_x)
  - Other nodes



On-demand processing

Procedure propagate($n_1 \to n_2$):
begin
  if $n_1 \to n_2 \in \textsc{PathEdge}$ **then**
    return
  $\textsc{PathEdge} := \textsc{PathEdge} \cup \{n_1 \to n_2\}$
  $\textsc{WorkList} := \textsc{WorkList} \cup \{n_1 \to n_2\}$
end

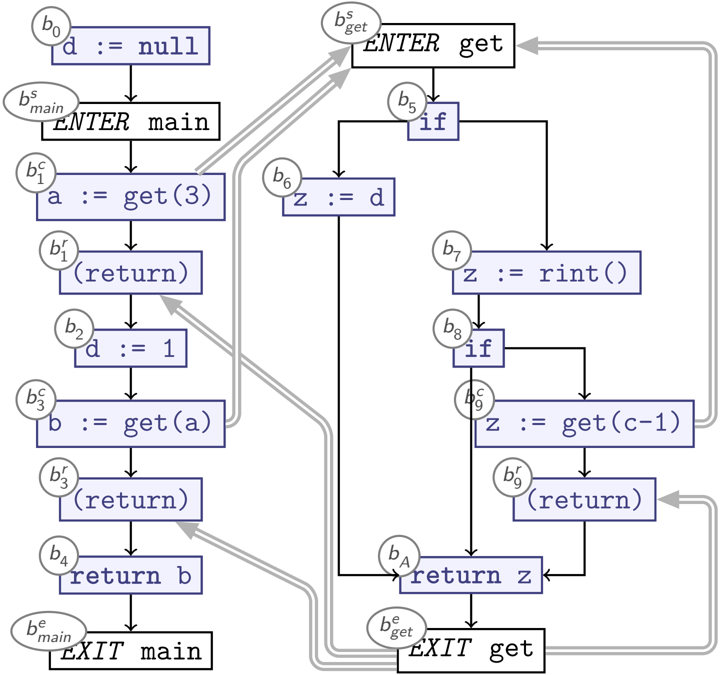
Running Example

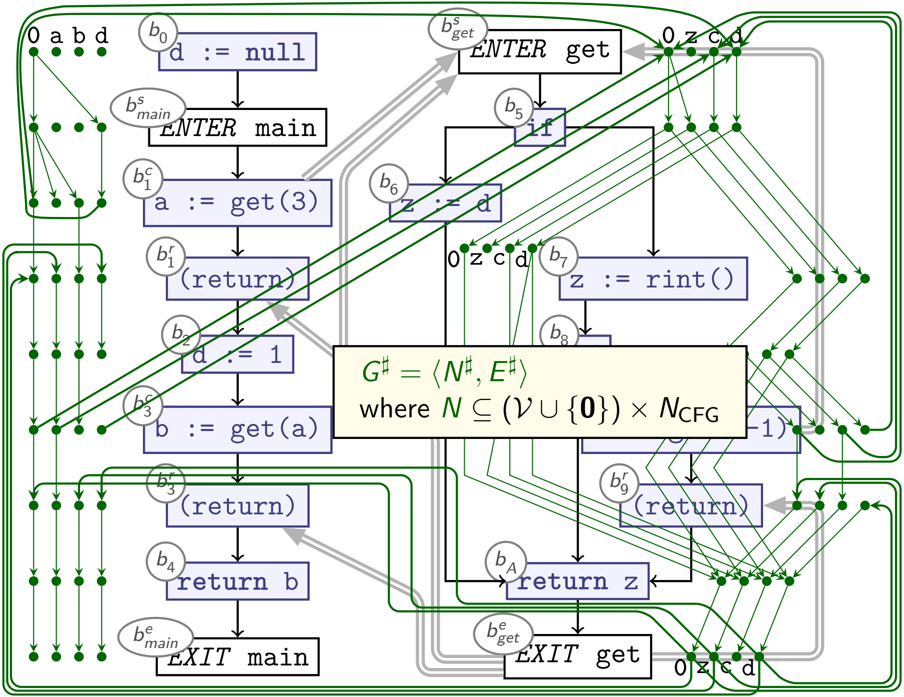
Teal-0: *main()*

```
var default := null;
fun main() = {
  var a := get(3);
  default := 1;
  var b := get(3);
  return b;
}
```

Teal-0: *get()*

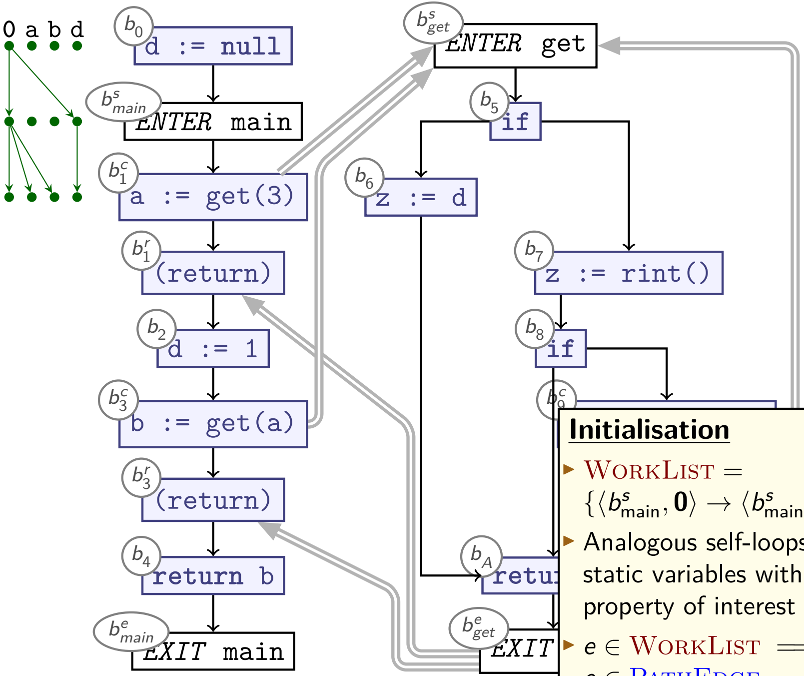
```
fun get(c) = {
  if c == 0 {
    z := default;
  } else {
    z := read_int();
    if z < 0 {
      z := get(c - 1);
    }
  }
  return z;
}
```


b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main
b^s_{get}
ENTER get
b_5
if
b_6
z := d
b_7
z := rint()
b_8
if
b^c_9
z := get(c-1)
b^r_9
(return)
b_A
return z
b^e_{get}
EXIT get



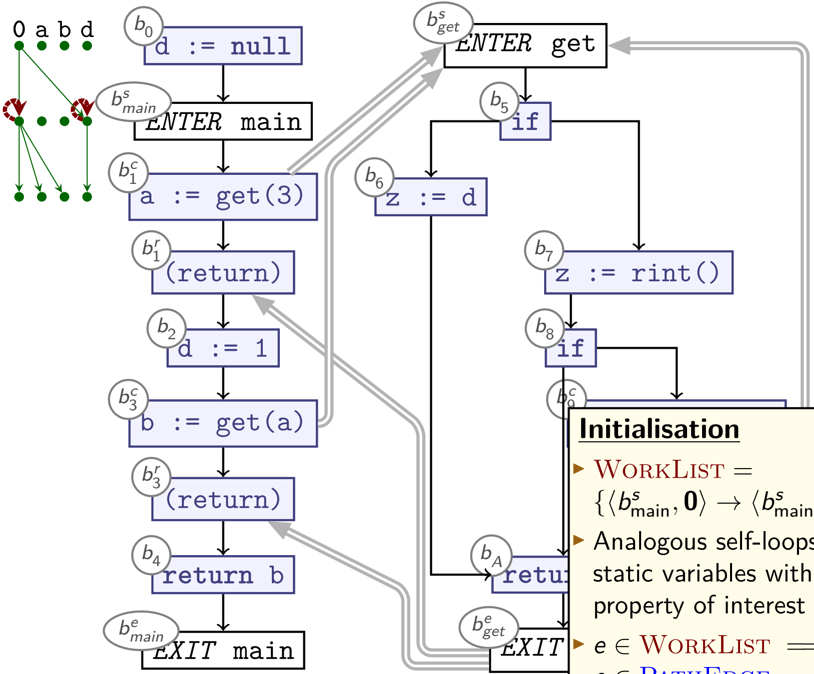
$G^\sharp = \langle N^\sharp, E^\sharp \rangle$

where $N \subseteq (\mathcal{V} \cup \{\mathbf{0}\}) \times N_{\mathsf{CFG}}$



Initialisation

- WorkList = $\{\langle b^s_{\text{main}}, \mathbf{0}\rangle \to \langle b^s_{\text{main}}, \mathbf{0}\rangle\}$
- Analogous self-loops for static variables with property of interest (d)
- $e \in$ WorkList $\implies$ $e \in$ PathEdge



