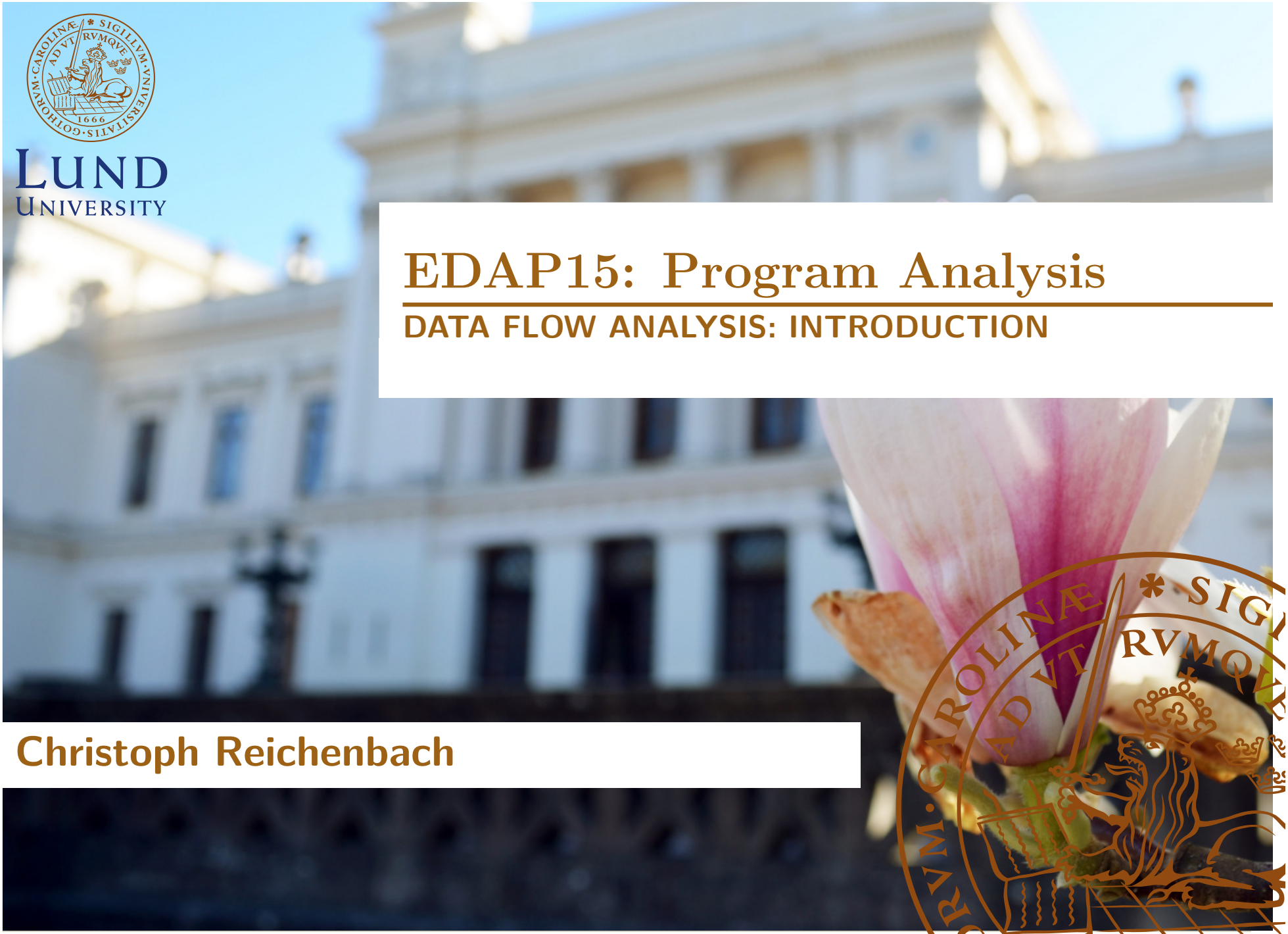

LUND UNIVERSITY

EDAP15: Program Analysis

DATA FLOW ANALYSIS: INTRODUCTION

Christoph Reichenbach

Towards Practical Program Analysis

Teal-0	Imperative and Procedural
Teal-1	Minor extensions on Teal-0
Teal-2	
Teal-3	

- Teal: Multi-layered language to exhibit program analysis challenges
- Small enough for homework exercises
- Big enough to exhibit real challenges
- Errors in Tealprograms trigger *failures*:
  - Build analyses to detect failures before they happen

Teal-0: A Procedural Language

```
module    ::= ⟨import⟩⋆ ⟨decl⟩⋆

import    ::= import ⟨qualified⟩ ;

qualified ::= id
            | ⟨qualified⟩ :: id

decl      ::= ⟨vardecl⟩ ;
            | fun id ( ⟨formals⟩? ) ⟨opttype⟩ = ⟨stmt⟩

vardecl   ::= var id ⟨opttype⟩
            | var id ⟨opttype⟩ := ⟨expr⟩;

formals   ::= id ⟨opttype⟩
            | id ⟨opttype⟩ , ⟨formal⟩

opttype   ::= : ⟨type⟩
            | ε

type      ::= int | string | any
            | array [ ⟨type⟩ ]

block     ::= { ⟨stmt⟩⋆ }
```

```
expr      ::= ⟨expr⟩ ⟨binop⟩ ⟨expr⟩
            | not ⟨expr⟩
            | ( ⟨expr⟩ ⟨opttype⟩ )
            | ⟨expr⟩ [ ⟨expr⟩ ]
            | id ( ⟨actuals⟩? )
            | [ ⟨actuals⟩? ]
            | new ⟨type⟩ ( ⟨expr⟩ )
            | int | string | null
            | id

actuals   ::= expr
            | expr, ⟨actuals⟩

binop     ::= + | - | * | / | %
            | == | != | < | <= | >= | >
            | or | and

stmt      ::= ⟨vardecl⟩
            | ⟨expr⟩ ;
            | ⟨expr⟩ := ⟨expr⟩ ;
            | ⟨block⟩
            | return ⟨expr⟩ ;
            | if ⟨expr⟩ ⟨block⟩ else ⟨block⟩
            | if ⟨expr⟩ ⟨block⟩
            | while ⟨expr⟩ ⟨block⟩
```

[1,2,3]

{ ... }

Teal-0: Example

Teal

```
var v := [0, 0];
print(x);
if z {
  v[0] := 2;
  v := null;
}
v[0] := 1;
```

A New Analysis Challenge

Teal

```
var x := [0, 0];
print(x);      // A
if z {
  x[0] := 2;  // B
  x := null;
}
x[0] := 1;     // C
```

null

- Analyse: Can there be a *failure* at B or C?

A New Analysis Challenge

Teal

```
var x := [0, 0];
print(x);     // A
if z {
  x[0] := 2; // B
  x := null;
}
x[0] := 1;    // C
```

- Analyse: Can there be a *failure* at `B` or `C`?
- Must distinguish between `x` at `A` vs. `x` at `B` and `C`

A New Analysis Challenge

Teal

```
var x := [0, 0];
print(x);     // A
if z {
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```

- Analyse: Can there be a *failure* at `B` or `C`?
- Must distinguish between `x` at `A` vs. `x` at `B` and `C`
- Need to model flow of information: **Flow-Sensitive Analysis**

A New Analysis Challenge

Teal

```
var x := [0, 0];
print(x);     // A
if z {
  x[0] := 2; // B
  x := null;
}
x[0] := 1;    // C
```

- Analyse: Can there be a *failure* at B or C?
- Must distinguish between x at A vs. x at B and C
- Need to model flow of information: **Flow-Sensitive Analysis**
- Type analysis is *not Flow-Sensitive* (normally)

Need analysis that can represent *data flow* through program

Evaluation Order

Teal-0

```
fun p(a) = { print(a); return 1; }
p(p(0) + p(1));
```

3 1 2

Evaluation Order

Teal-0

```
fun p(a) = { print(a); return 1; }
p(p(0) + p(1));
```

Every analysis must remember the evaluation order rules!

Evaluation Order

Teal-0

```
fun p(a) = { print(a); return 1; }
p(p(0) + p(1));
```

Teal-0 with explicit order

```
var tmp1 := p(0);
var tmp2 := p(1);
var tmp3 := tmp1 + tmp2;
var tmp4 := p(tmp3);
```

Every analysis must remember the evaluation order rules!

Evaluation Order

Teal-0

```
fun p(a) = { print(a); return 1; }
p(p(0) + p(1));
```

Teal-0 with explicit order

```
var tmp1 := p(0);
var tmp2 := p(1);
var tmp3 := tmp1 + tmp2;
var tmp4 := p(tmp3);
```

Java or C or C++

```
// Many challenging constructions:
a[i++] = b[i > 10 ?  i-- :  i++] + c[f(i++, --i)];
```

Every analysis must remember the evaluation order rules!

Eliminating Nested Expressions

- No nested expressions

$\Longrightarrow$ Evaluation order is explicit

$\Longrightarrow$ Fewer patterns to analyse

- We still have nested statements

Multiple Paths

Teal

```
v := new array[int](1);
if condition {
  v := null;
} else {
  print(v);
}
v[0] := 1;
```

Teal

```
v := new array[int](1);
while condition {
  v := null;
}
v[0] := 1;
```

Multiple Paths

Teal

```
v := new array[int](1);
if condition {
  v := null;
} else {
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}
v[0] := 1;
```

Teal

```
v := new array[int](1);
while condition {
  v := null;
}
v[0] := 1;
```

Need to reason about the order of execution of *statements*, too

Summary

- Understanding variable updates requires **Flow-Sensitive Analysis**
- Type analysis is *not* flow sensitive
- "Flow" is complicated, influenced by:
  - Expression evaluation order
  - Short-circuit evaluation
  - Statement execution order
- Best analysed with special intermediate representation:
  - Flatten nested expressions
  - Introduce temporary variables as needed
  - ...do something about statement execution? (up next!)

Control-Flow Graphs (CFGs)

Teal

```
var v := [0, 0];
if condition {
  v := null;
} else {
  print(v);
}
v[0] := 1;
```

Control-Flow Graphs (CFGs)

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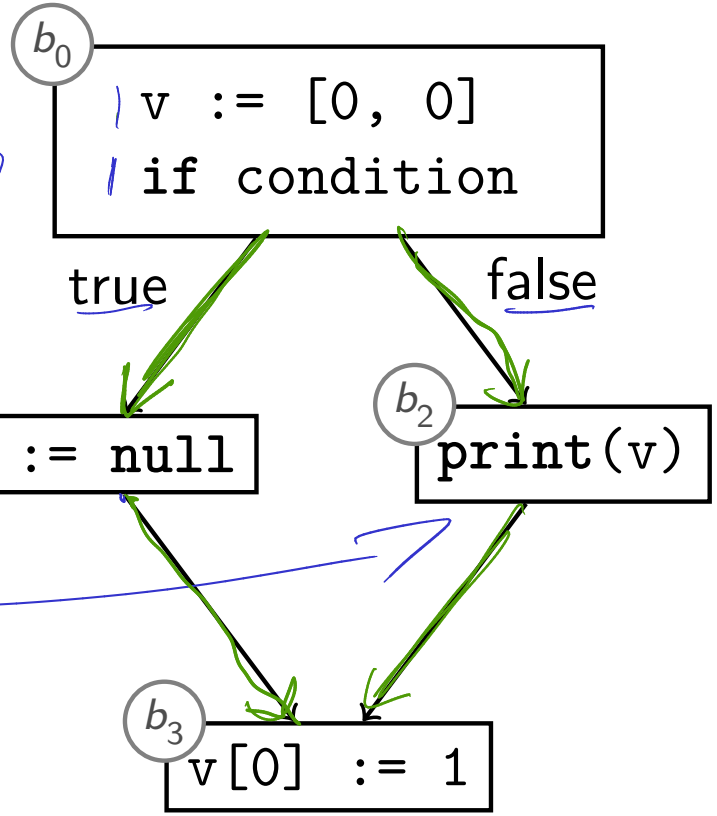

Control-Flow Graphs (CFGs)

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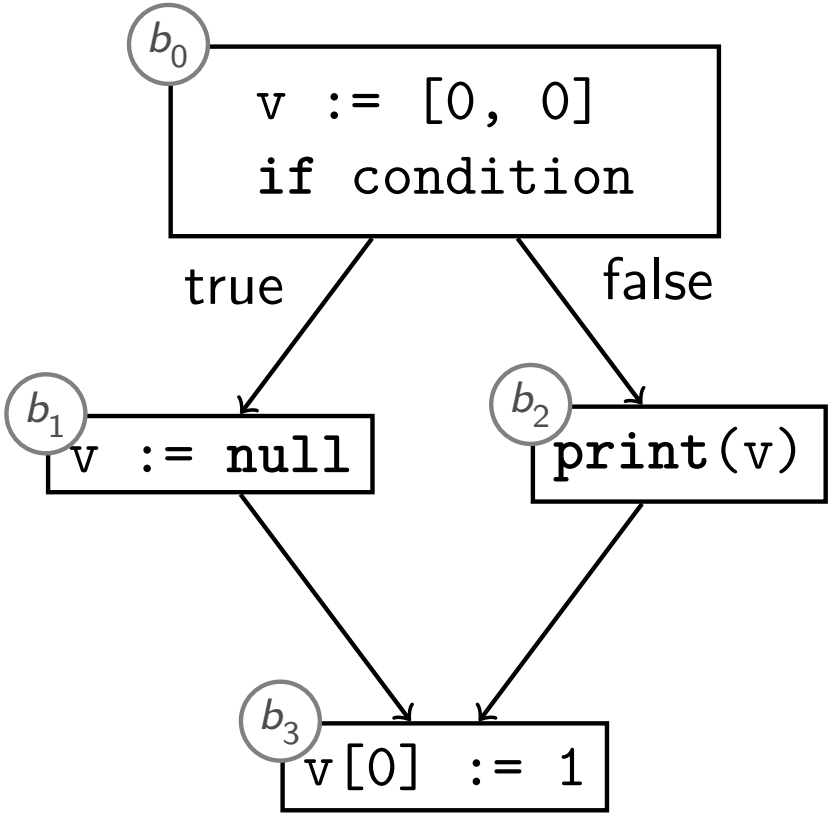
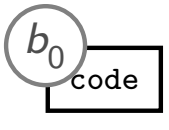
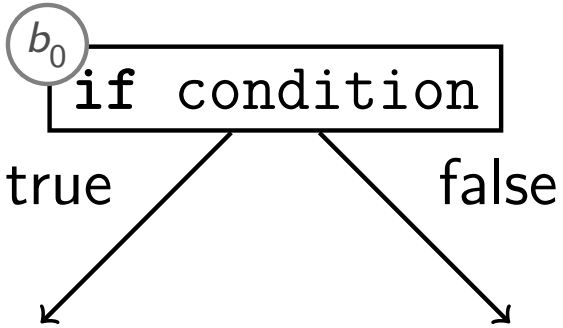


Control Flow Graphs encode statement execution order

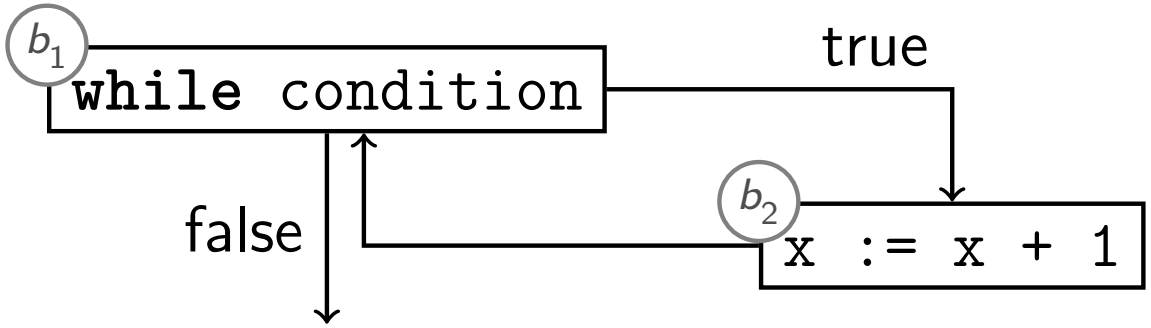
Control-Flow-Graphs

- Encode statement order by *nodes* 

and edges →
- *Multiple* outgoing edges (branches): Add label:

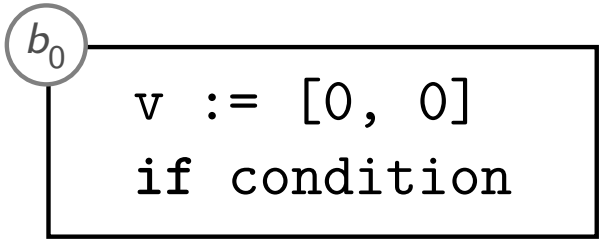


- Uniform representation for control statements:

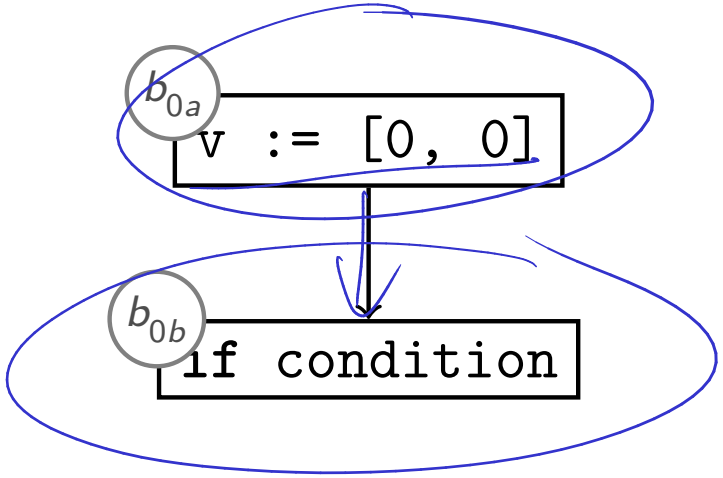


Basic Blocks

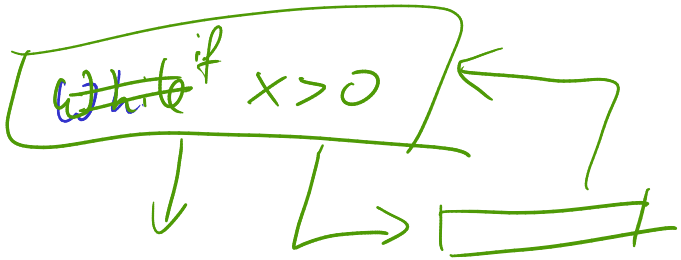
Can group statements into **Basic Blocks** or keep them separate:

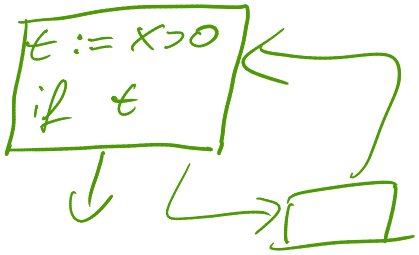


Basic Block



- A **Basic Block** is a sequence of statements
- Last statement is *always* return, branch, or jump
- Other statements are *never always* return, branch, or jump





Summary

- Different **Intermediate Representations** (IRs) to pick
- Usually eliminate nested expressions
  - Make evaluation order explicit
- **Control-Flow Graph** (CFG):
  - Represent control flow as **Blocks** and **Control-Flow Edges**
  - Edges represent control flow, **labelled** to identify conditionals
  - Blocks can be single statements or **Basic Blocks**
    - Basic blocks are sequences of statements without branches

Control Flow

Understanding data flow requires understanding control flow:

Teal