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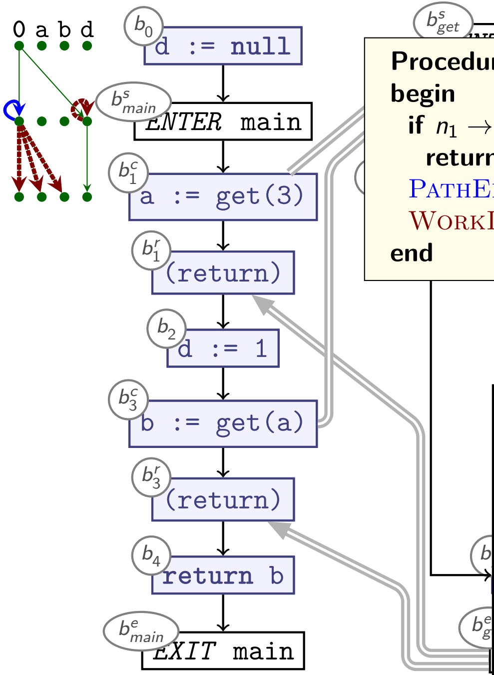

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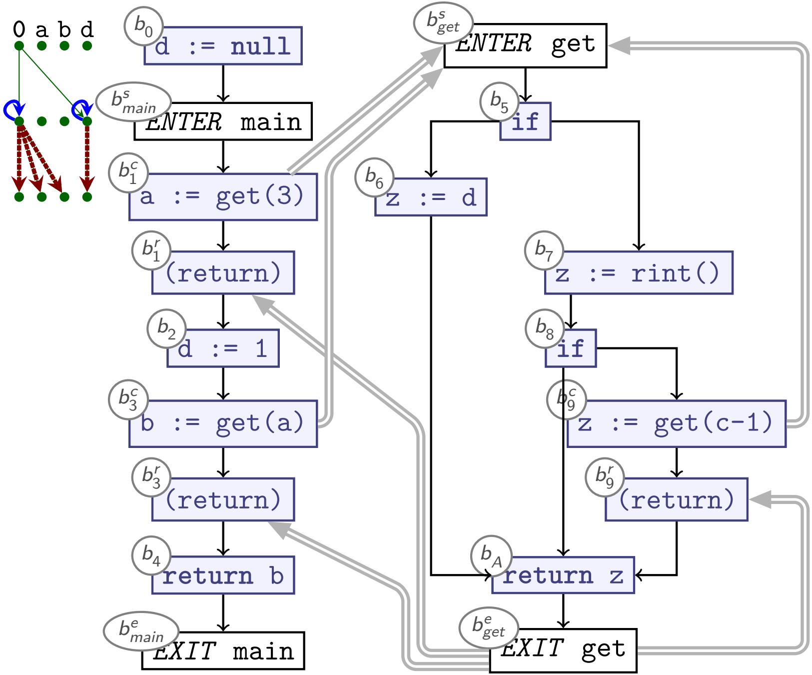
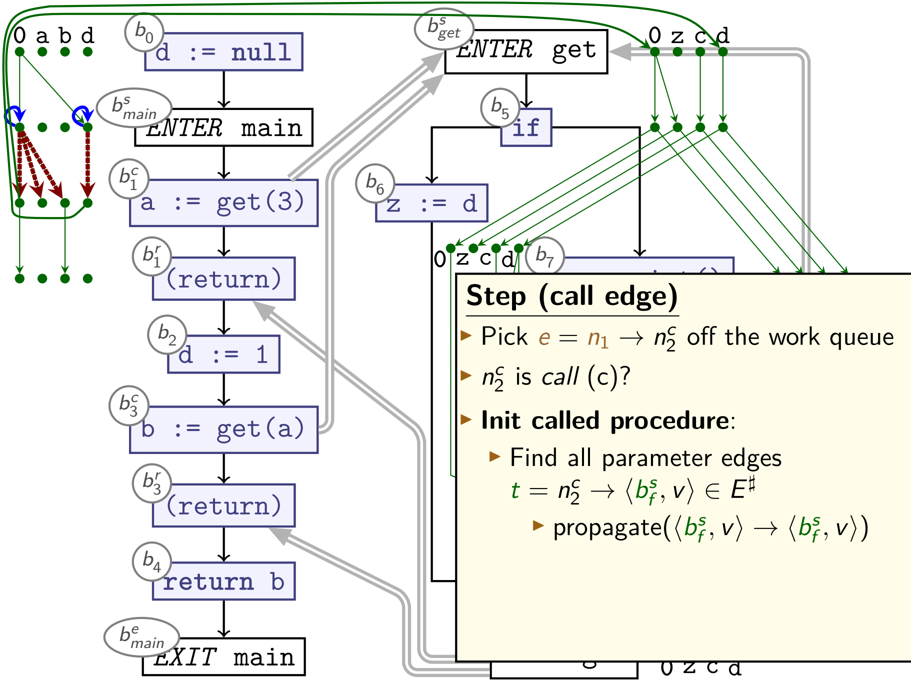
begin
  if $n_1 \to n_2 \in \textsc{PathEdge}$ **then**
    return
  $\textsc{PathEdge} := \textsc{PathEdge} \cup \{n_1 \to n_2\}$
  $\textsc{WorkList} := \textsc{WorkList} \cup \{n_1 \to n_2\}$
end

Step (regular edge)

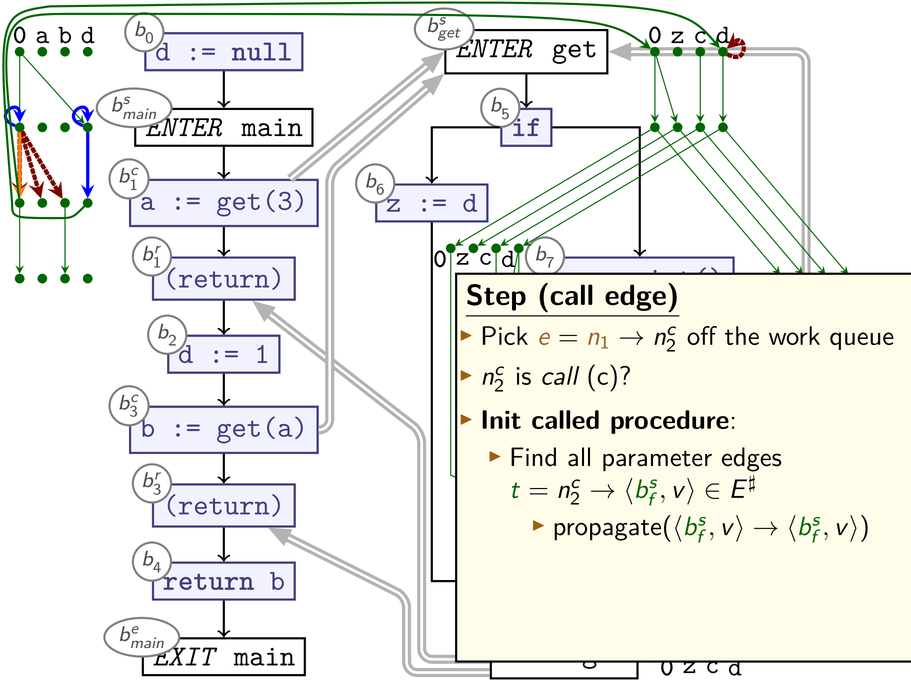
- Pick e off the work queue
  $e = n_1 \to n_2$
- n_2 neither call (c) nor exit (e)?
- Find all $n_2 \to n_3$
  propagate($n_1 \to n_3$)
- Remove e from $\textsc{WorkList}$
- e remains in $\textsc{PathEdge}$


0 a b d
b_0 d := null
b^s_{main} ENTER main
b^c_1 a := get(3)
b^r_1 (return)
b_2 d := 1
b^c_3 b := get(a)
b^r_3 (return)
b_4 return b
b^e_{main} EXIT main
b^s_{get} ENTER get
b_5 if
b_6 z := d
b_7 z := rint()
b_8 if
b^c_9 z := get(c-1)
b^r_9 (return)
b_A return z
b^e_{get} EXIT get



Step (call edge)

- Pick $e = n_1 \to n_2^c$ off the work queue
- n_2^c is *call* (c)?
- **Init called procedure**:
  - Find all parameter edges $t = n_2^c \to \langle b_f^s, v \rangle \in E^\sharp$
    - propagate$(\langle b_f^s, v\rangle \to \langle b_f^s, v\rangle)$



Step (call edge)

- Pick $e = n_1 \to n_2^c$ off the work queue
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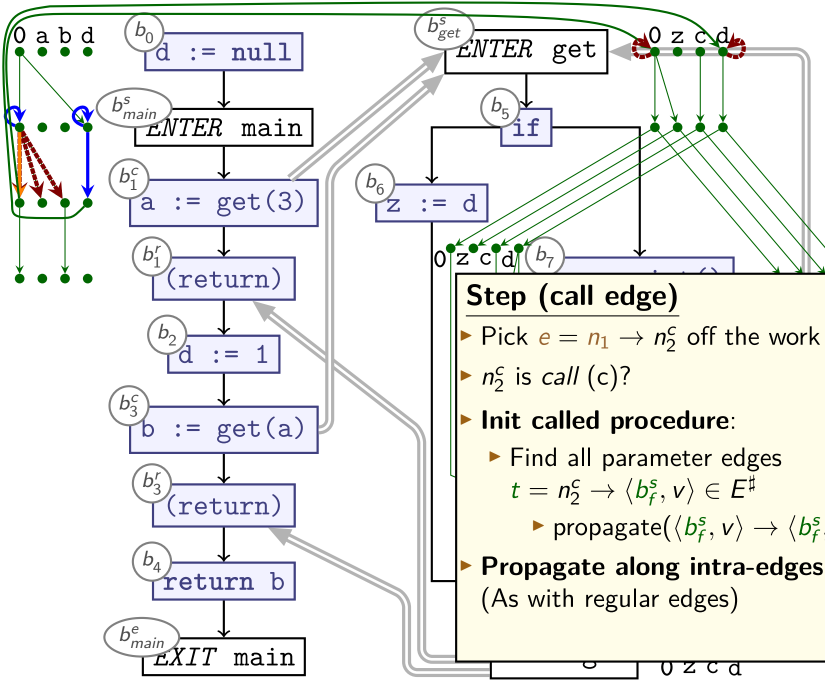
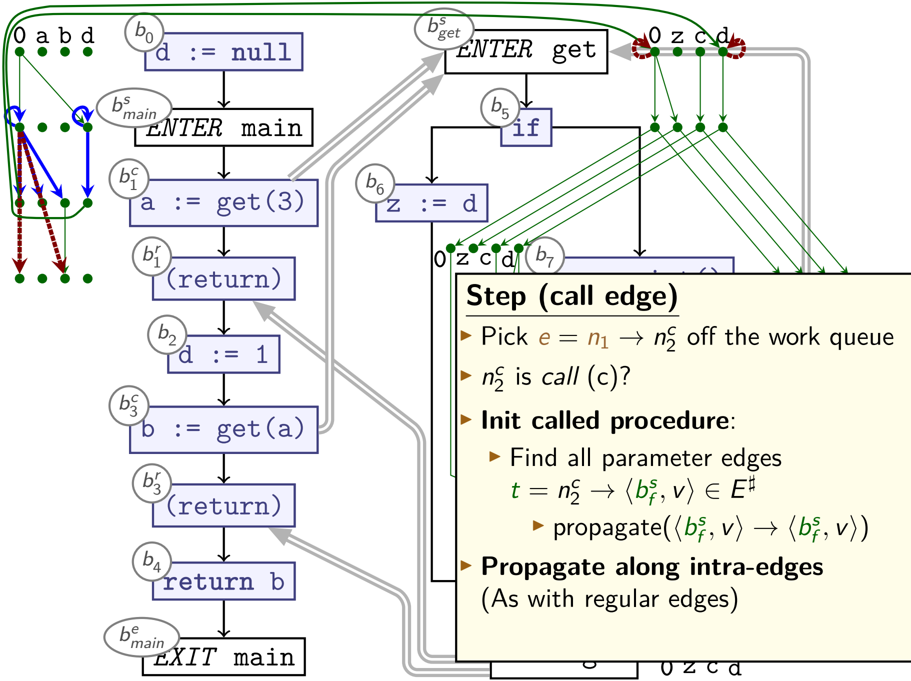
    - propagate$(\langle b_f^s, v\rangle \to \langle b_f^s, v\rangle)$



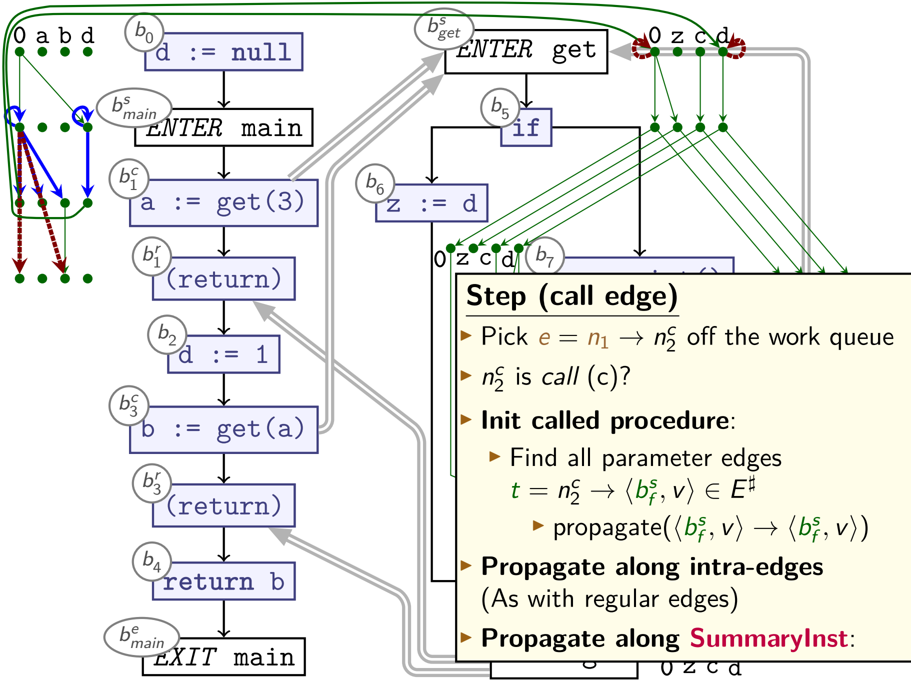
Step (call edge)

- Pick $e = n_1 \to n_2^c$ off the work queue
- n_2^c is *call* (c)?
- **Init called procedure**:
  - Find all parameter edges $t = n_2^c \to \langle b_f^s, v \rangle \in E^\sharp$
    - propagate($\langle b_f^s, v \rangle \to \langle b_f^s, v \rangle$)
- **Propagate along intra-edges**
  (As with regular edges)



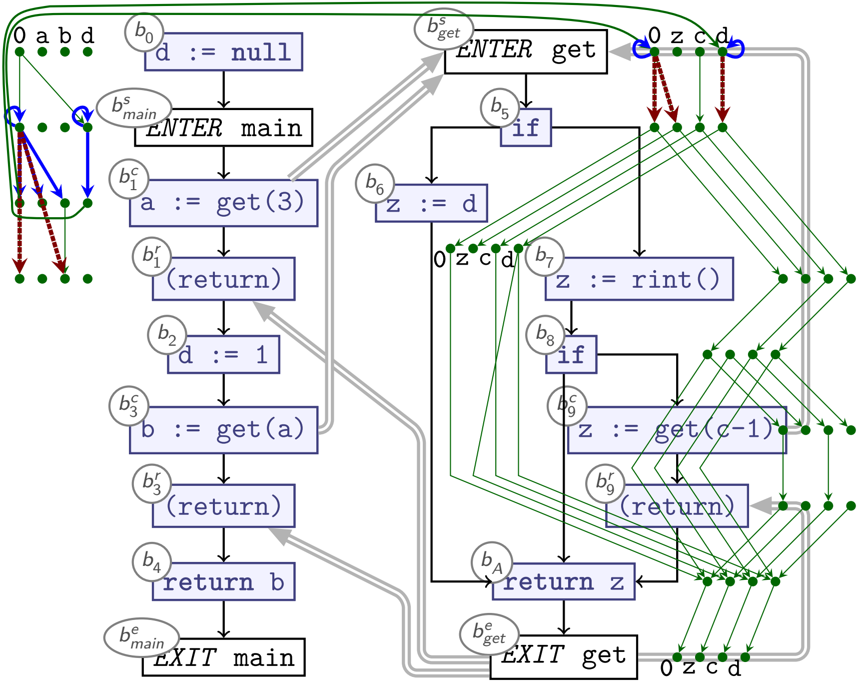
Step (call edge)