```
var v := [0, 0];
print(v);
if z {
  v[0] := 2;
  v := null;
}
v[0] := 1;
```

Control Flow

Understanding data flow requires understanding control flow:

Teal

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var v := [0, 0];
print(v);
if z {
  v[0] := 2;
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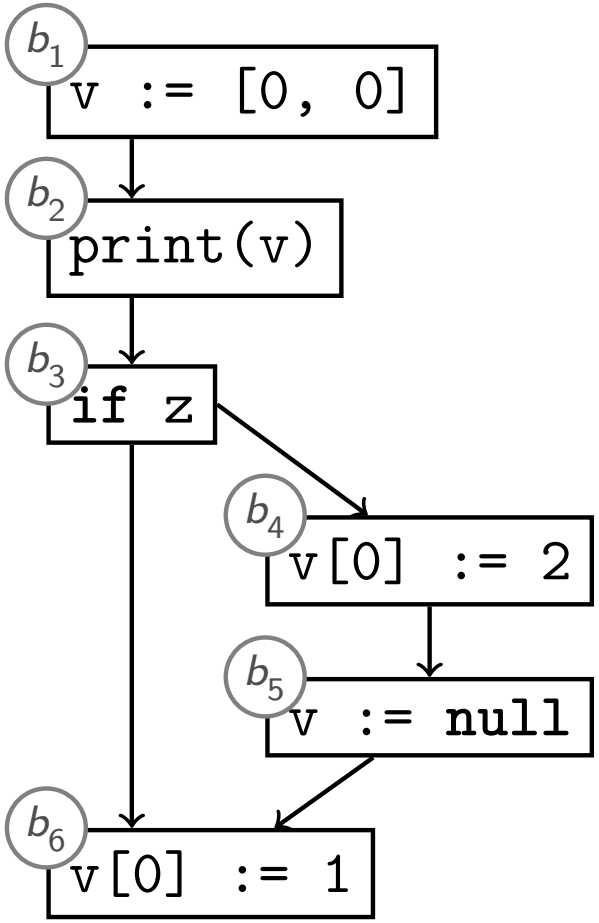
```



⟶ Control flow

Control Flow

Understanding data flow requires understanding control flow:

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print(v);
if z {
  v[0] := 2;
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```
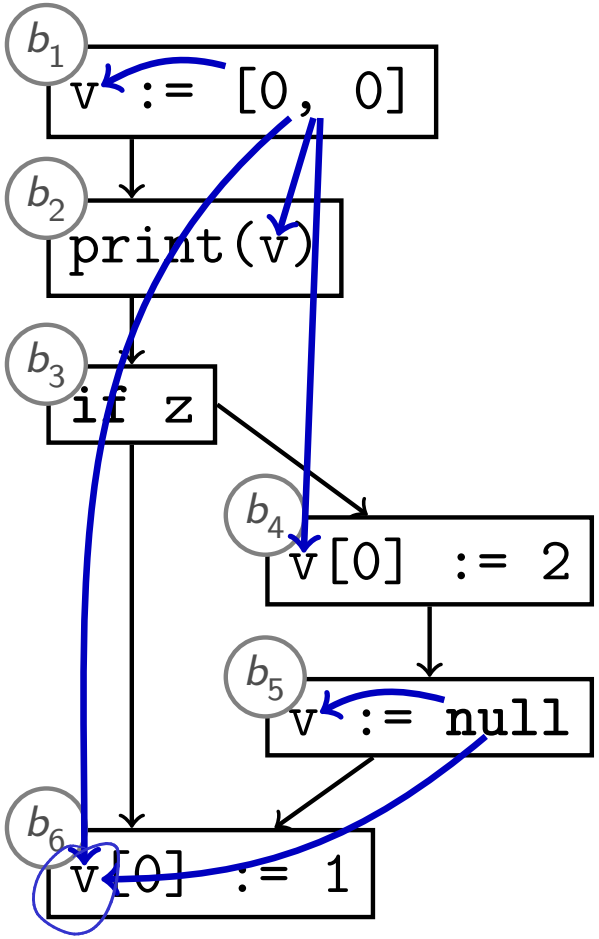
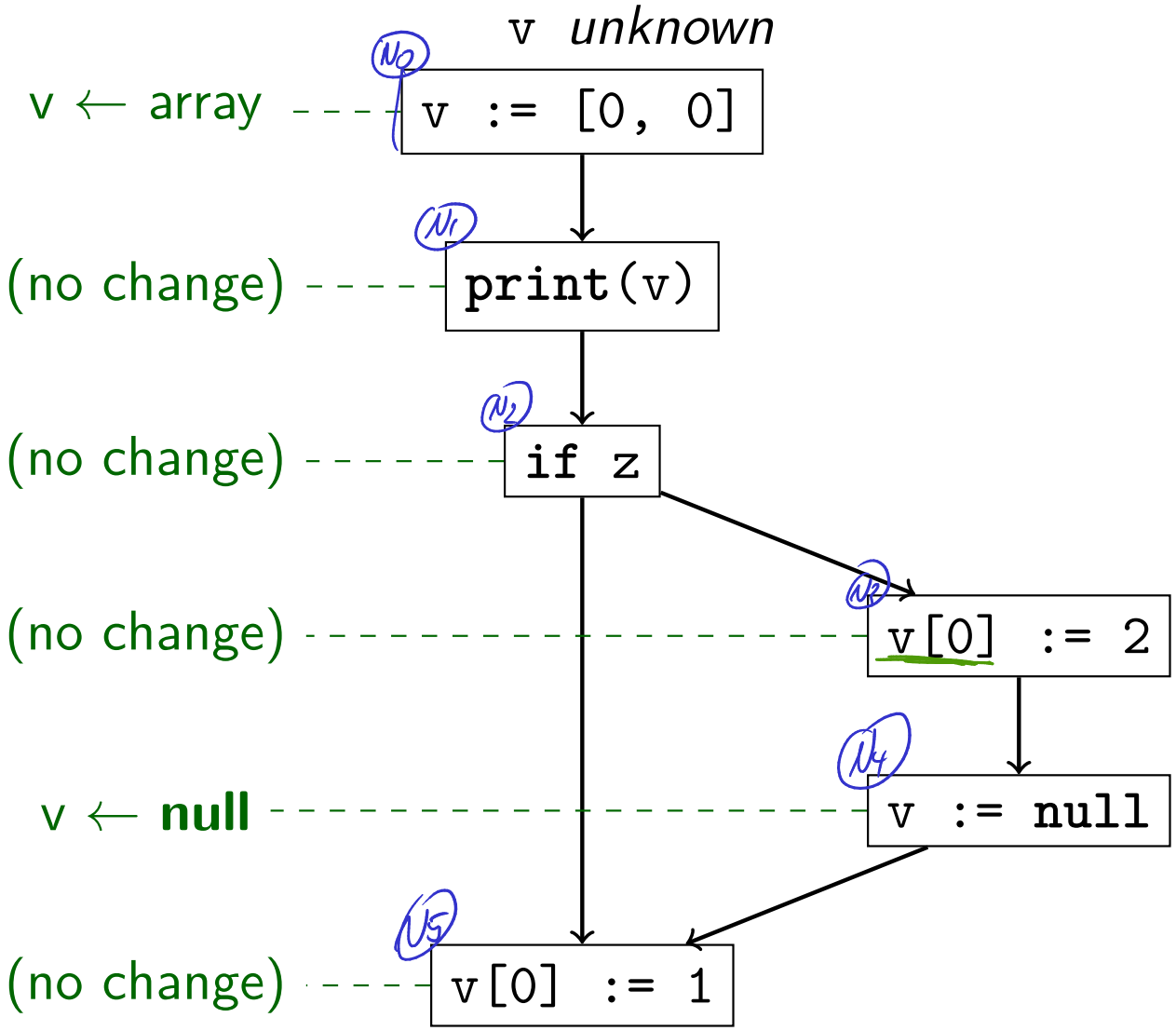



⟶ Control flow

⟶ Data flow (Def-Use chains)

Basic Ideas of Data Flow Analysis



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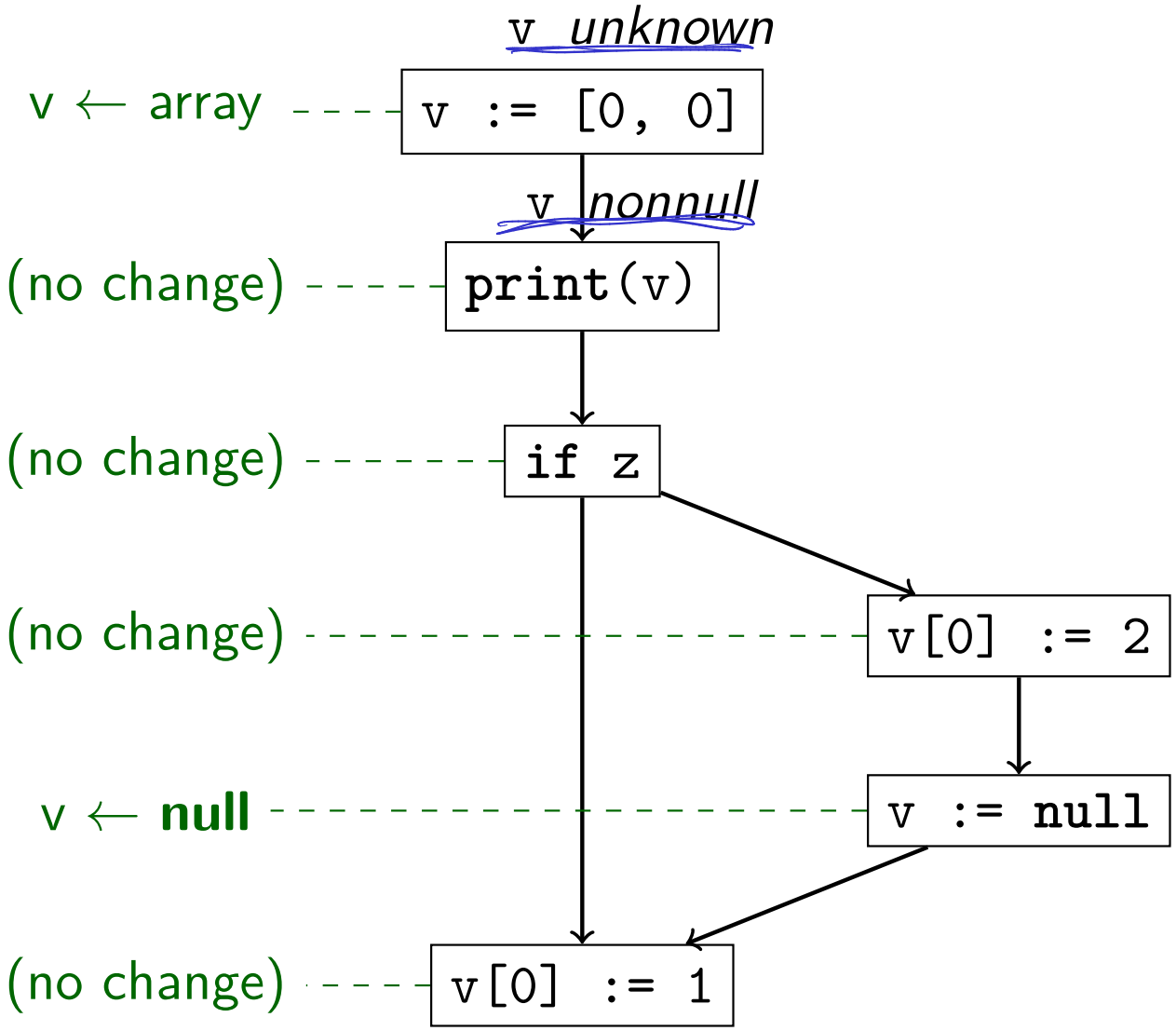

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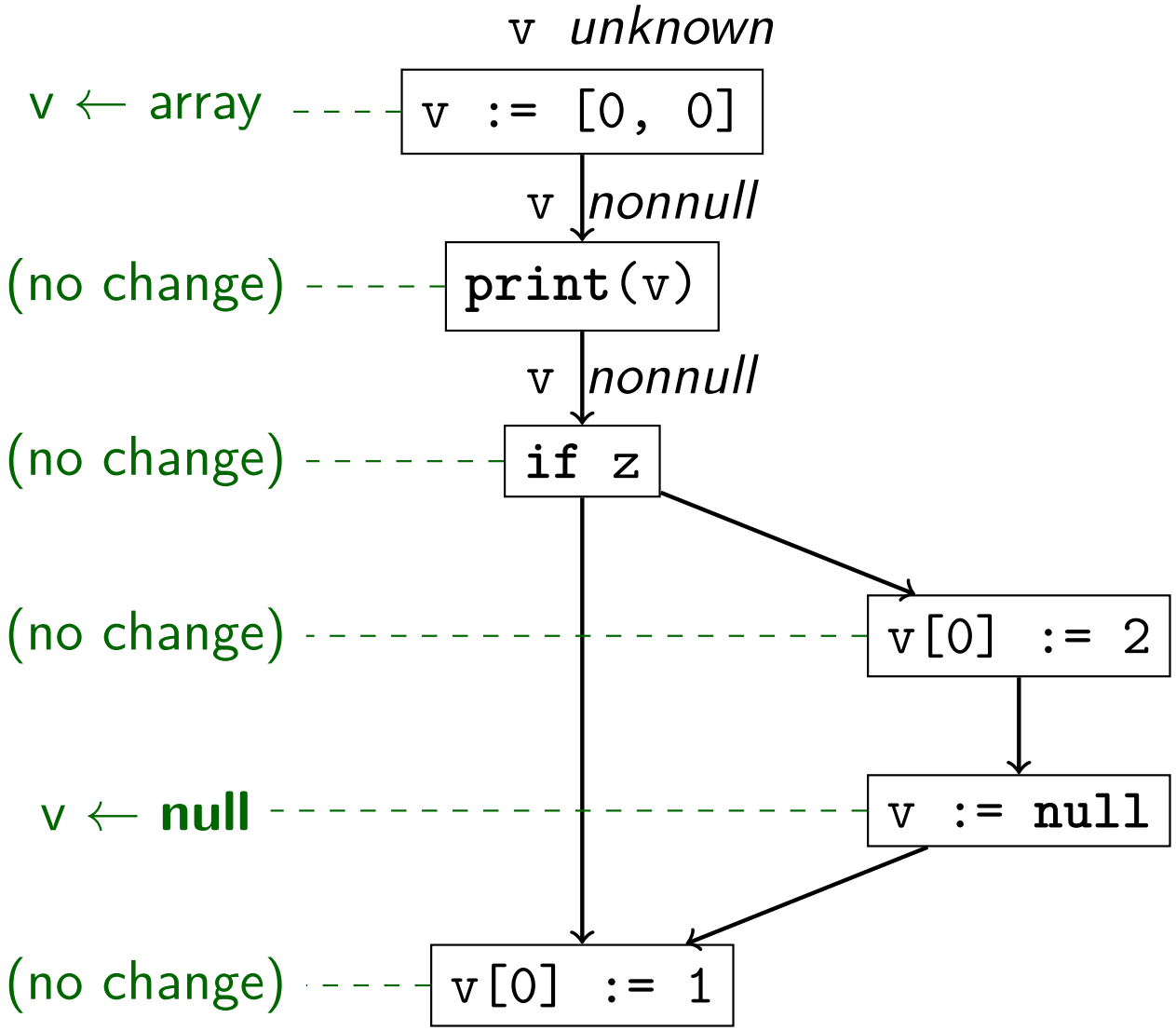

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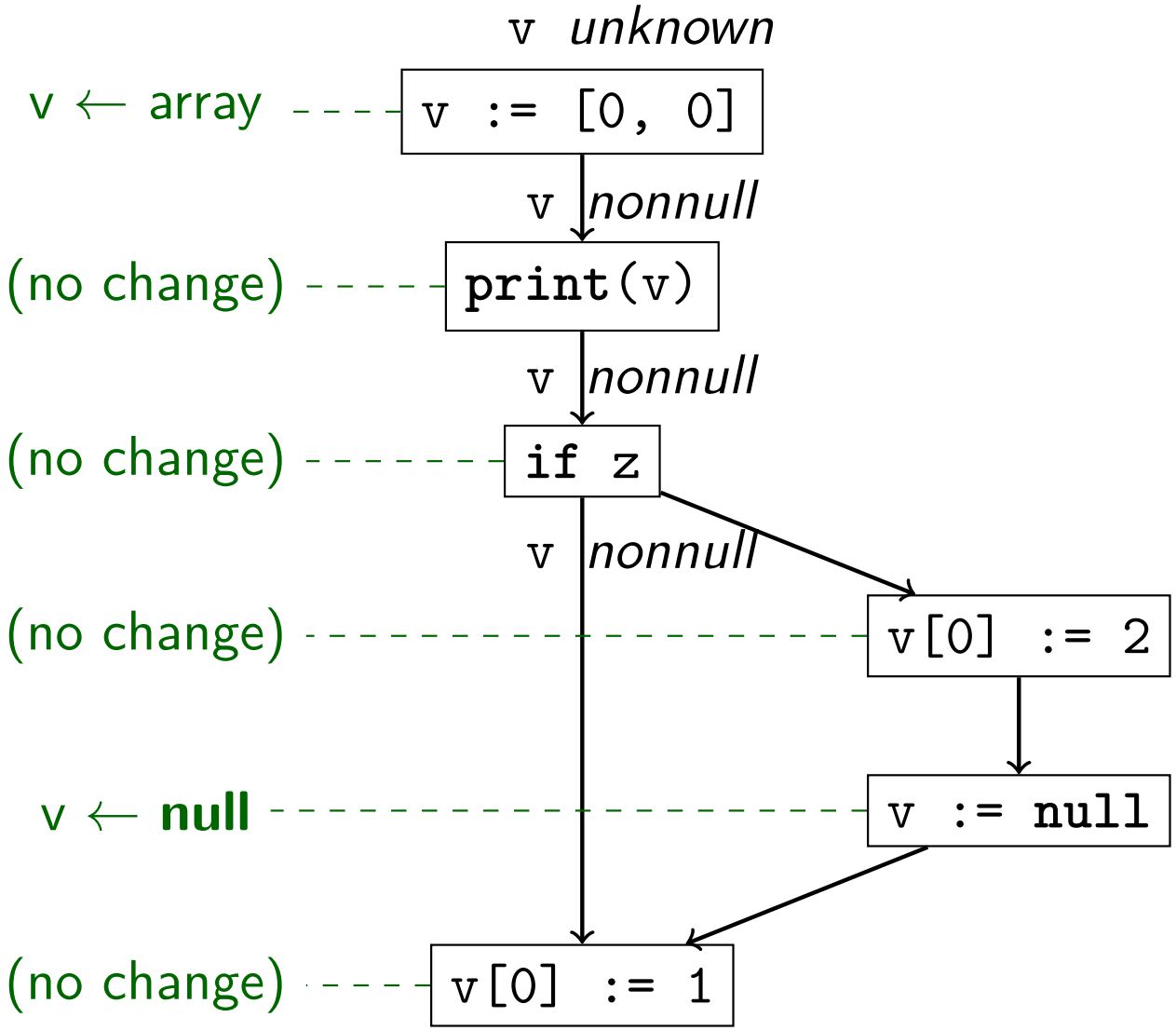

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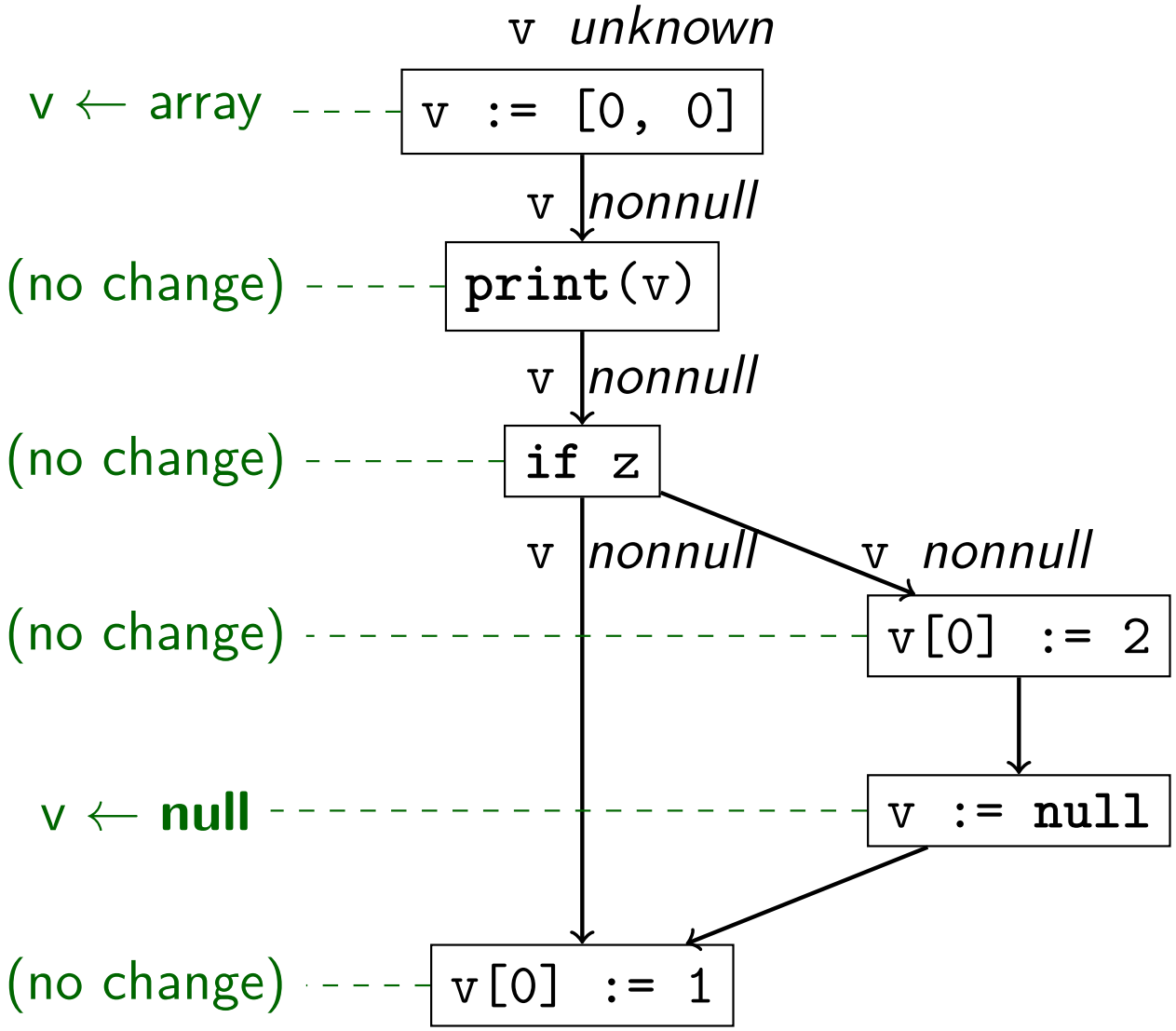

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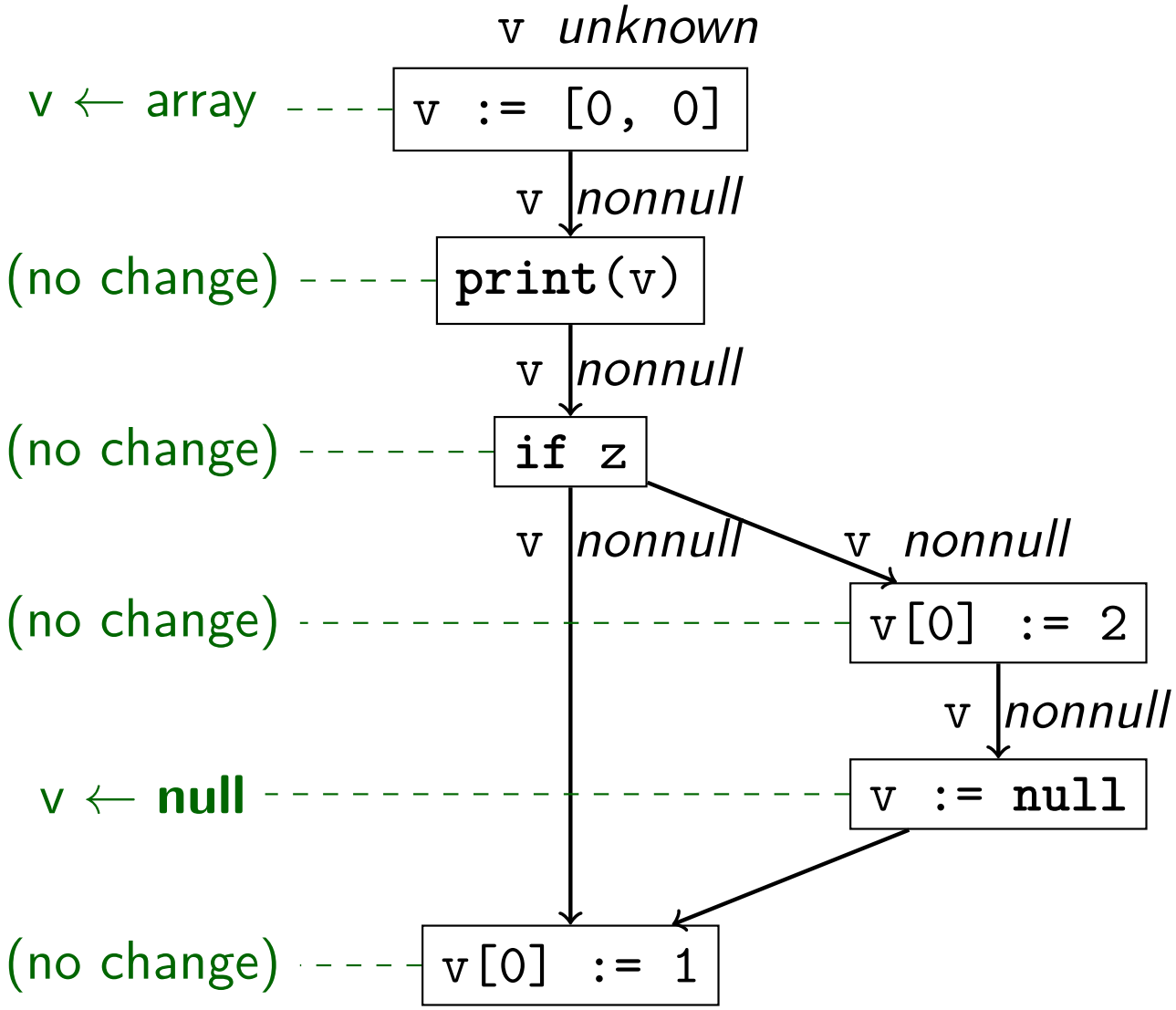

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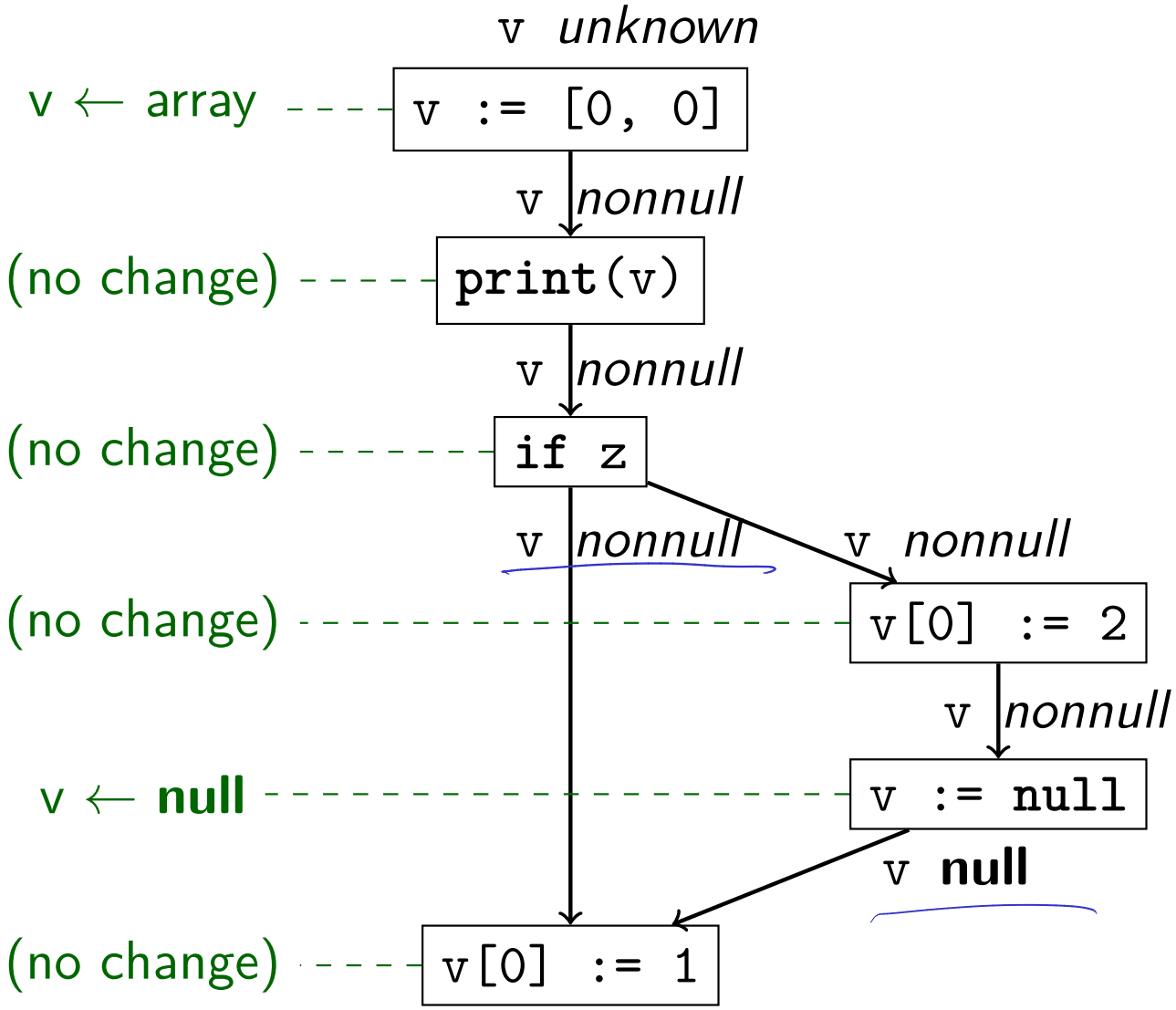
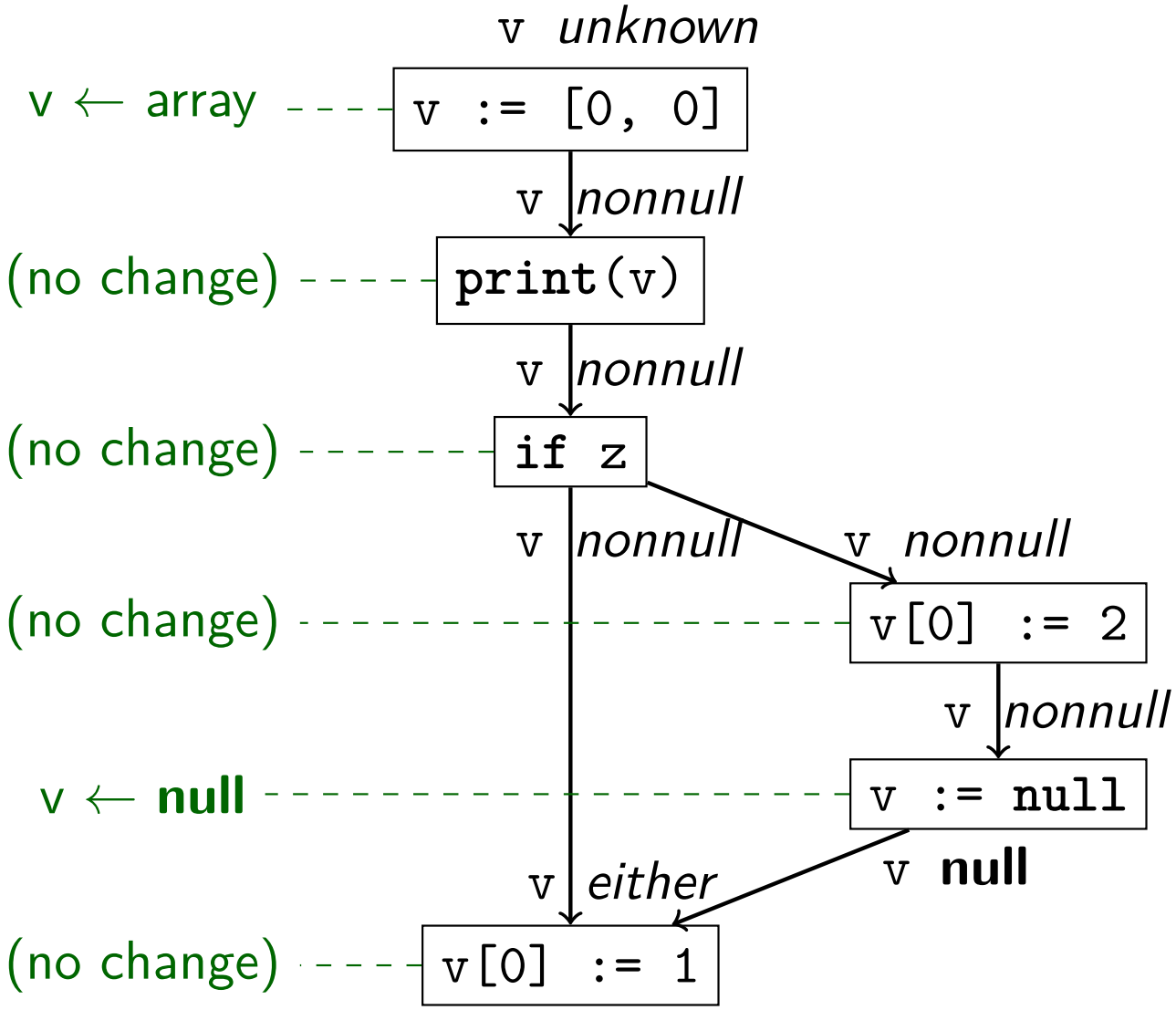


Basic Ideas of Data Flow Analysis



Another Analysis

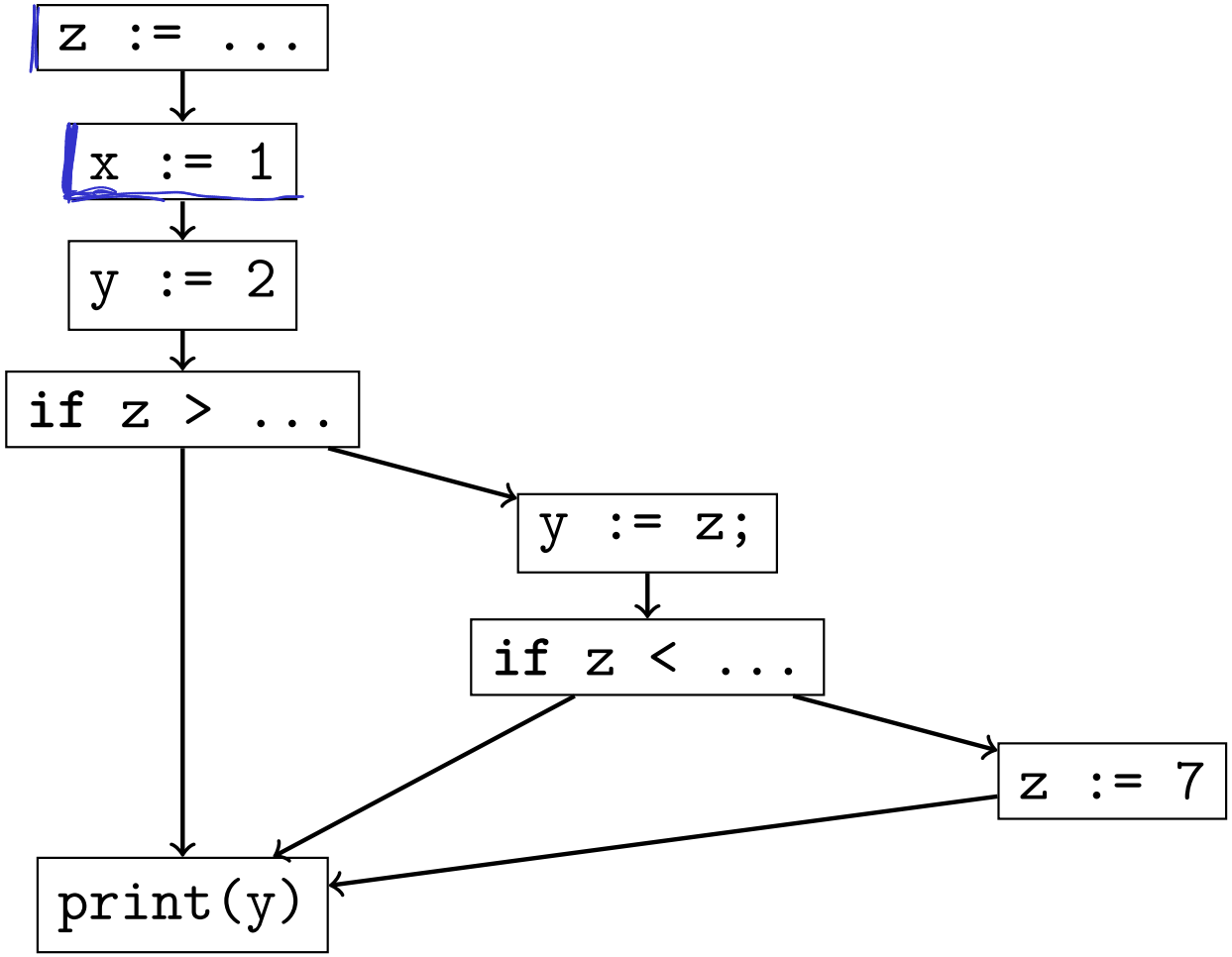
Teal

```
z := ...
x := 1;
y := 2;
if z > ...  {
  y := z
  if z < ...  {
    z := 7
  }
}
print(y);
```

- Which assignments are unnecessary?