- Pick $e = n_1 \to n_2^c$ off the work queue
- n_2^c is *call* (c)?
- **Init called procedure**:
  - Find all parameter edges $t = n_2^c \to \langle b_f^s, v\rangle \in E^\sharp$
    - propagate($\langle b_f^s, v\rangle \to \langle b_f^s, v\rangle$)
- **Propagate along intra-edges**
  (As with regular edges)

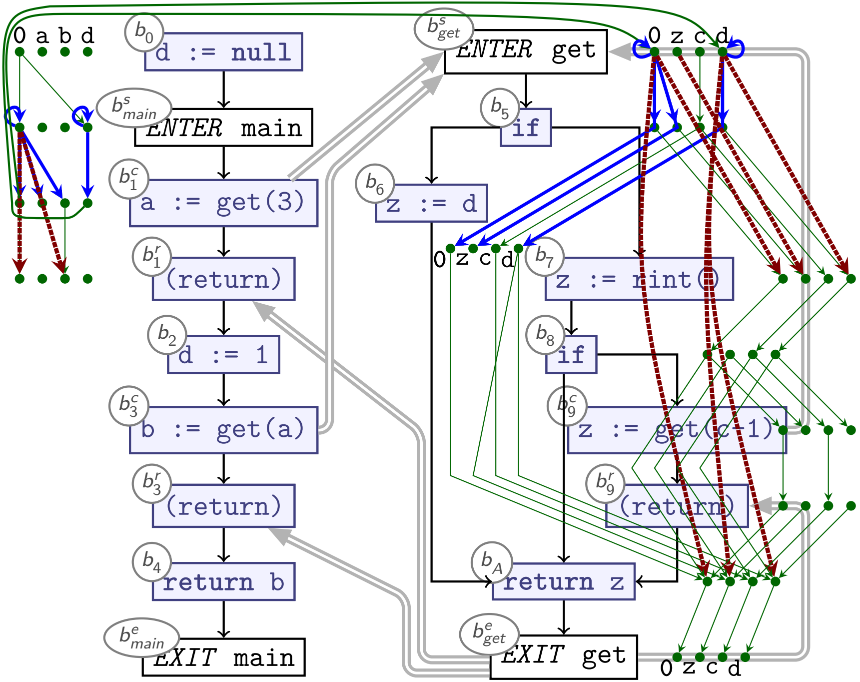


Step (call edge)

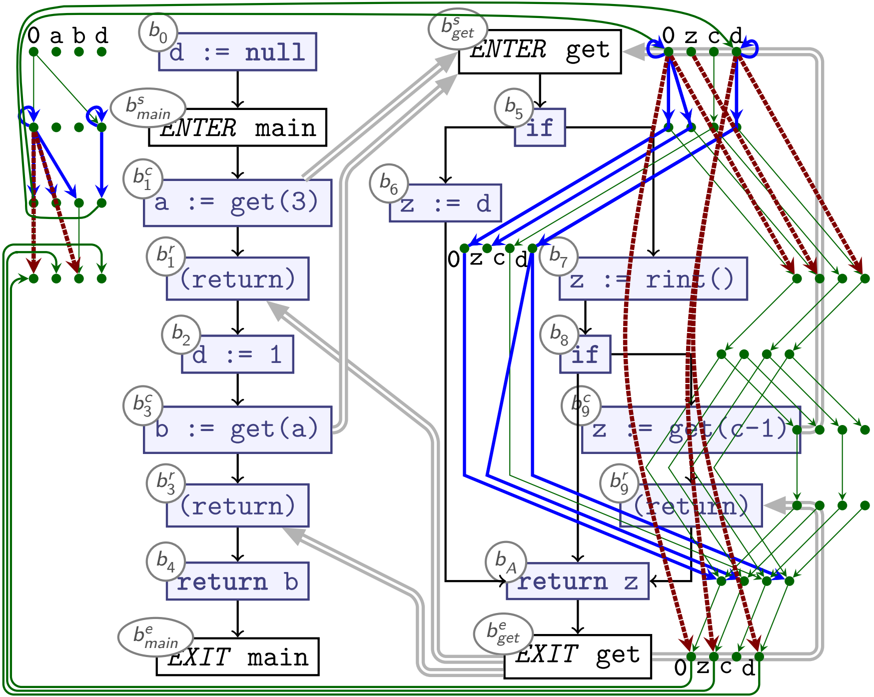
- Pick $e = n_1 \to n_2^c$ off the work queue
- n_2^c is *call* (c)?
- **Init called procedure**:
  - Find all parameter edges $t = n_2^c \to \langle b_f^s, v \rangle \in E^\sharp$
    - propagate($\langle b_f^s, v \rangle \to \langle b_f^s, v \rangle$)
- **Propagate along intra-edges**
  (As with regular edges)
- **Propagate along SummaryInst**:

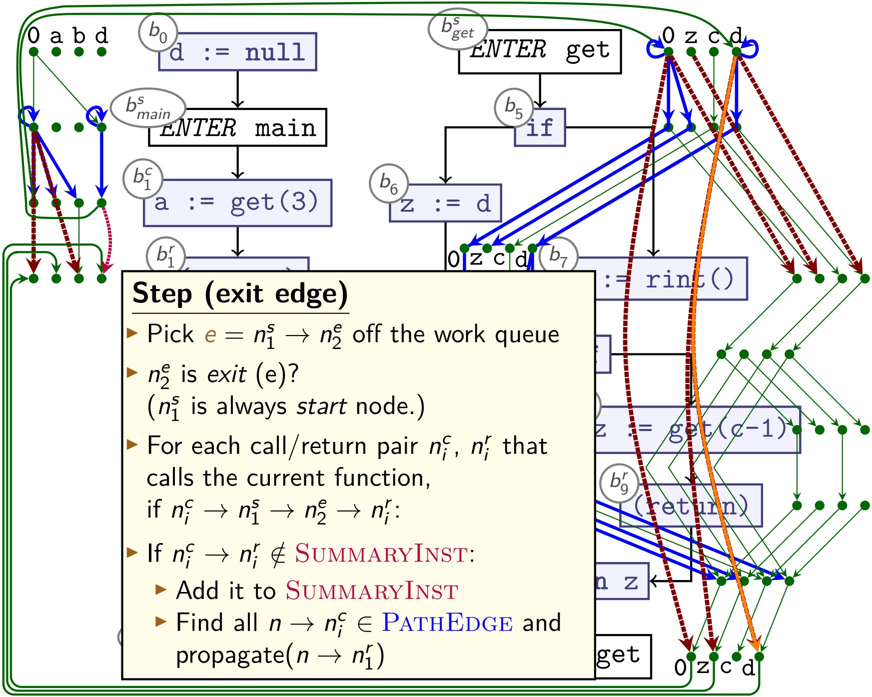


0 a b d
d := null
ENTER main
a := get(3)
(return)
d := 1
b := get(a)
(return)
return b
EXIT main
0 z c d
ENTER get
if
z := d
0 z c d
z := rint()
if
z := get(c-1)
(return)
return z
EXIT get
0 z c d