⇒ Possible oversights / bugs
(*Live Variables Analysis*)

Unnecessary Assignments: Intuition



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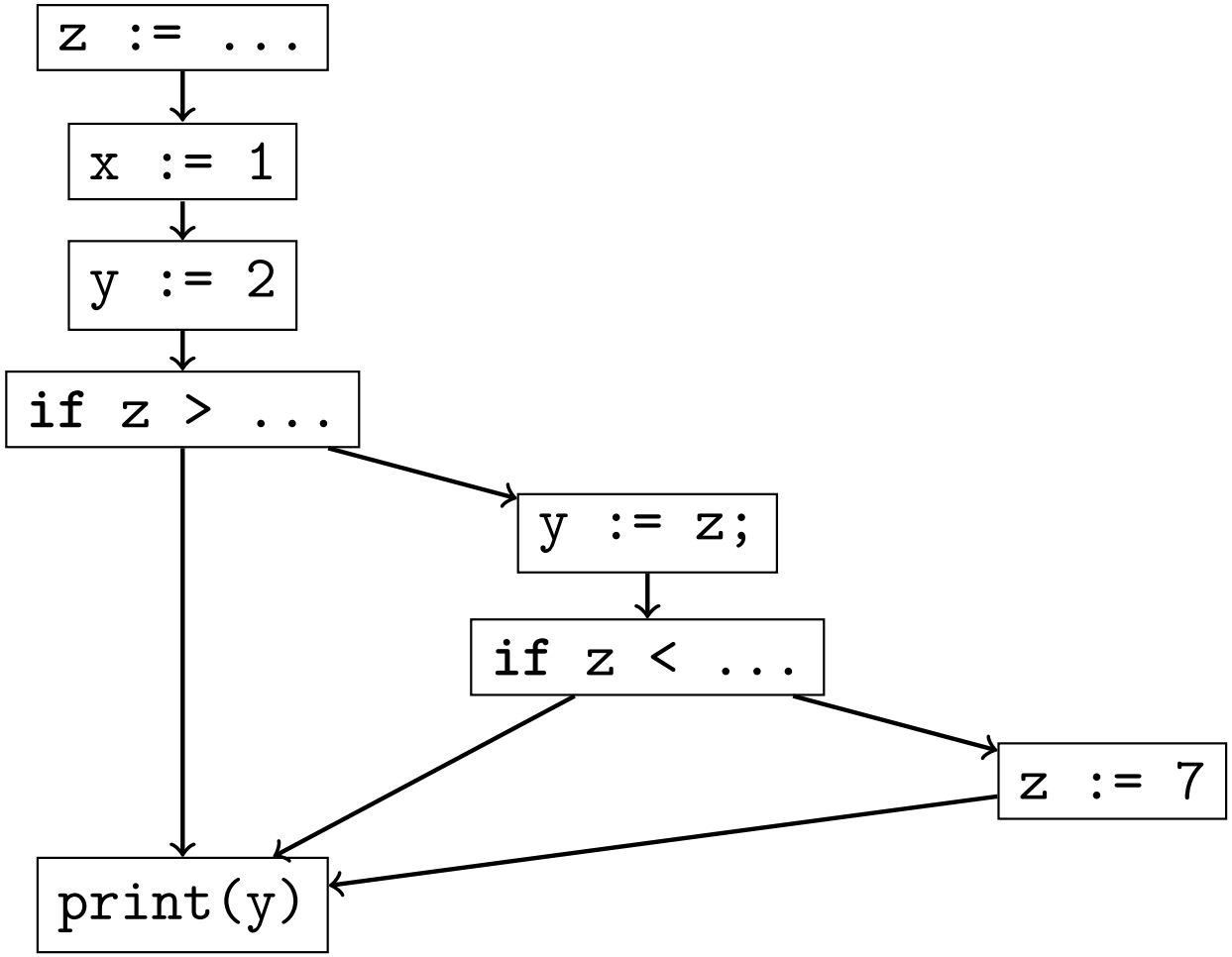

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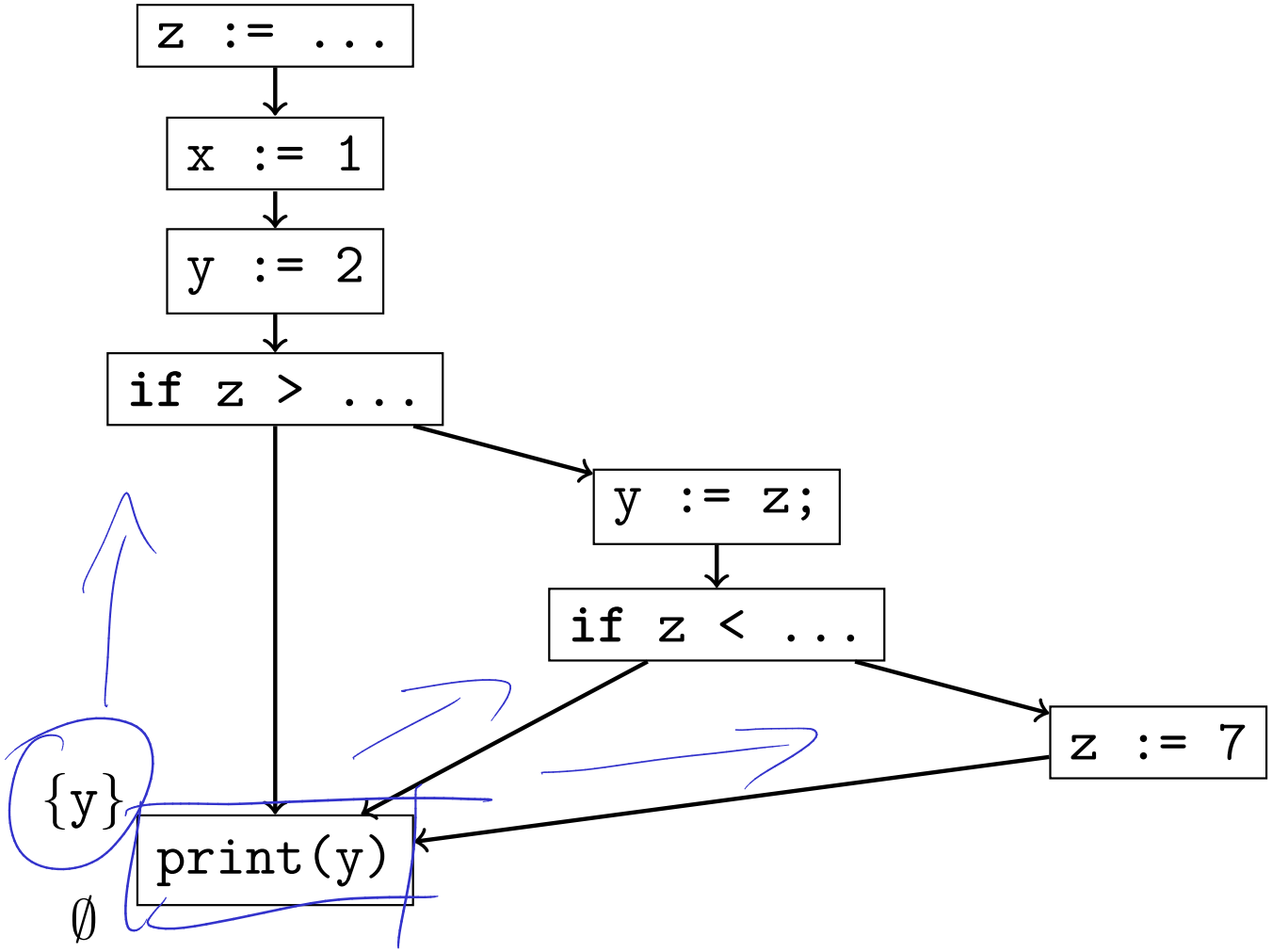
Unnecessary Assignments: Intuition



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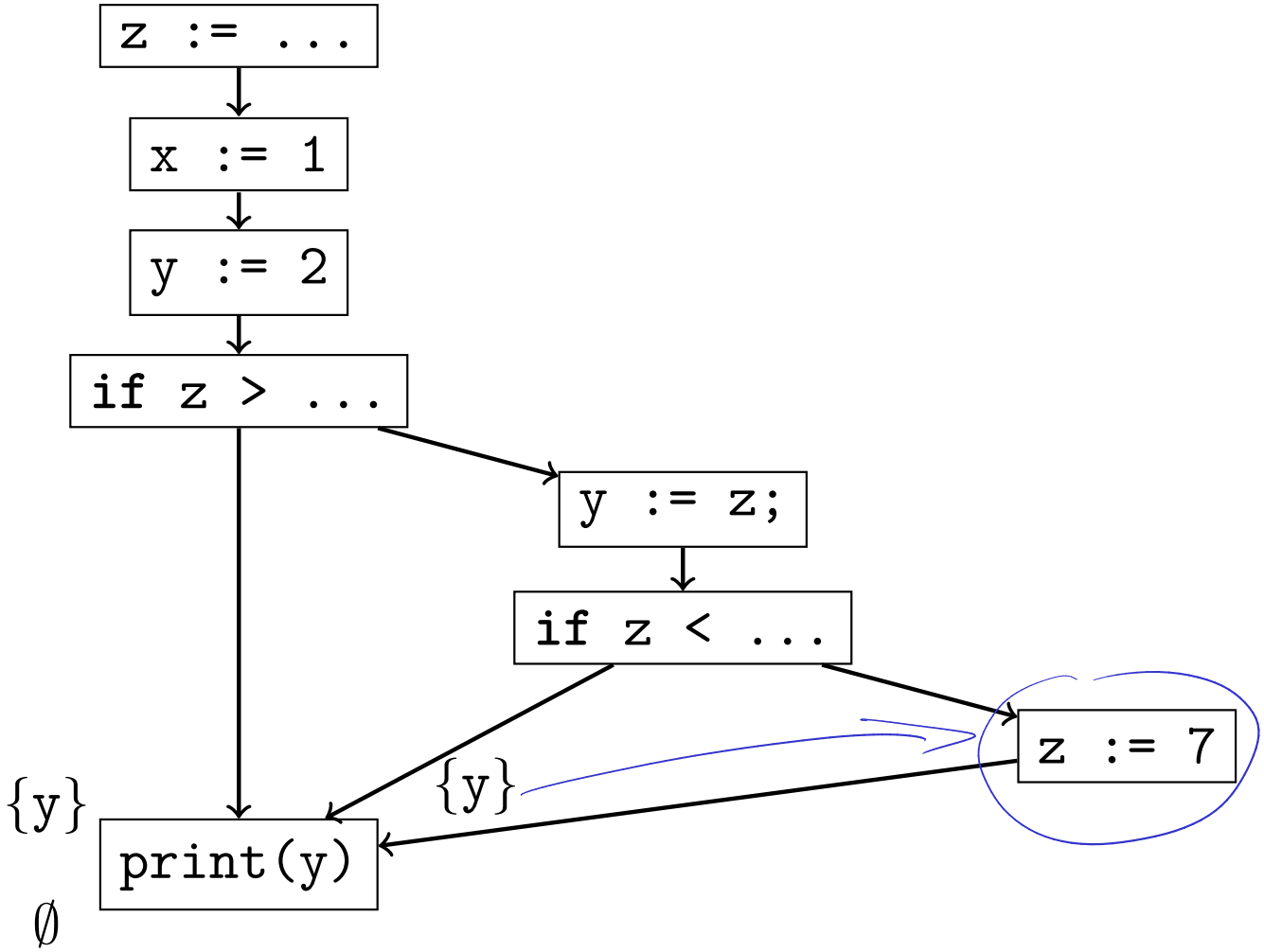

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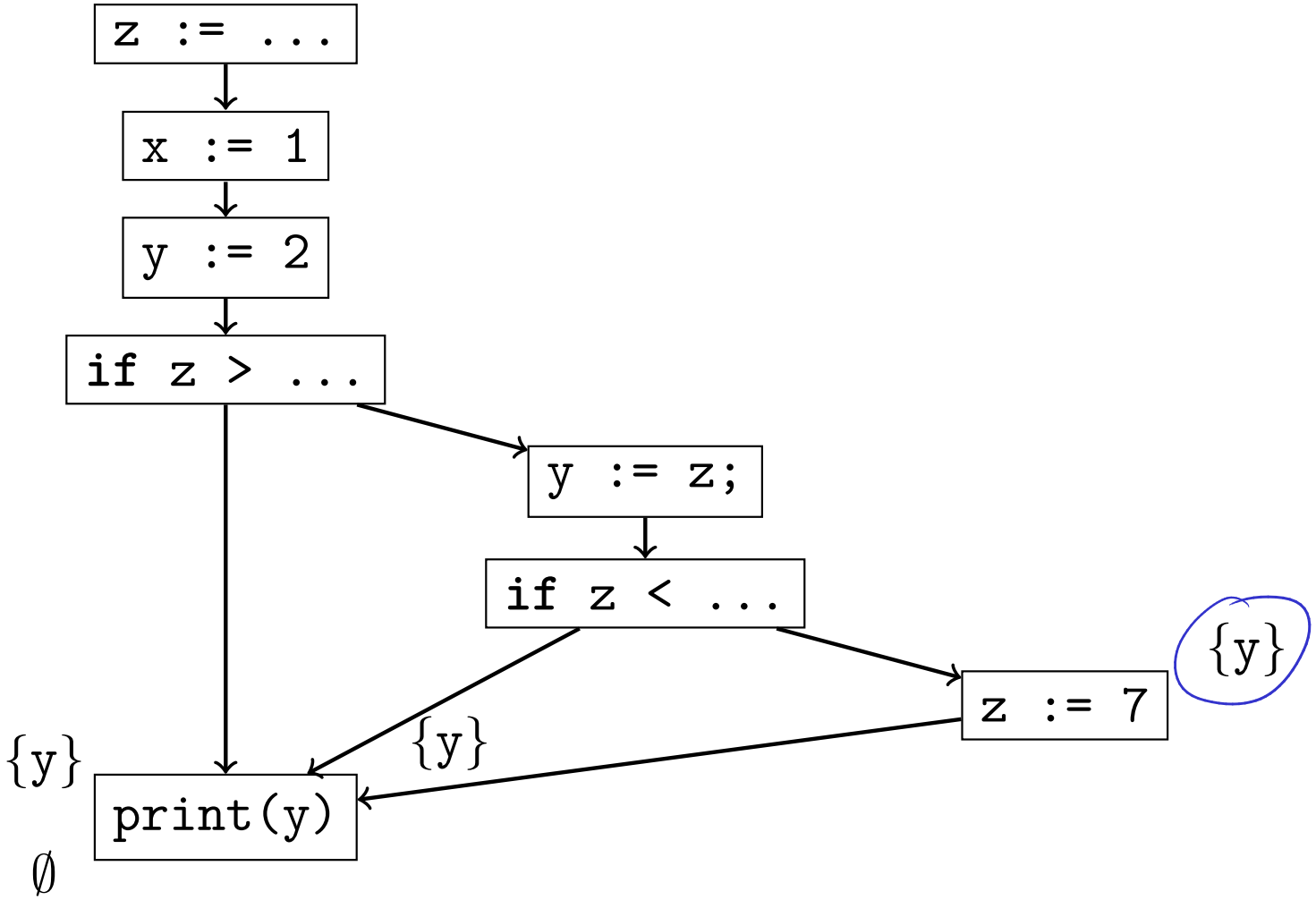

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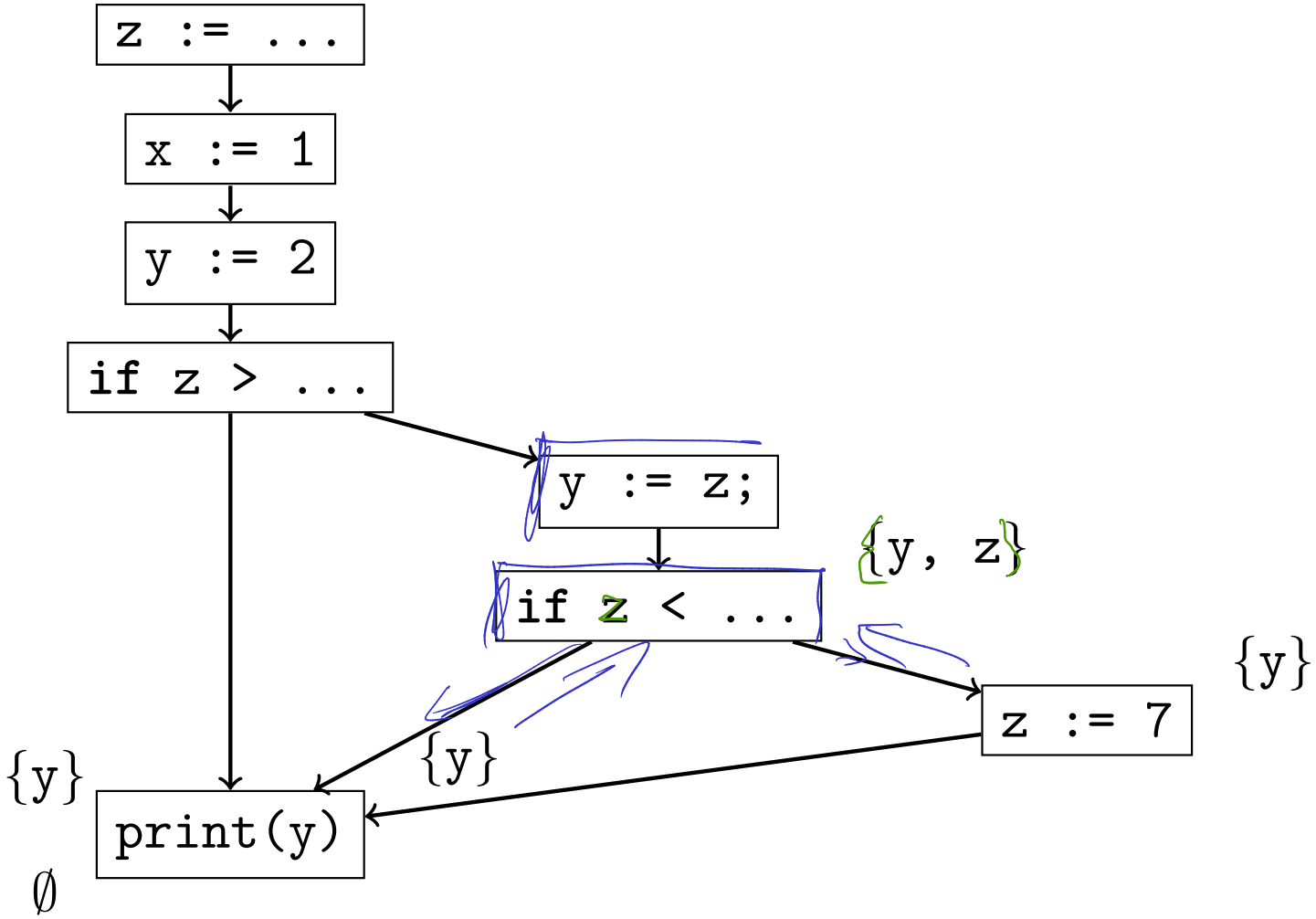