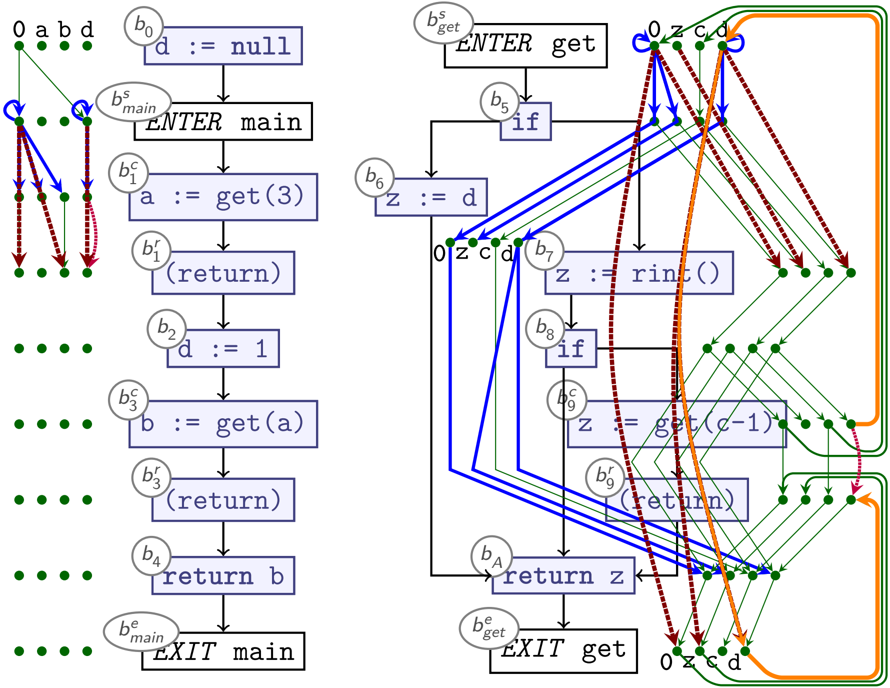
0 a b d
b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main
b^s_{get}
ENTER get
0 z c d
b_5
if
b_6
z := d
0 z c d
b_7
z := rint()
b_8
if
b^c_9
z := get(c-1)
b^r_9
(return)
b_A
return z
b^e_{get}
EXIT get
0 z c d


0 a b d
b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main
b^s_{get}
ENTER get
0 z c d
b_5
if
b_6
z := d
0 z c d
b_7
z := rint()
b_8
if
b^c_9
z := get(c-1)
b^r_9
(return)
b_A
return z
b^e_{get}
EXIT get
0 z c d

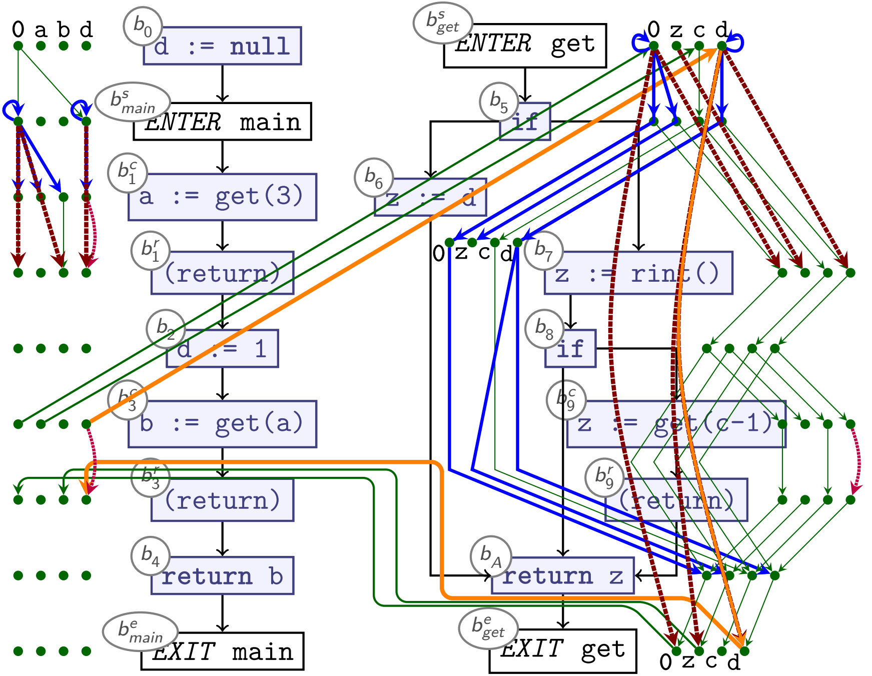


Step (exit edge)

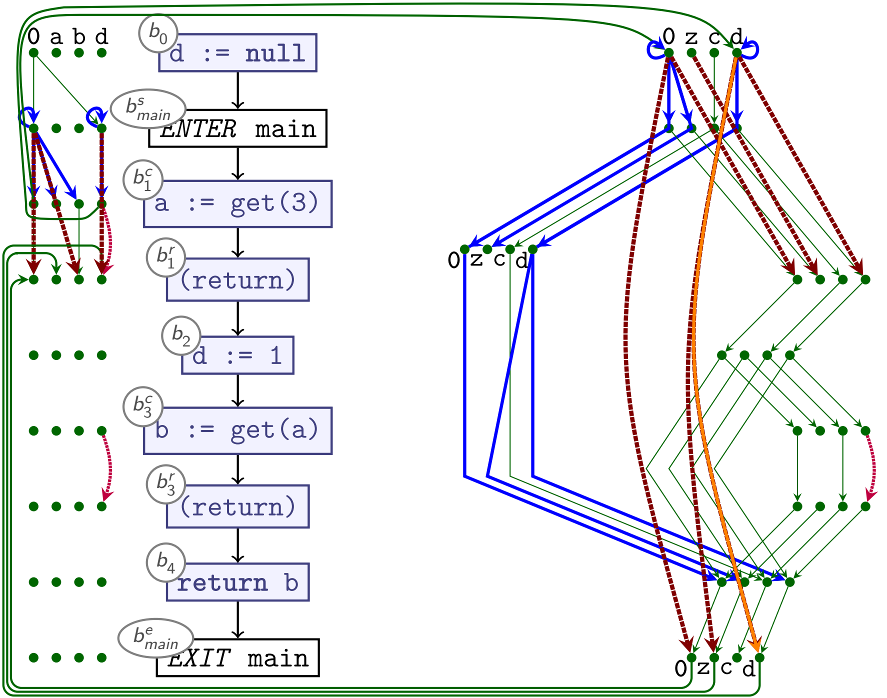
- Pick $e = n_1^s \to n_2^e$ off the work queue
- n_2^e is *exit* (e)?
  (n_1^s is always *start* node.)
- For each call/return pair n_i^c, n_i^r that calls the current function,
  if $n_i^c \to n_1^s \to n_2^e \to n_i^r$:
- If $n_i^c \to n_i^r \notin$ SummaryInst:
  - Add it to SummaryInst
  - Find all $n \to n_i^c \in$ PathEdge and propagate($n \to n_1^r$)



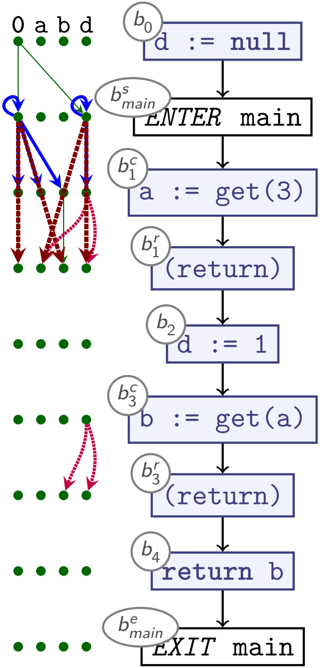
0 a b d
b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main
b^s_{get}
ENTER get
0 z c d
b_5
if
b_6
z := d
0 z c d
b_7
z := rint()
b_8
if
b^c_9
z := get(c-1)
b^r_9
(return)
b_A
return z
b^e_{get}
EXIT get
0 z c d

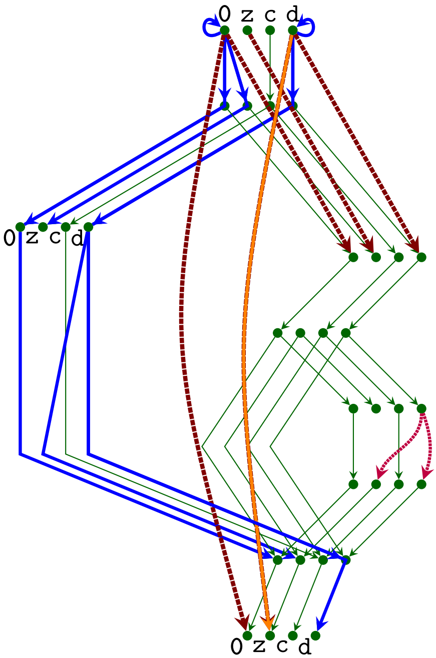


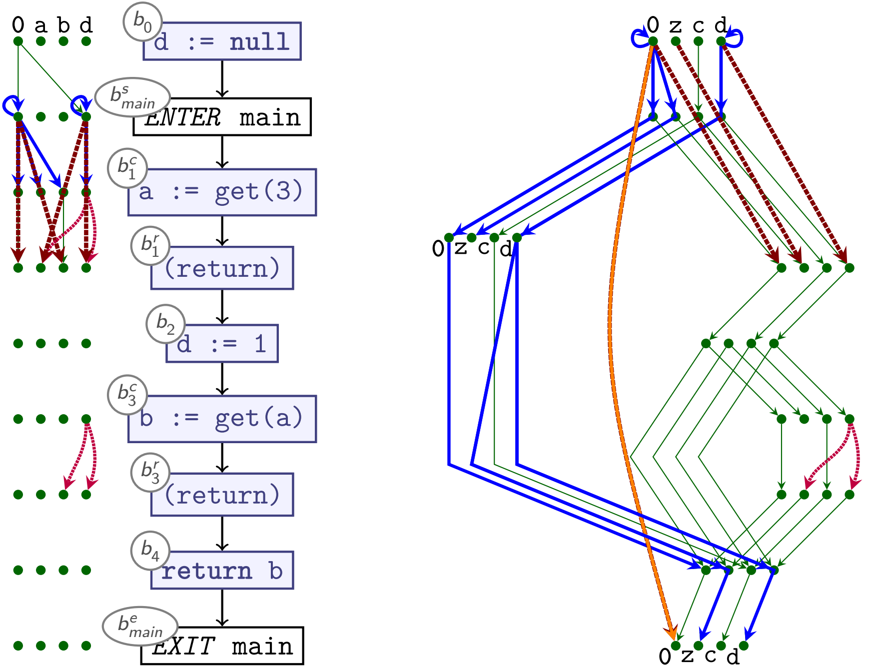
0 a b d
d := null
ENTER main
a := get(3)
(return)
d := 1
b := get(a)
(return)
return b
EXIT main
ENTER get
if
z := d
0 z c d
z := rint()
if
z := get(c-1)
(return)
return z
EXIT get

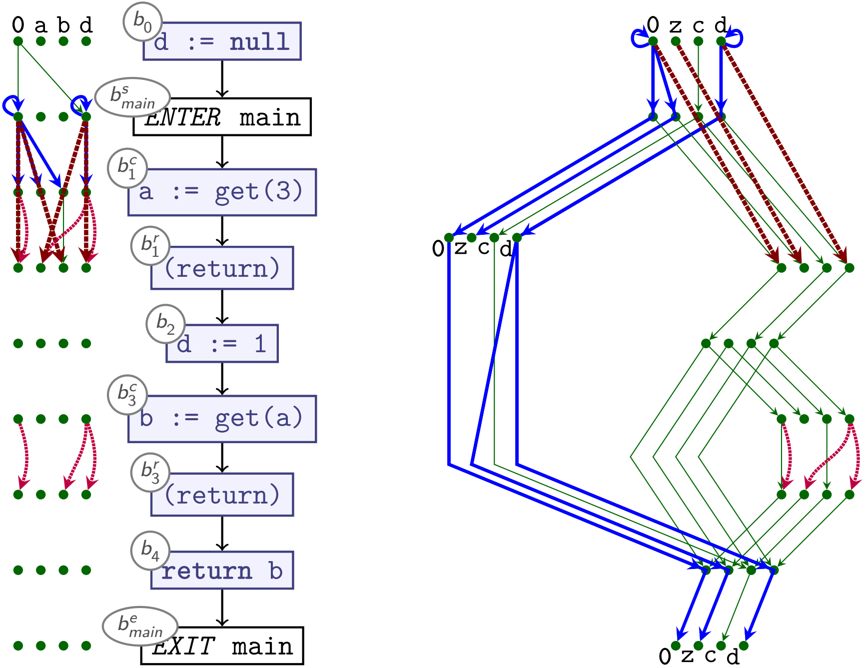


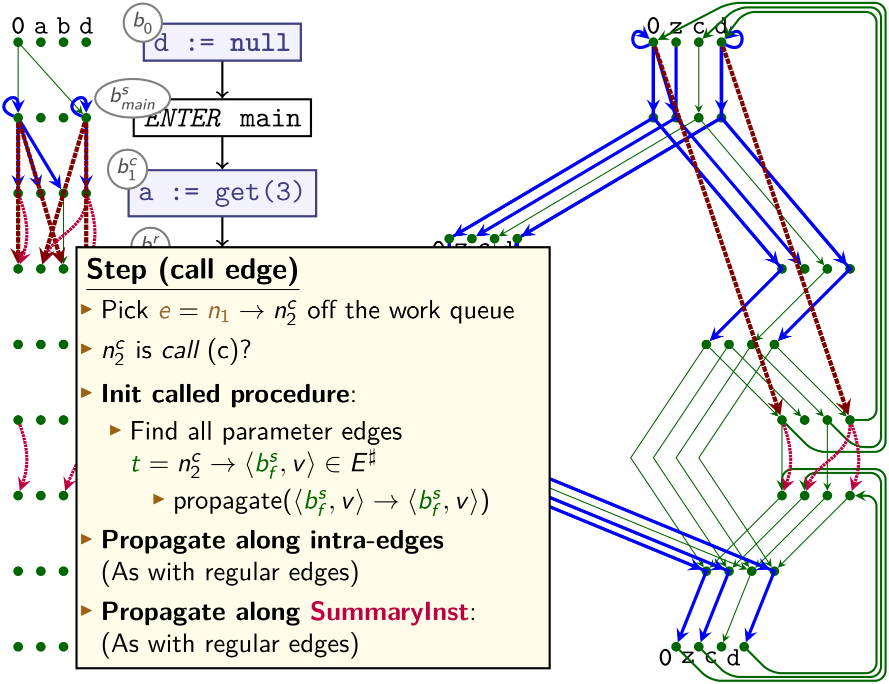
0 a b d
b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main
0 z c d
0 z c d
0 z c d


0 a b d
b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main


0 z c d
0 z c d
0 z c d

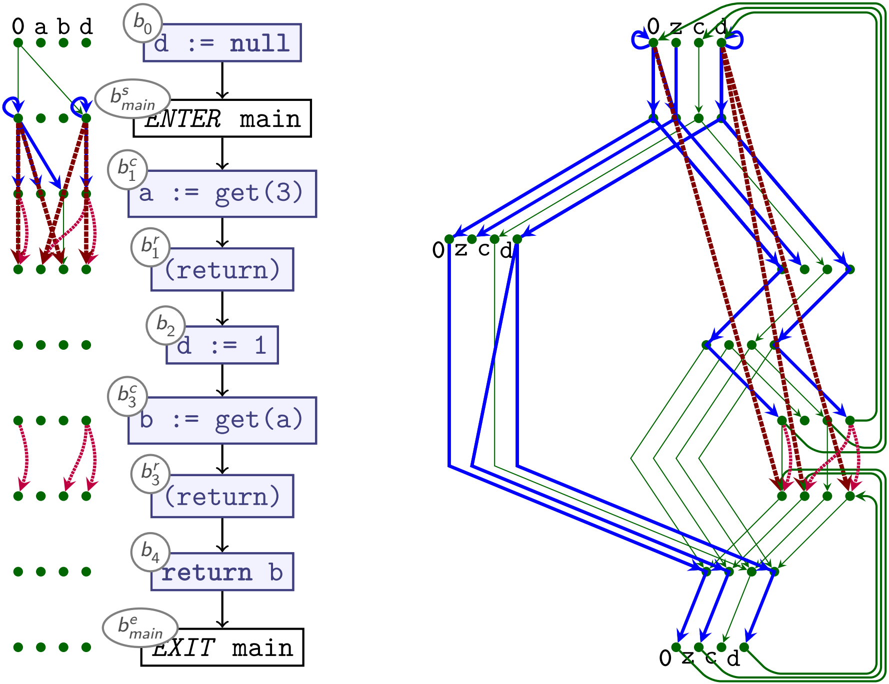

0 a b d
b0
d := null
b^s_main
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b4
return b
b^e_main
EXIT main
0 z c d
0 z c d
0 z c d