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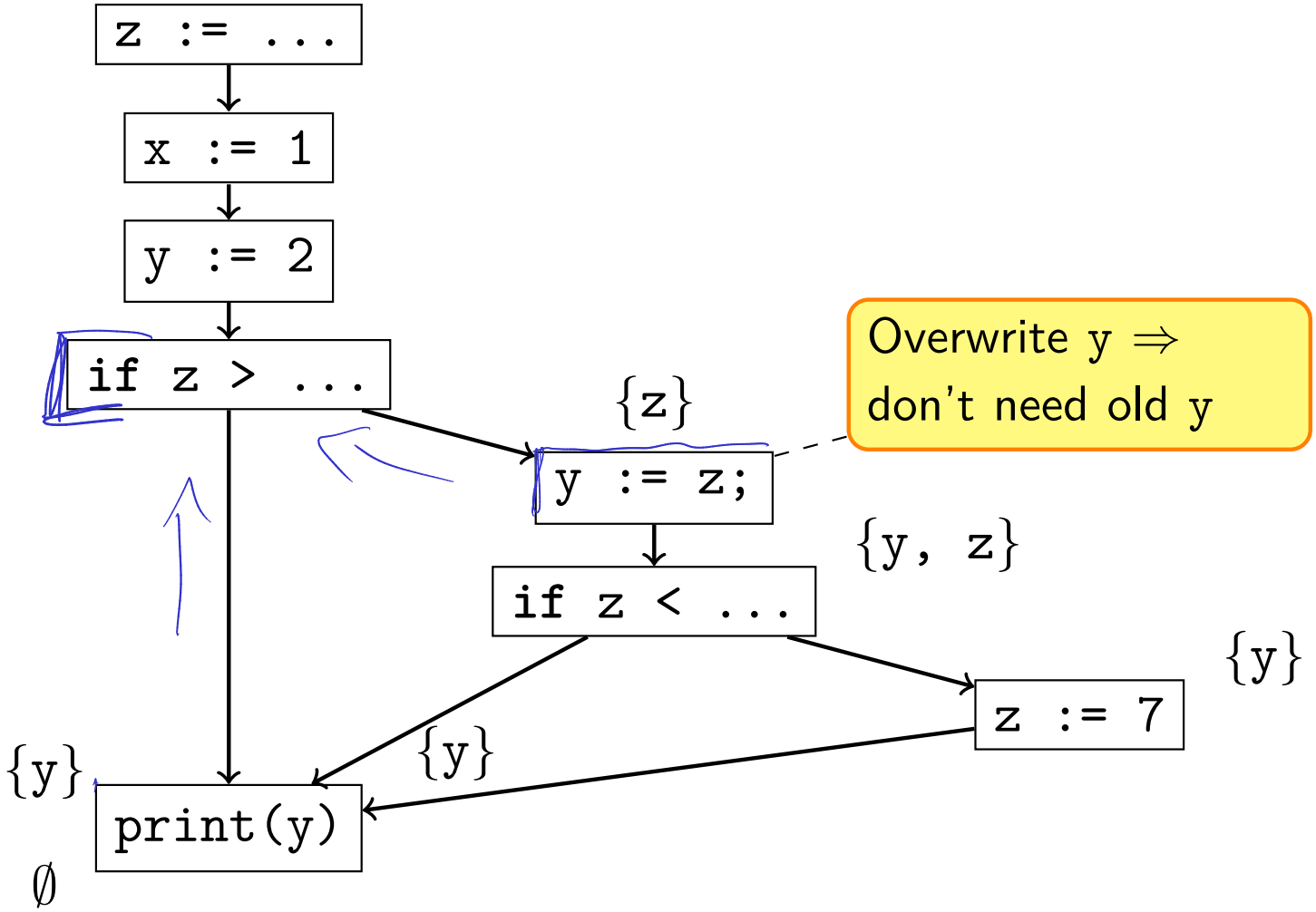

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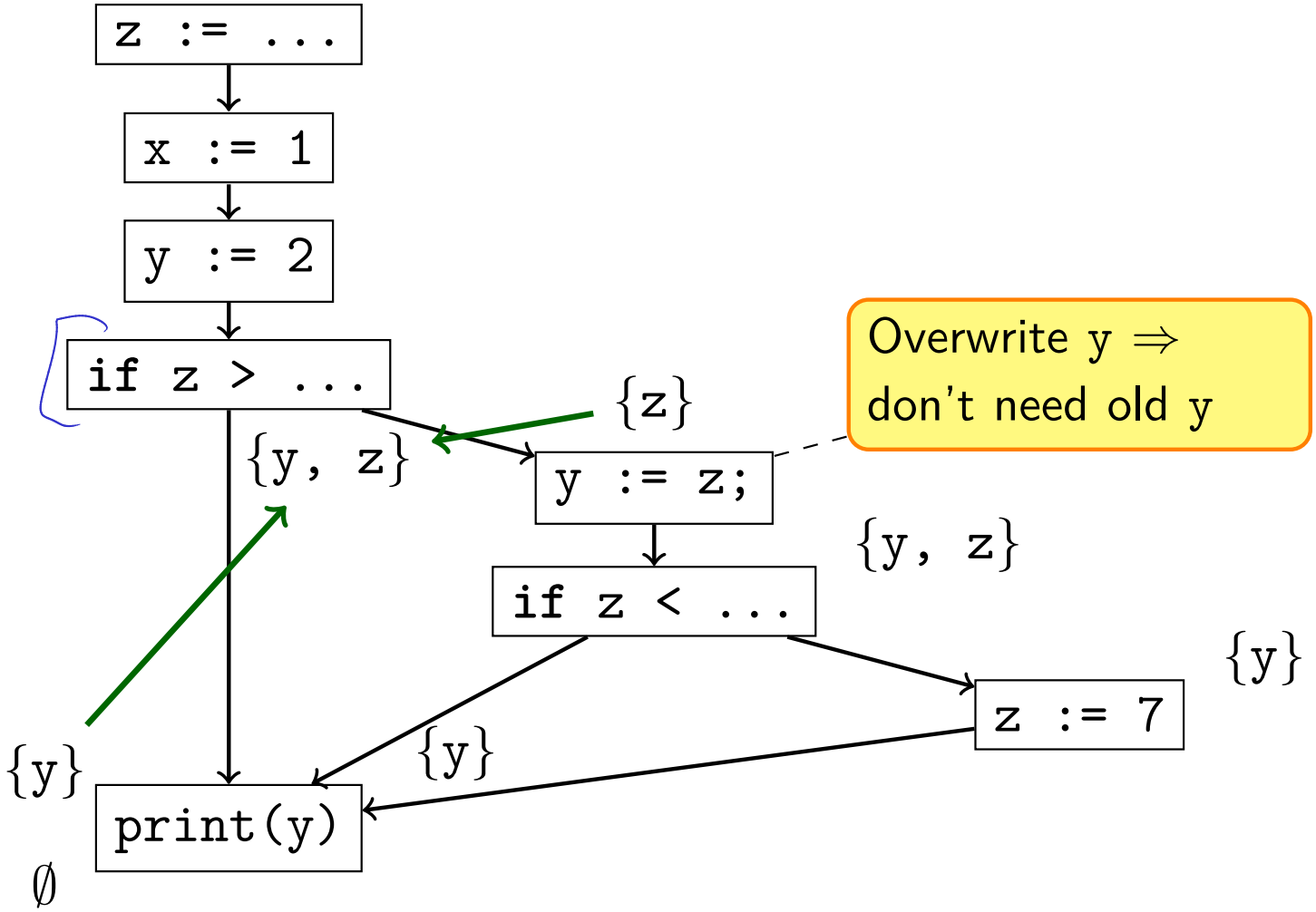

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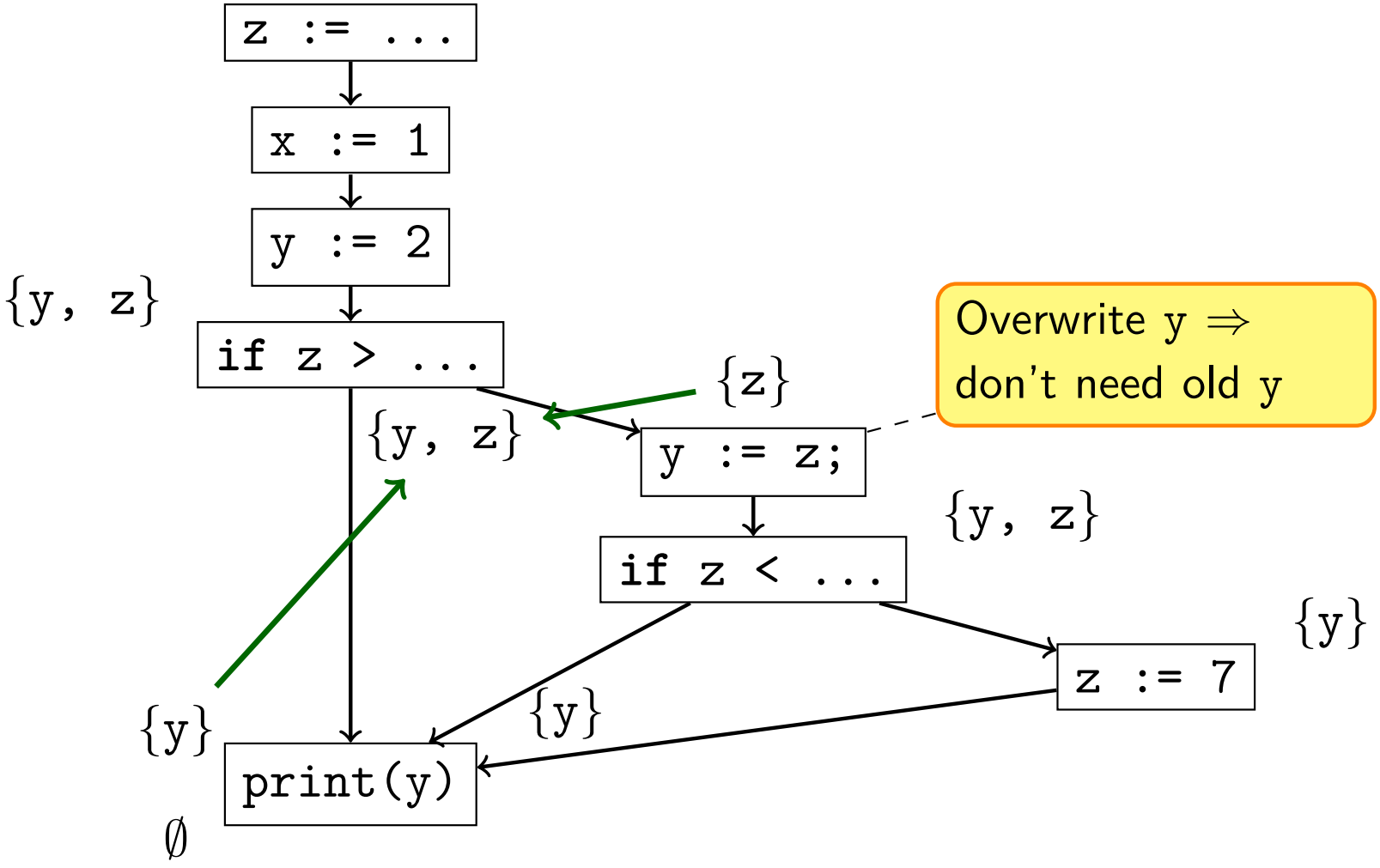

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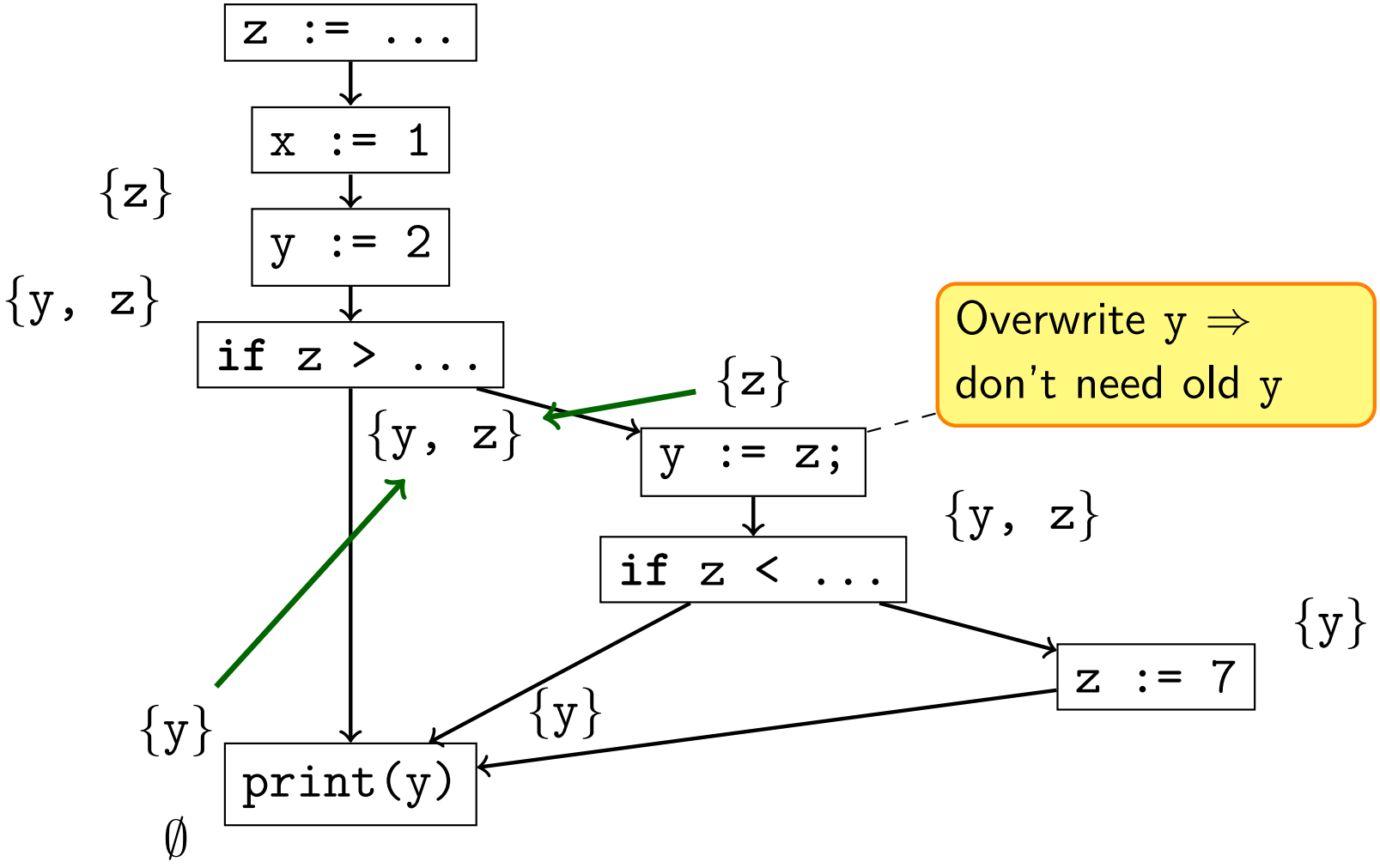

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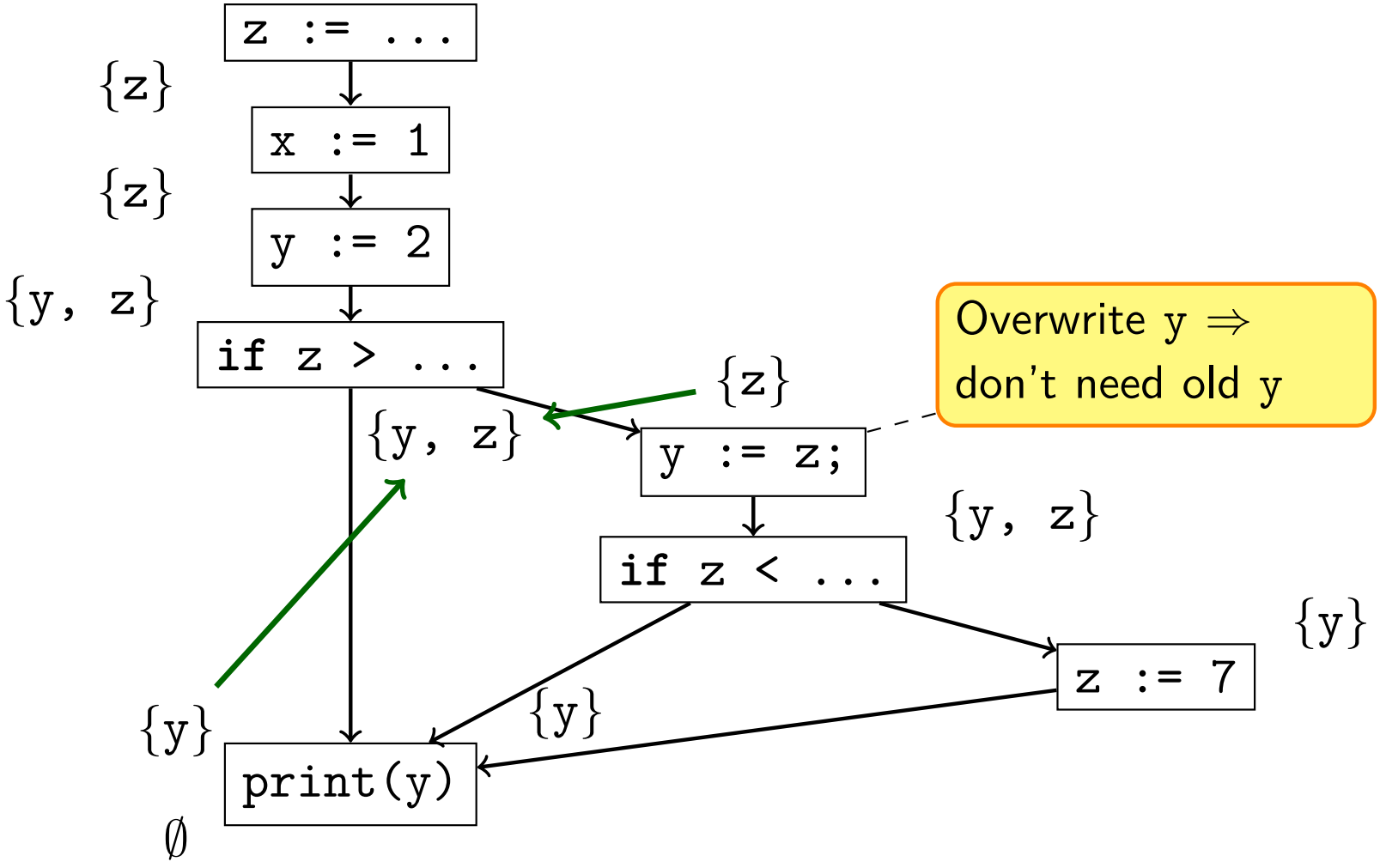