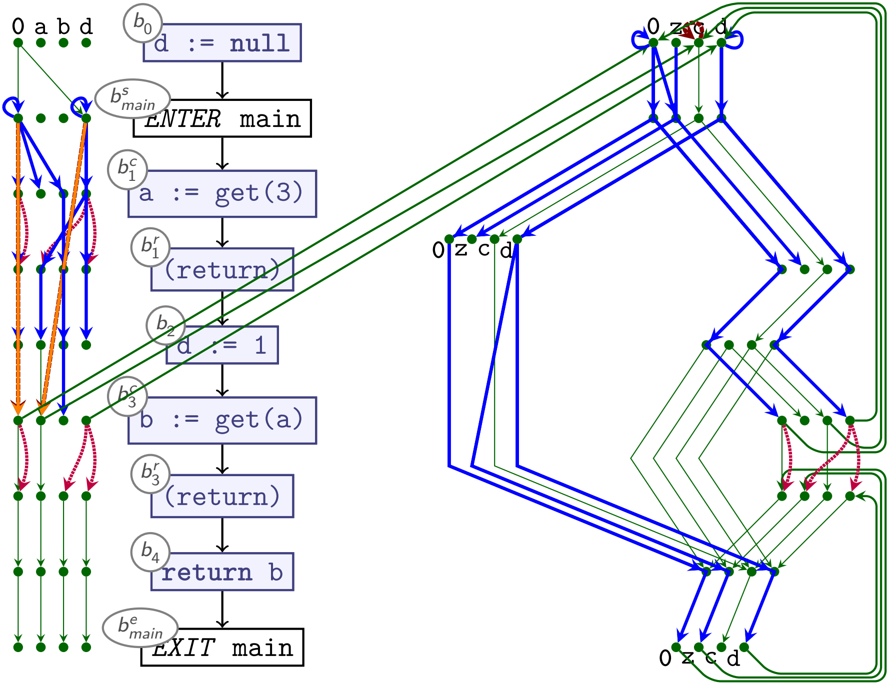

0 a b d
b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main
0 z c d
0 z c d
0 z c d



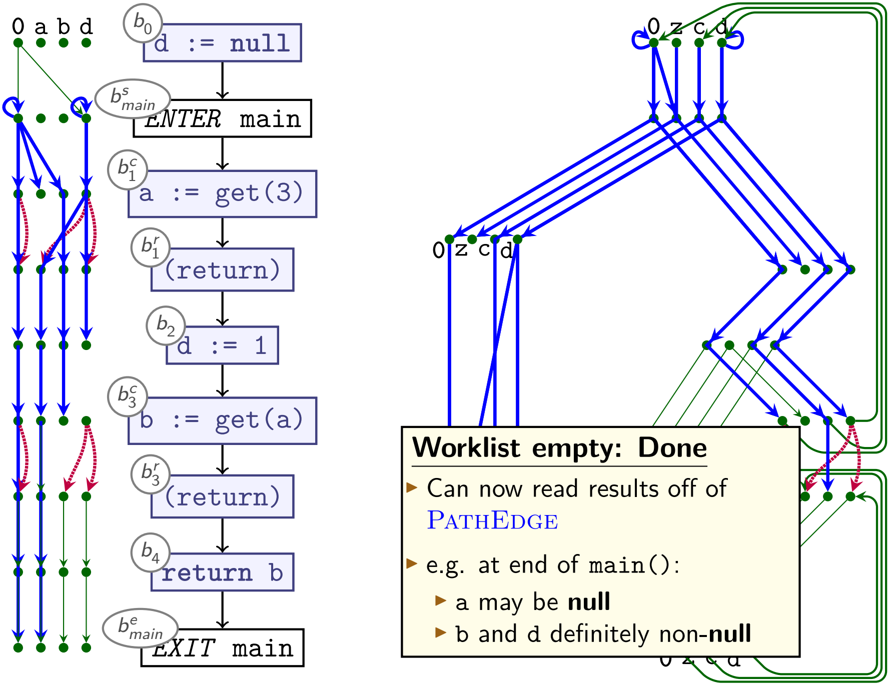
Step (call edge)

- Pick $e = n_1 \to n_2^c$ off the work queue
- n_2^c is *call* (c)?
- **Init called procedure**:
  - Find all parameter edges $t = n_2^c \to \langle b_f^s, v\rangle \in E^\sharp$
    - propagate$(\langle b_f^s, v\rangle \to \langle b_f^s, v\rangle)$
- **Propagate along intra-edges** (As with regular edges)
- **Propagate along SummaryInst**: (As with regular edges)


0 a b d
b0
d := null
bsmain
ENTER main
bc1
a := get(3)
br1
(return)
b2
d := 1
bc3
b := get(a)
br3
(return)
b4
return b
bemain
EXIT main
0 z c d
0 z c d
0 z c d



0 a b d
b_0
d := null
b^s_{main}
ENTER main
b^c_1
a := get(3)
b^r_1
(return)
b_2
d := 1
b^c_3
b := get(a)
b^r_3
(return)
b_4
return b
b^e_{main}
EXIT main
0 z c d
0 z c d
0 z c d



Worklist empty: Done

- Can now read results off of PATHEDGE
- e.g. at end of `main()`:
  - `a` may be **null**
  - `b` and `d` definitely non-**null**

The IFDS Algorithm: Initialisation and Propagation)

Procedure Init():
begin
  $\textsc{WorkList} := \textsc{PathEdge} := \emptyset$
  propagate($\langle b^s_{\text{main}}, \mathbf{0}\rangle \to \langle b^s_{\text{main}}, \mathbf{0}\rangle$)
  ForwardTabulate()
end

Procedure propagate($n_1 \to n_2$):
begin
  if $n_1 \to n_2 \in \textsc{PathEdge}$ **then**
    return
  $\textsc{PathEdge} := \textsc{PathEdge} \cup \{n_1 \to n_2\}$
  $\textsc{WorkList} := \textsc{WorkList} \cup \{n_1 \to n_2\}$
end

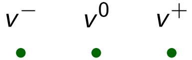
IFDS: Forward Tabulation

Procedure ForwardTabulate():
begin
  while $n_0 \to n_1 \in \textsc{WorkList}$ **do**
    $\textbf{WorkList} := \textbf{WorkList} \setminus \{n_0 \to n_1\}$
    $\langle b_0, v_0 \rangle = n_0$; $\langle b_1, v_1 \rangle = n_1$
    if b_1 is neither *Call* nor *Exit* node **then**
      foreach $n_1 \to n_2 \in E^\sharp$:
        propagate$(n_0 \to n_2)$
    else if b_1 is *Call* node **then begin**
      foreach call edge $n_1 \to n_2 \in E^\sharp$:
        propagate$(n_2 \to n_2)$
      foreach non-call edge $n_1 \to n_2 \in E^\sharp \cup \textsc{SummaryInst}$:
        propagate$(n_0 \to n_2)$
    end else if b_1 is *Exit* node **then begin**
      foreach caller/return node pair b_i^c, b_i^r that calls b_0 **and** vars v_0, v_1 **do**
        $n_s = \langle b_i^c, v_0 \rangle$; $n_r = \langle b_i^c, v_1 \rangle$
        if $\{n_s \to n_0, n_0 \to n_1, n_1 \to n_r\} \subseteq E^\sharp$ **and not** $n_s \to n_r \in \textsc{SummaryInst}$ **then**
          $\textsc{SummaryInst} := \textsc{SummaryInst} \cup \{n_s \to n_r\}$
          foreach $n_z \to n_s \in \textsc{PathEdge}$:
            propagate(n_z, n_r)
end done end done end

Summary: IFDS Algorithm

- Computes yes-or-no analysis on all variables
  - Original notion of 'variables' is slightly broader)
- Represents facts-of-interest as nodes $\langle b, v \rangle$:
  - b is node (basic block) in CFG
  - v is variable that we are interested in
- Uses
  - *'Exploded Supergraph'* $G^\sharp$
    - All CFGs in program in one graph
    - Plus interprocedural call edges
  - *Representation relations*
  - *Graph reachability*
  - A *worklist*
- Distinguishes between *Call* nodes, *Exit* nodes, others
- **Demand-driven**: only analyses what it needs
- **Whole-program analysis**
- **Computes Least Fixpoint on distributive frameworks**

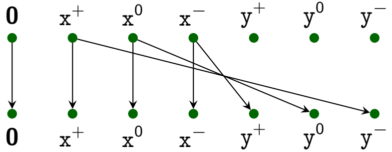
Beyond True and False



- What if abstract domain is not boolean?
  - e.g., $\{\top, A^+, A^-, A^0, \bot\}$
- Multiple boolean properties per variable
  - easy for powerset lattice $\mathcal{P}(\{+, -, 0\})$
- *Limitation*: Transfer functions only depend on one variable
- Some problems not representable, others must adapt lattice

Consider $b_1 = \boxed{\texttt{y := 0 - x}}$:

$trans_{b_1} =$



This is how the algorithm was originally proposed

BONUS SLIDES

Extending IFDS?

- Not all analyses map well to IFDS
- Core ideas are appealing:
  - Automatically compute procedure summaries
  - Exploit graph reachability + worklist for *dependency tracking*

It is possible to extend this to other classes of problems

Linear Reaching Values

Statement	$\mathbf{in}_b$	$\mathbf{out}_b$
`x := 42`	M	M with $[x \mapsto 42]$
`x := y + 1`	$M = \{[y \mapsto c], \ldots\}$	M with $[x \mapsto c + 1]$
`x := y * 7`	$M = \{[y \mapsto c], \ldots\}$	M with $[x \mapsto c \times 7]$
`x := y + z`	M	M with $[x \mapsto \top]$

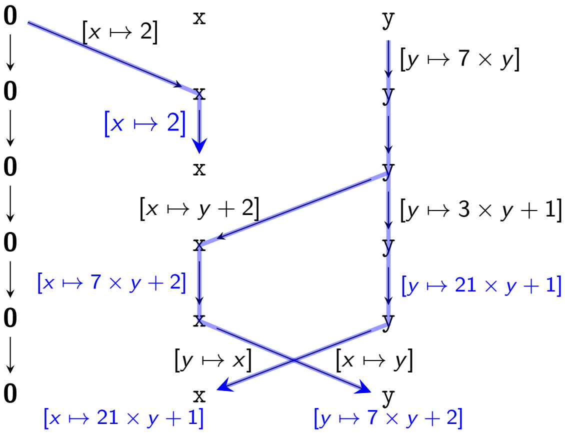
- "M with $[x \mapsto e]$" means "Remove from M any $[x \mapsto \ldots]$ if it exists, and then add $[x \mapsto e]$".
- The above sketches a *distributive* reaching values analysis
  - Each annotation of form $v_1 \mapsto c_1 \times v_2 + c_2$
  - Tradeoff: no support for adding / multiplying / ... (multiple variables)
- Encode in IFDS?

Labelling Graph Edges

```
x := 2;
y := y * 7;
```

```
x := y + 2;
y := 3 * y + 1;
```

```
{ t := x;
  x := y;
  y := t; }
```



- Extending IFDS to support information processing
- Carrying over key techniques:
  - *Track dependencies*
  - *Generate procedure summaries on the fly*

Representation

$$\left\{\begin{array}{l}[x \mapsto c_{x,1} \times x + d_{x,1}] \\ [y \mapsto c_{y,1} \times y + d_{y,1}]\end{array}\right\} \circ \left\{\begin{array}{l}[x \mapsto c_{x,2} \times v_1 + d_{x,2}] \\ [y \mapsto c_{y,2} \times v_2 + d_{y,2}]\end{array}\right\}$$

$$=$$

$$\left\{\begin{array}{l}[x \mapsto (c_{x,2} \times c_{x,1}) \times v_1 + (d_{x,2} + c_{x_1} \times d_{x_1})] \\ [y \mapsto (c_{y,2} \times c_{y,1}) \times v_1 + (d_{y,2} + c_{y_1} \times d_{y_1})]\end{array}\right\}$$

- c_i, d_i: constants
- v_i: program variables
- (Maps of) linear functions are closed under composition
- Must support $\sqcup$ to merge, map to $\top$ on mismatch

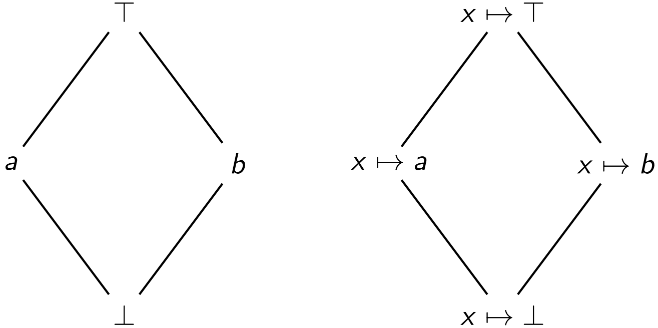
$$\left\{\begin{array}{l}[x \mapsto c_{x,1} \times v_1 + d_{x,1}] \\ [y \mapsto c_{y,1} \times v_3 + d_{y,1}]\end{array}\right\} \sqcup \left\{\begin{array}{l}[x \mapsto c_{x,1} \times v_1 + d_{x,1}] \\ [y \mapsto c_{y,2} \times v_2 + d_{y,2}]\end{array}\right\}$$

$$=$$

$$\left\{\begin{array}{l}[x \mapsto c_{x,1} \times x + d_{x,1}] \\ [y \mapsto \perp]\end{array}\right\}$$

Micro-Functions and Lattices

- Extend lattices to such 'Micro-Functions':



Micro-Functions, Efficient Representation

- Micro-Functions must support:

Encoding		$O(1)$ space
Computation	$f(x)$	$O(1)$ time
Equality testing	$f = f'$	$O(1)$ time
Composition	$f \circ f'$	$O(1)$ time
Meet	$f \sqcup f'$	$O(1)$ time

- Micro-functions are efficiently representable if they satisfy space / time constraints
  - Required for the algorithm's time bounds
- Other examples:
  - IFDS problems
  - Value bounds analysis

The IDE Algorithm (1/1)

- **I**nterprocedural **D**istributive **E**nvironments algorithm
- Extends IFDS to 'labelled' edges as described above
- Assumes distributive framework over micro-functions
- Algorithmic changes:
  - First phase analogous to IFDS
  - Second phase applies computed functions to read out results
- Maintain/update mapping from path edges to micro-functions f:

$$\textsc{PathEdge} = \{\langle b_0, v_0\rangle \xrightarrow{f_0} \langle b_1, v_1\rangle, \ldots\}$$

- 'Missing edges' equivalent to $x \mapsto \bot$
- Initialise:

$$\textsc{PathEdge} = \{\langle b_0, v_0\rangle \xrightarrow{v_1 \mapsto \bot} \langle b_1, v_1\rangle, \ldots\}$$

- Always exactly one f per $\{\langle b_0, v_0\rangle \xrightarrow{f} \langle b_1, v_1\rangle\} \in \textsc{PathEdge}$

The IDE Algorithm (2/2)

Procedure propagate($n_1 \to n_2$): `-- IFDS version`
begin
  if $n_1 \to n_2 \in \textsc{PathEdge}$ **then**
    return
  $\textsc{PathEdge} := \textsc{PathEdge} \cup \{n_1 \to n_2\}$
  $\textsc{WorkList} := \textsc{WorkList} \cup \{n_1 \to n_2\}$
end

$$\Downarrow$$

Procedure $\text{propagate}_{\text{IDE}}(n_1 \xrightarrow{f} n_2)$: `-- IDE version`
begin
  let $n_1 \xrightarrow{f'} n_2 \in \textsc{PathEdge}$
  $f_{\text{upd}} := f \sqcup f'$
  if $f_{\text{upd}} = f'$ **then**
    return
  $\textsc{PathEdge} := (\textsc{PathEdge} \setminus \{n_1 \xrightarrow{f'} n_2\}) \cup \{n_1 \xrightarrow{f_{\text{upd}}} n_2\}$
  $\textsc{WorkList} := \textsc{WorkList} \cup \{n_1 \to n_2\}$
end

Summary

- IDE strictly generalises IFDS
- Utilises **Micro-Functions** to ensure efficient summaries:
  - Intra-procedural summaries via PathEdge
  - Inter-procedural procedure summaries via SummaryInst
- Runtime is $O(LED^3)$ if micro-functions are **efficiently representable**
  - L: Lattice height
    - IFDS: 1
    - IDE: length of longest descending chain
  - E: Number of control-flow edges
  - D: Number of variables
- IFDS supported by many popular dataflow frameworks