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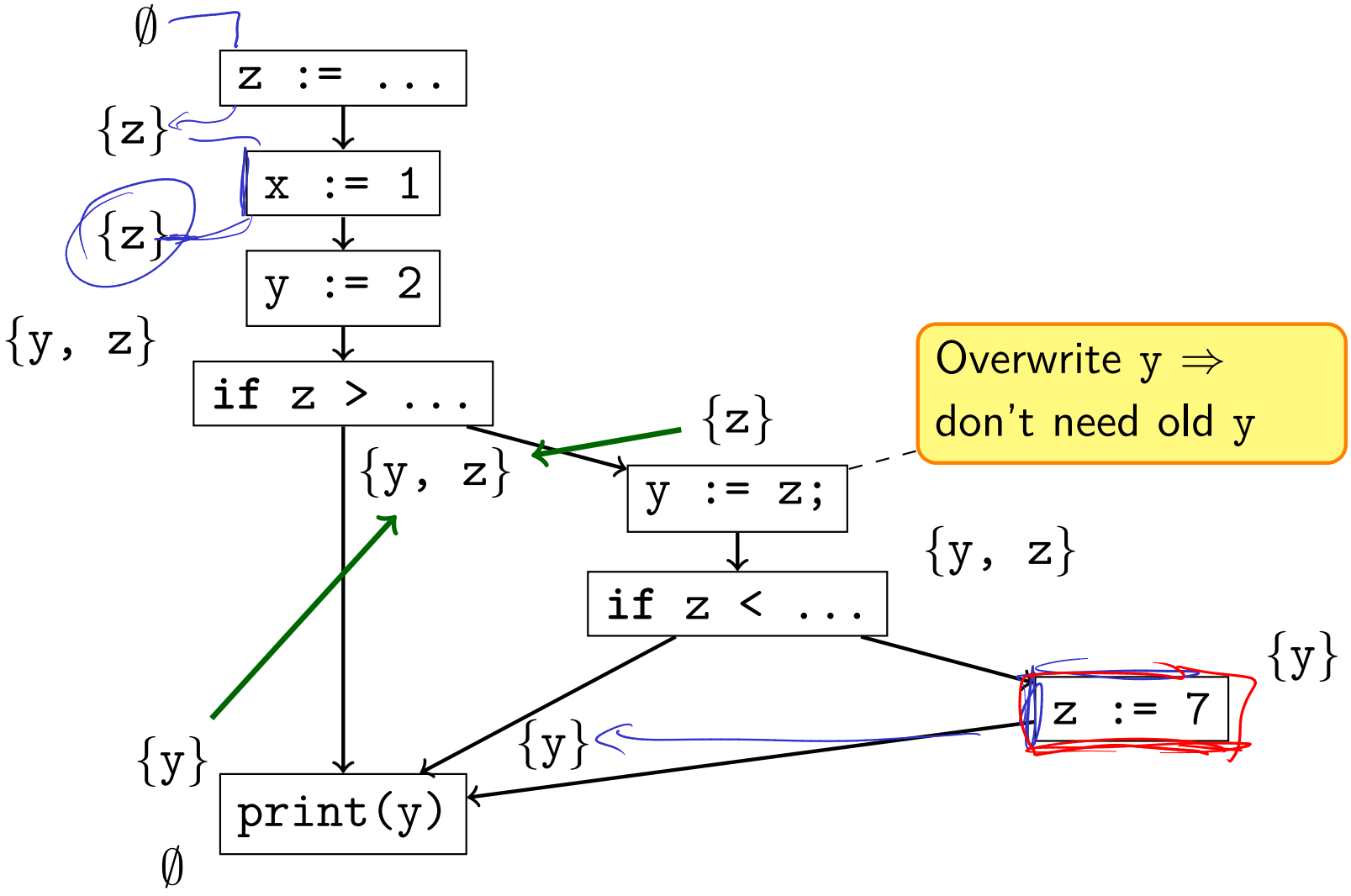

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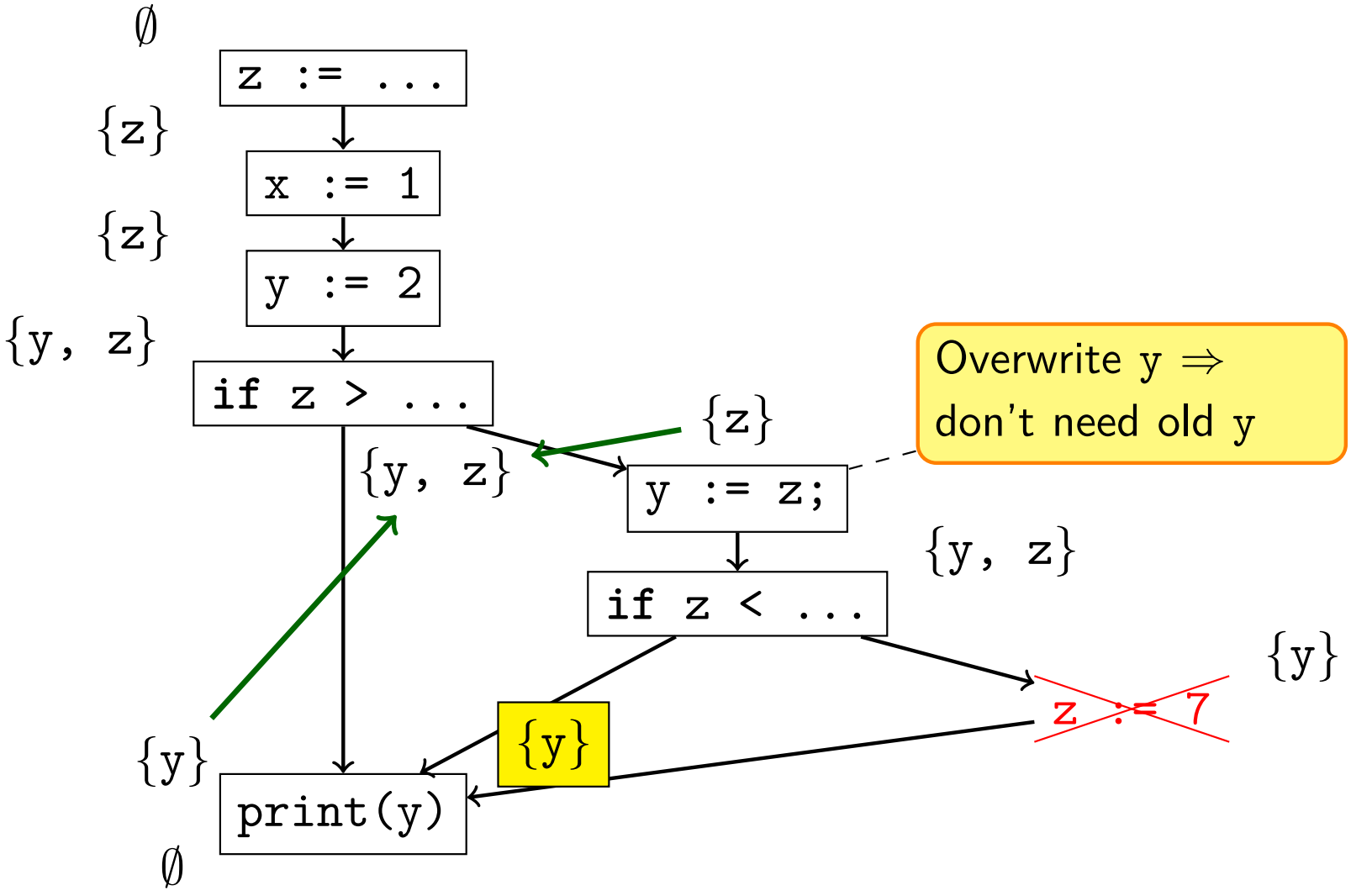

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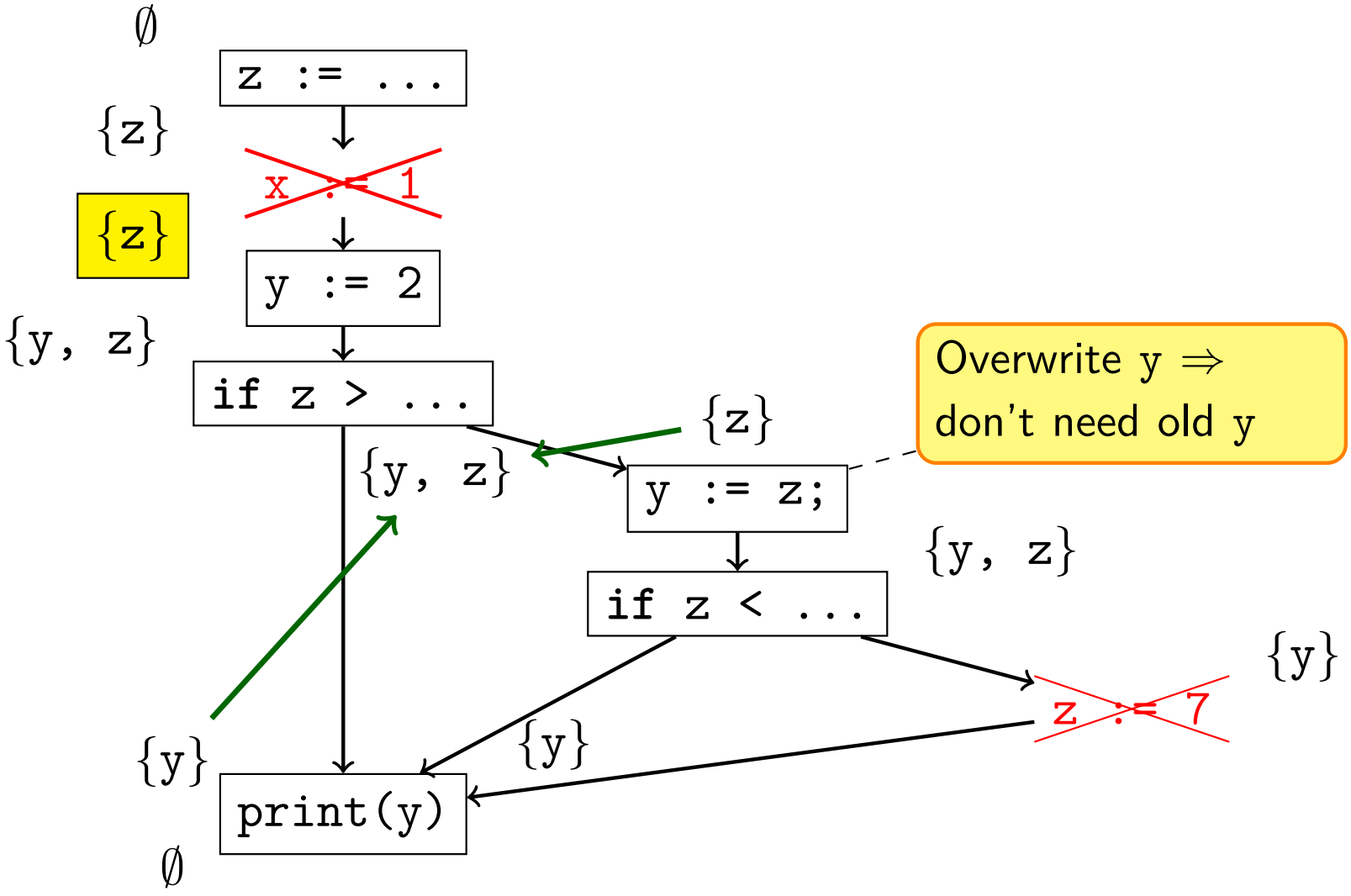

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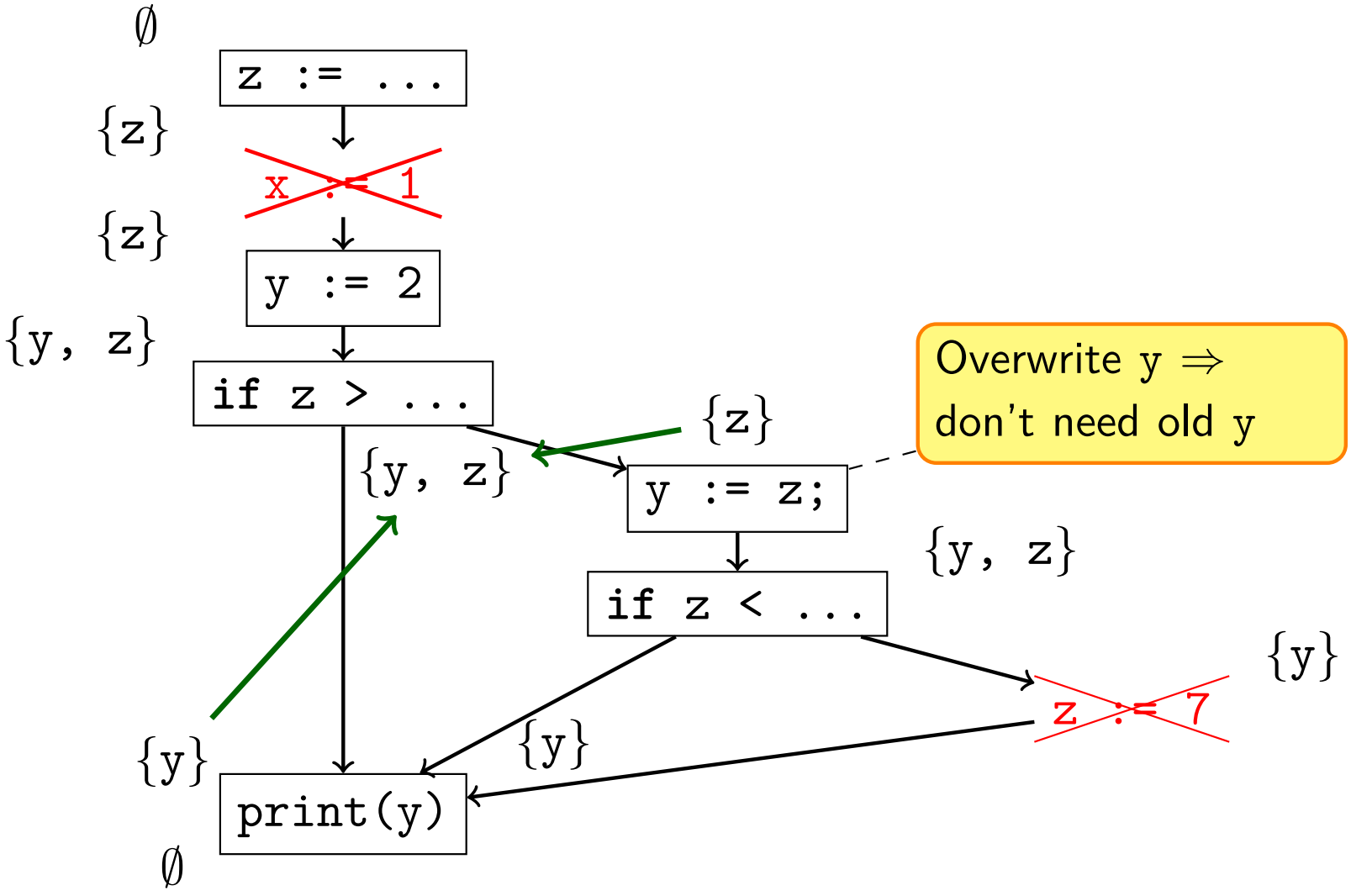
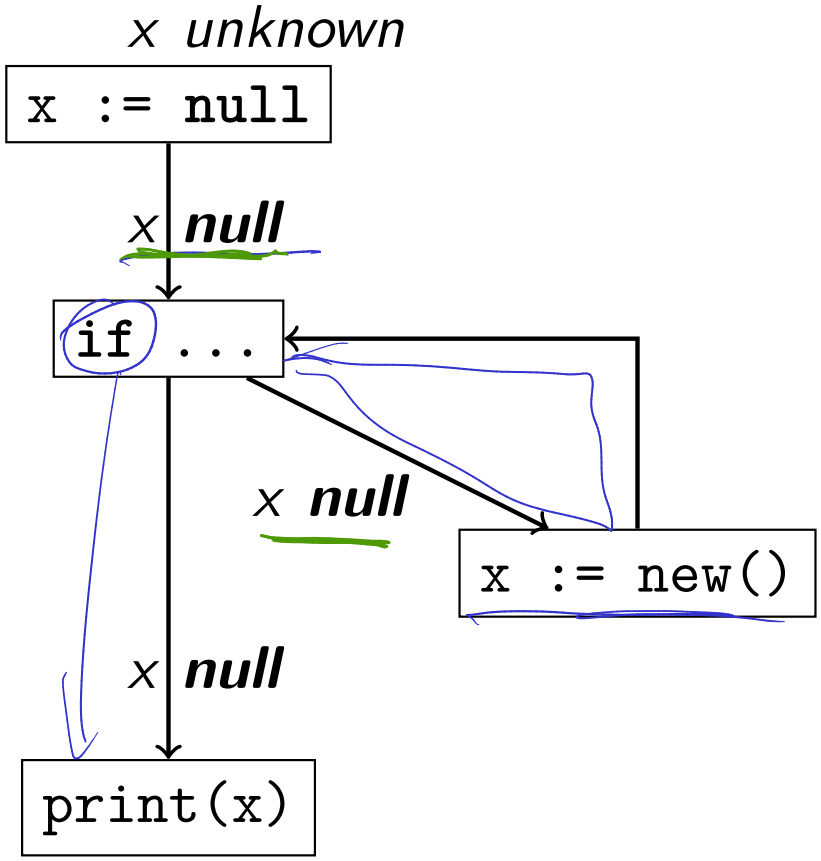


Analysis effective: found useless assignments to z and x

Observations

1. Data Flow analysis can be run *forward* or *backward*
2. May have to *join* results from multiple sources
3. Some analyses may need multiple "passes" (steps)

What about Loops? (1/2)



- Analysis: *Null Pointer Dereference*

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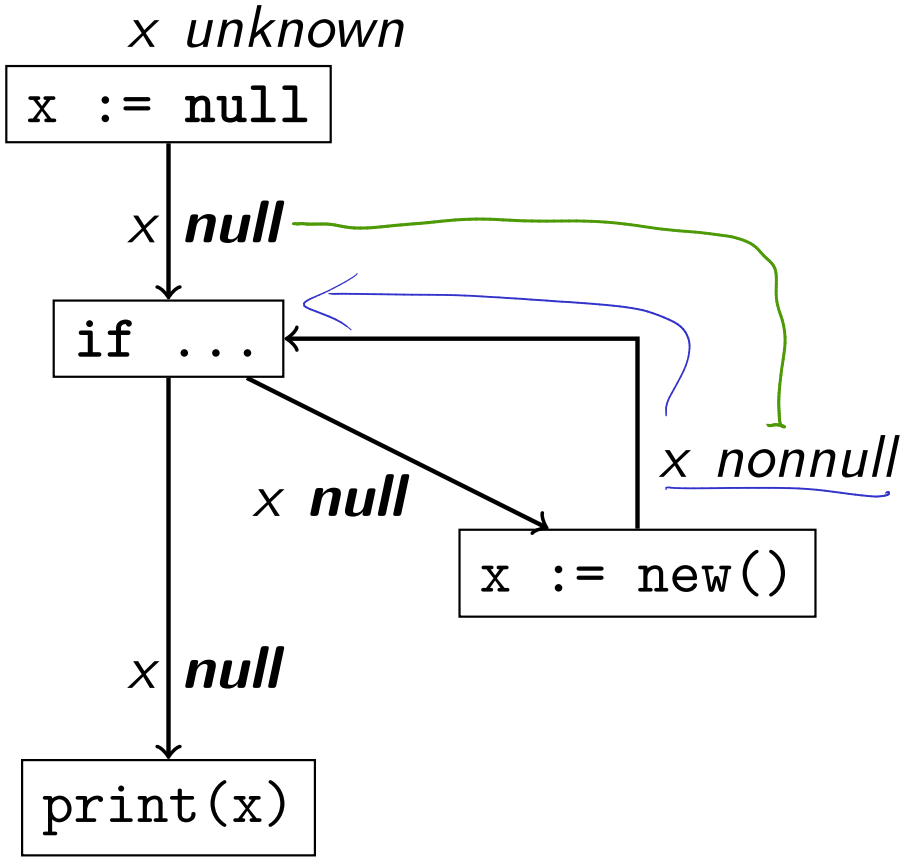

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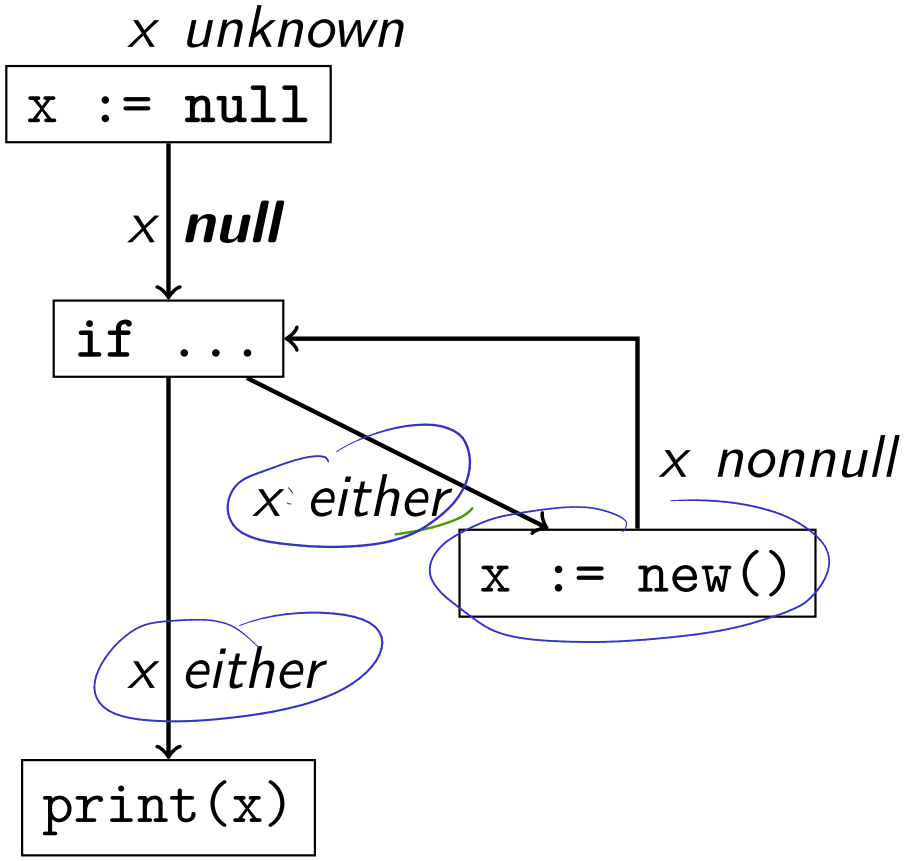

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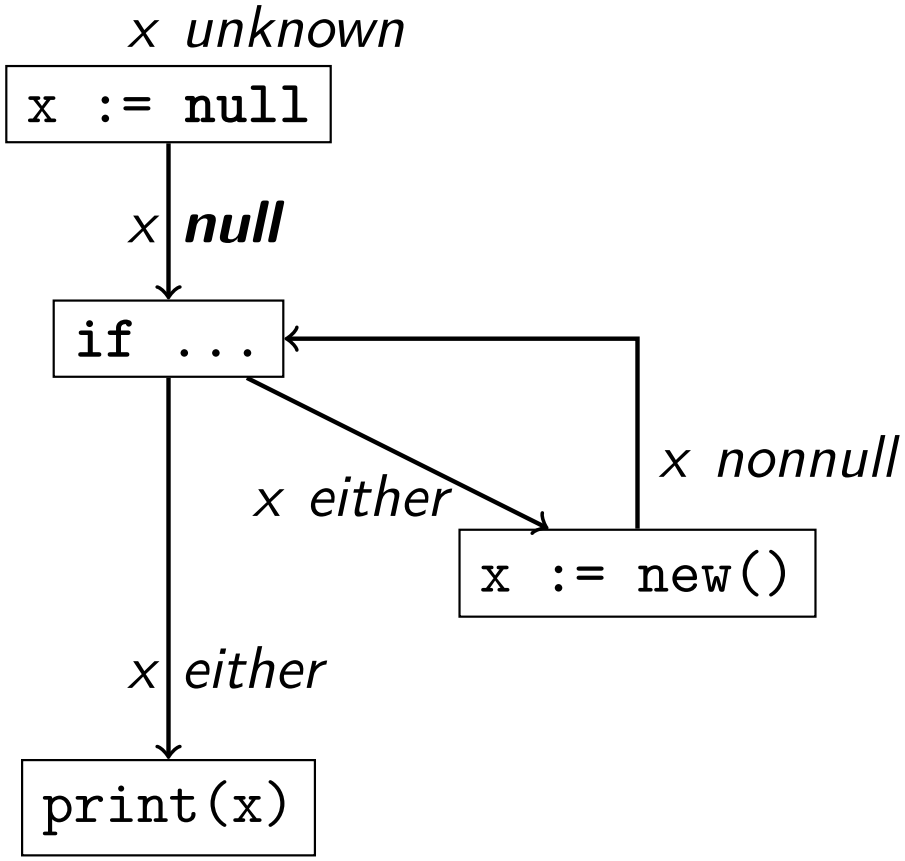

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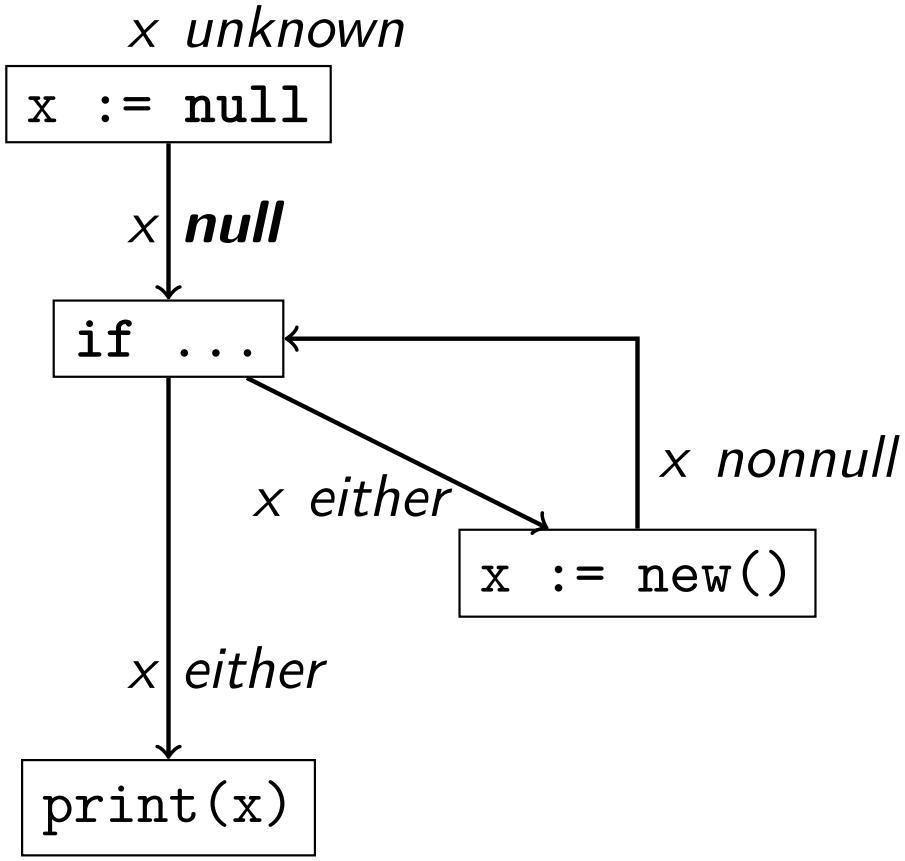
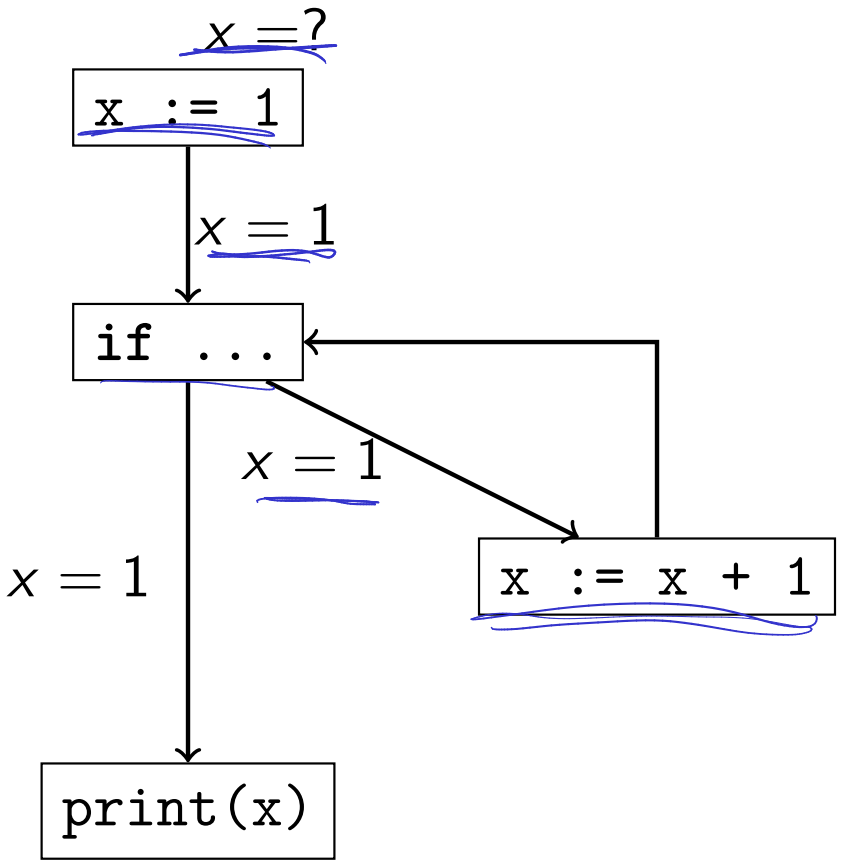


- Analysis: *Null Pointer Dereference*
- Stop when we're not learning anything new any more
- Works fine

What about Loops? (2/2)



- Analysis: *Reaching Definitions*

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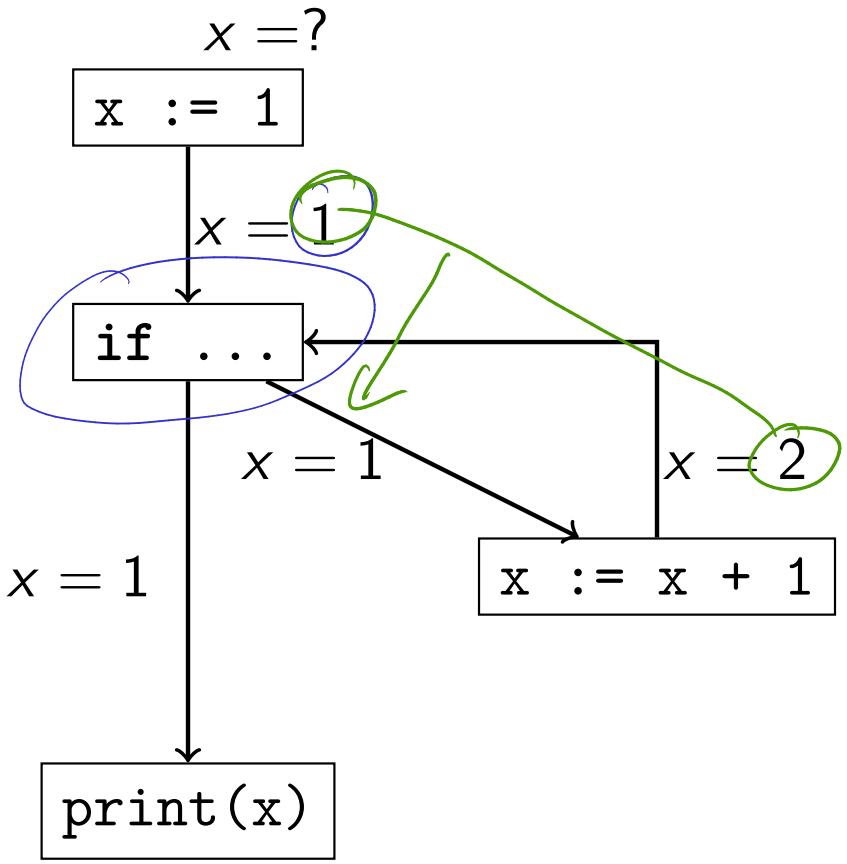

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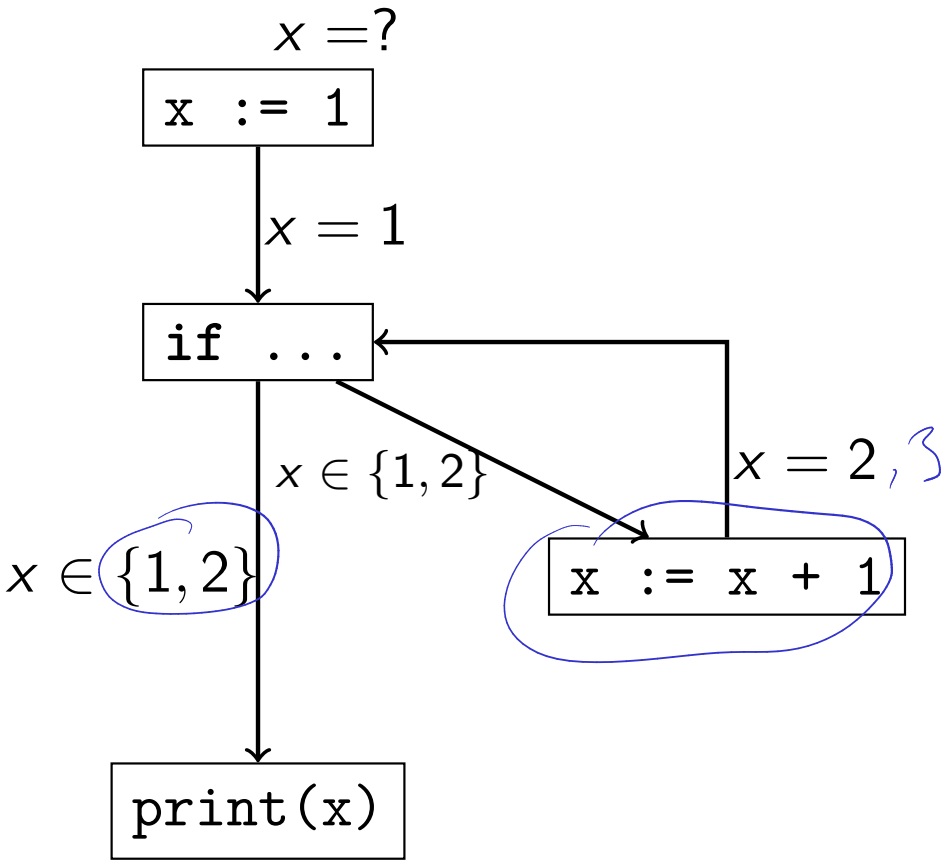

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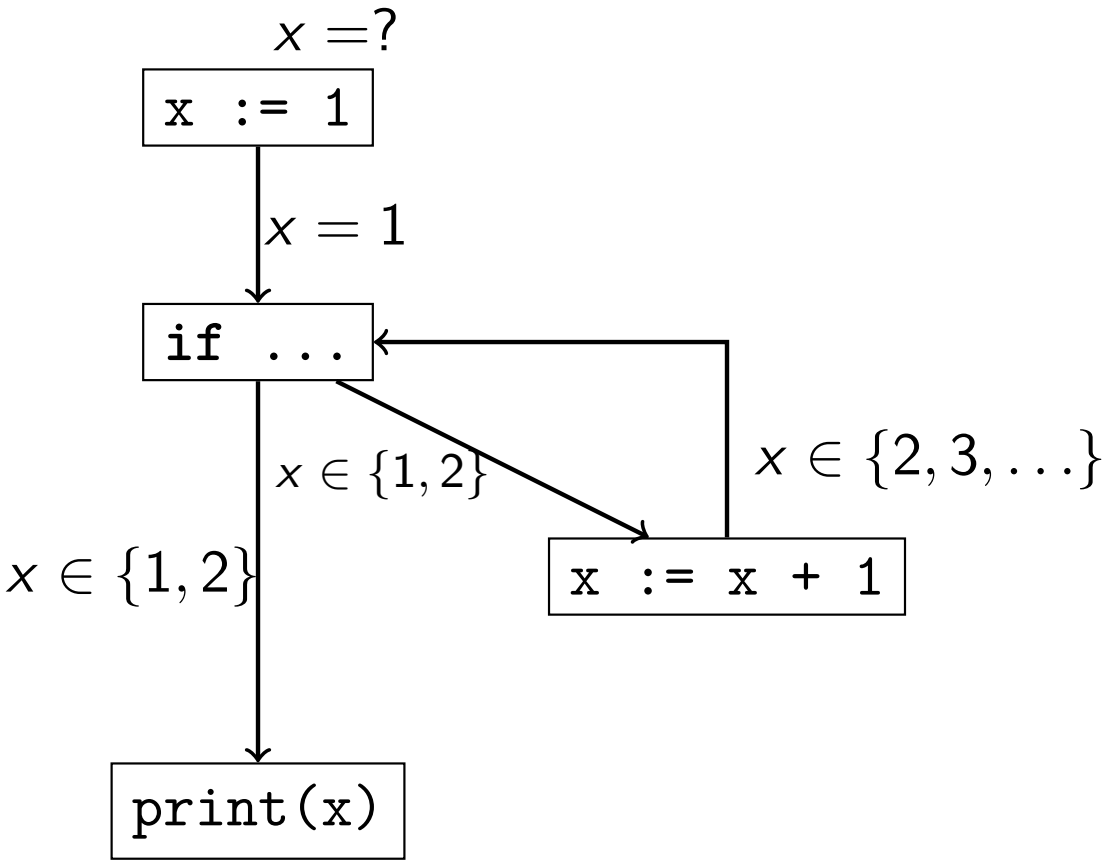

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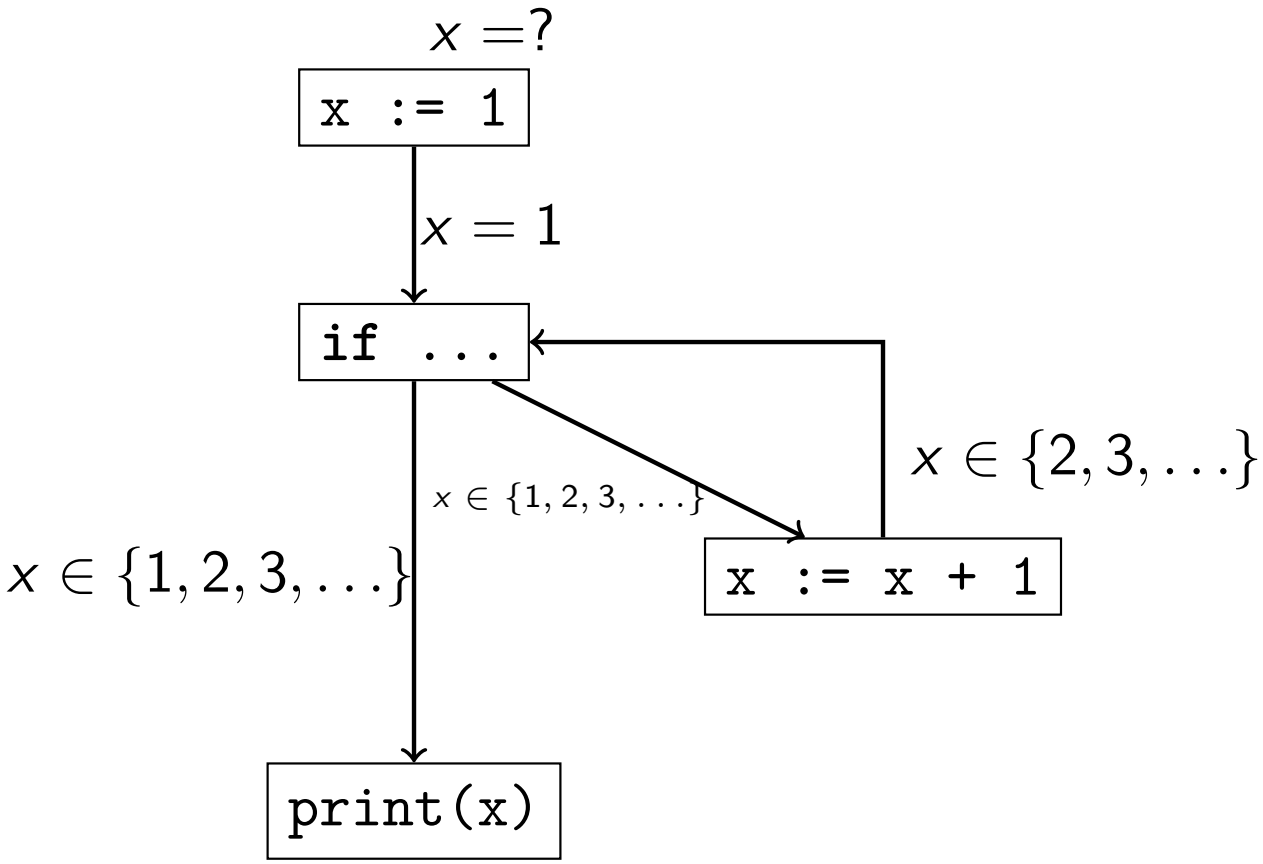

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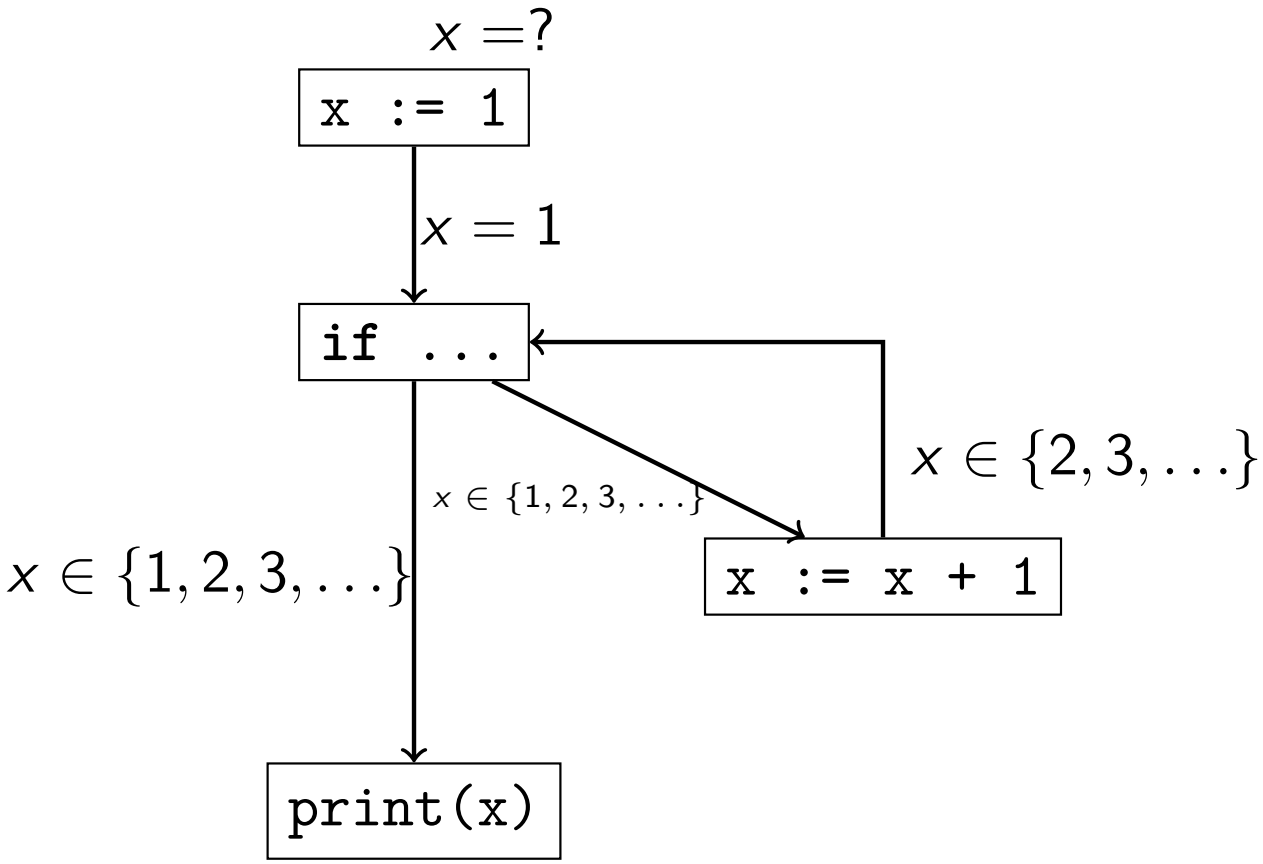

- Analysis: *Reaching Definitions*

We need to bound repetitions!

Summary: Data-Flow Analysis (Introduction)

- Data flow depends on *control flow*
- Data flow analysis examines how variables or other program state change across control-flow edges
- May have to join multiple results
- Can run *forward* or *backward* relative to control flow edges
- Handling loops is nontrivial

Engineering Data Flow Algorithms

Engineering Data Flow Algorithms

1. General Algorithm

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2. Termination

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3. (Correctness)

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   - Keep updating until nothing changes
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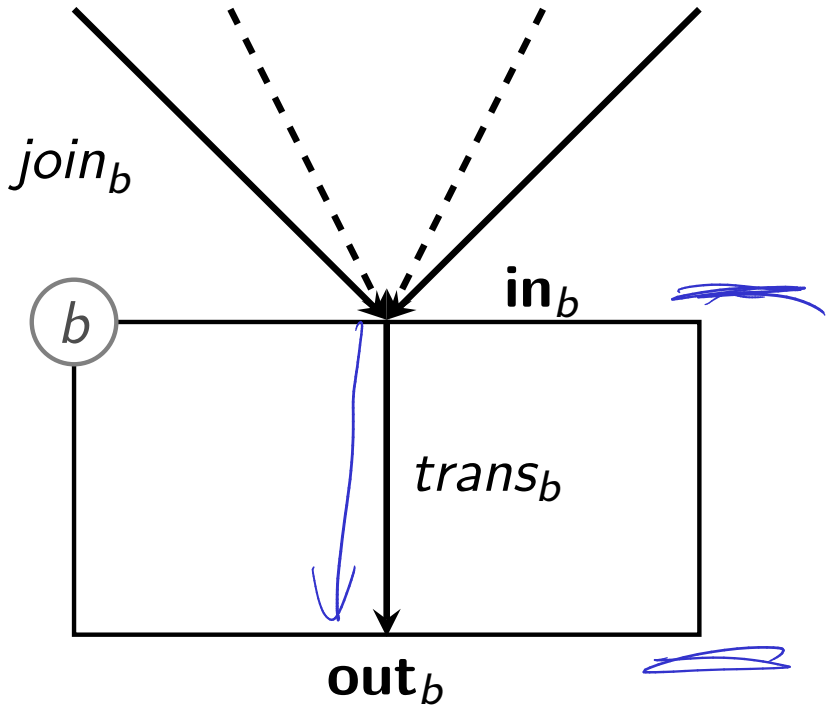
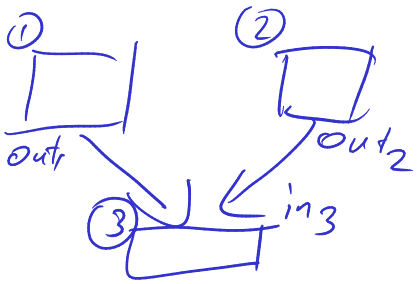
Engineering Data Flow Algorithms

1. General Algorithm
   - Keep updating until nothing changes
2. Termination
   - Assumption: Operate on Control Flow Graph
   - Theory: Ensure termination
3. (Correctness)

Data Flow Analysis on CFGs

- $\mathbf{in}_b$: knowledge at entrance of basic block b
- $\mathbf{out}_b$: knowledge at exit of basic block b
- $join_b$: combines all $\mathbf{out}_{b_i}$ for all basic blocks b_i that flow into b "**Join Function**"
- $trans_b$: updates $\mathbf{out}_b$ from $\mathbf{in}_b$ "**Transfer Function**"





Characterising Data Flow Analyses

Characteristics:

- *Forward* or *backward* analysis
- L: Abstract Domain (the 'analysis domain')
- $trans_b : L \rightarrow L$
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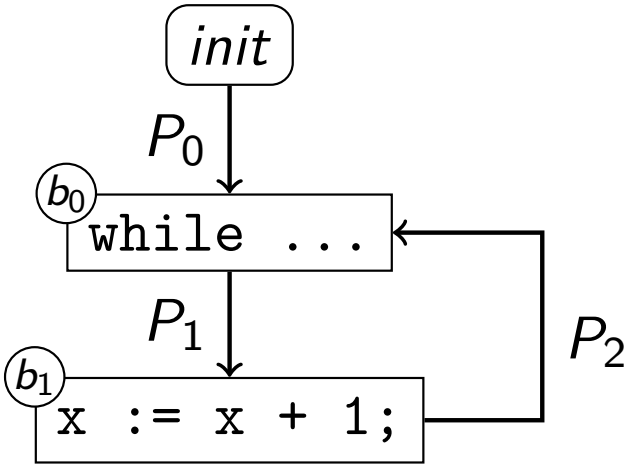
Require properties of L, $trans_b$, $join_b$ to ensure termination

Limiting Iteration



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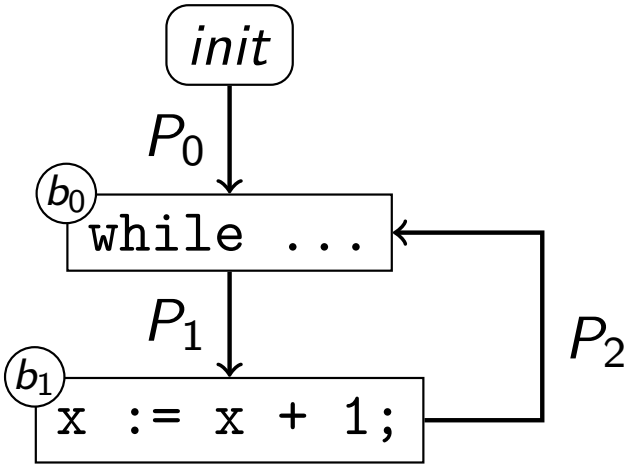

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$$\mathbf{in}_{b_0} = join_{b_0}(P_0, P_2)$$

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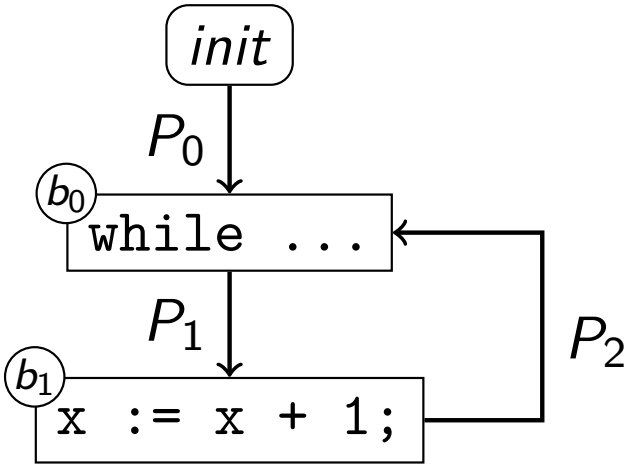

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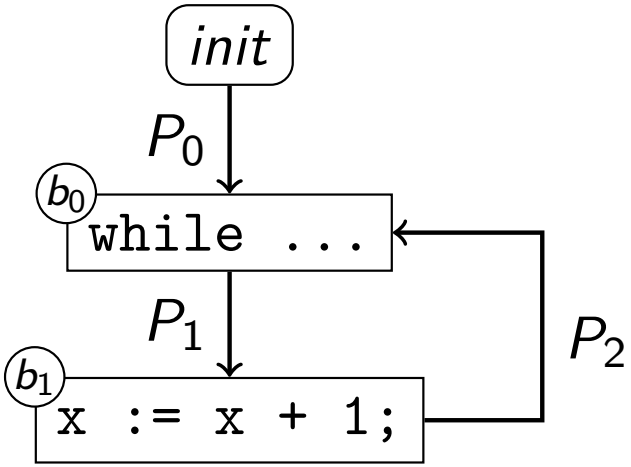

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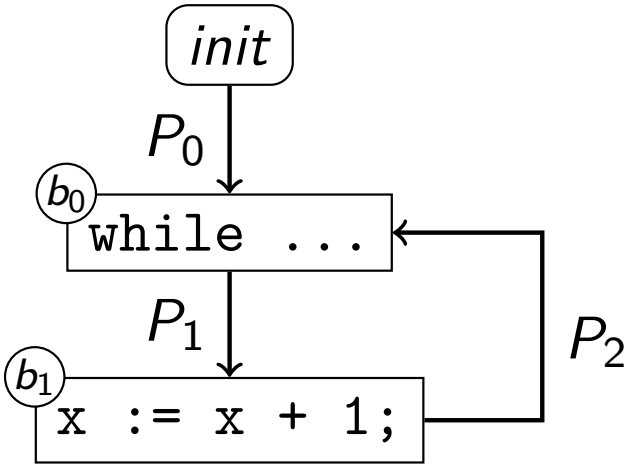

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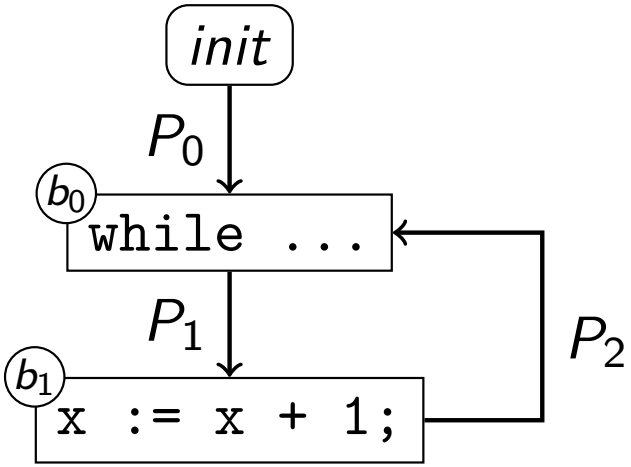

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Eventually, either nothing changes or we hit growth limit

Ordering Knowledge

$$B \sqsupseteq A$$

$$A \sqsubseteq B$$

$$A \sqsubset B$$

$$\begin{array}{c} B \\ | \\ A \end{array}$$

- B describes at least as much knowledge as A
- Either:
  - $A = B$ (i.e., $A \sqsupseteq B \sqsupseteq A$), or
  - B has strictly more knowledge than A

Intuition: Knowing Less, Knowing More

Structure of L:

A B $\cdots$ Y Z

Intuition: Knowing Less, Knowing More

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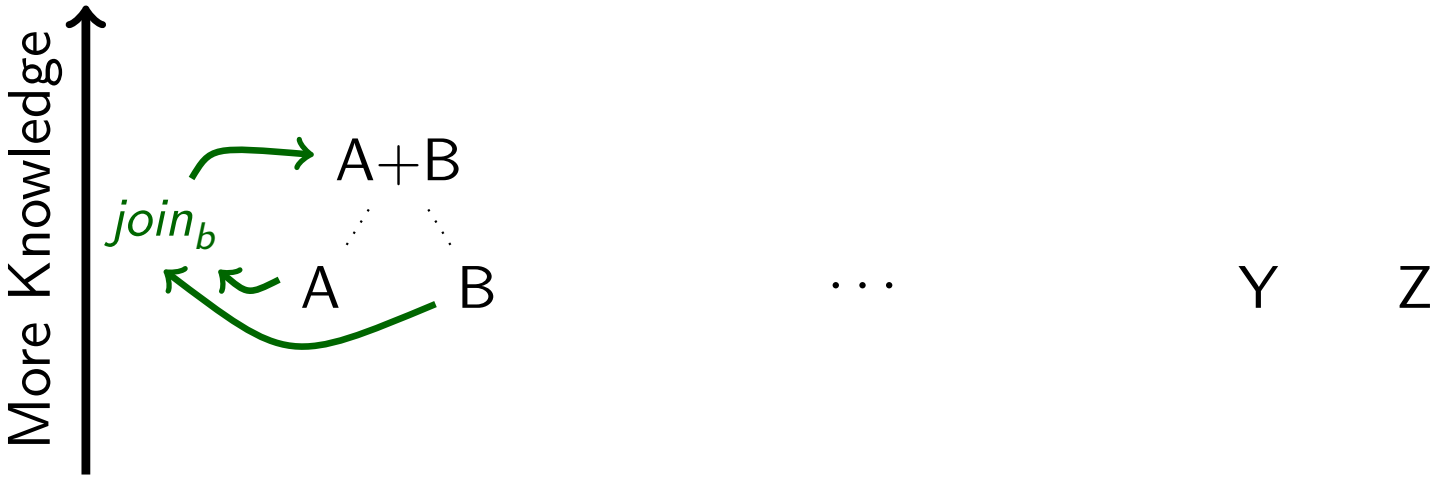

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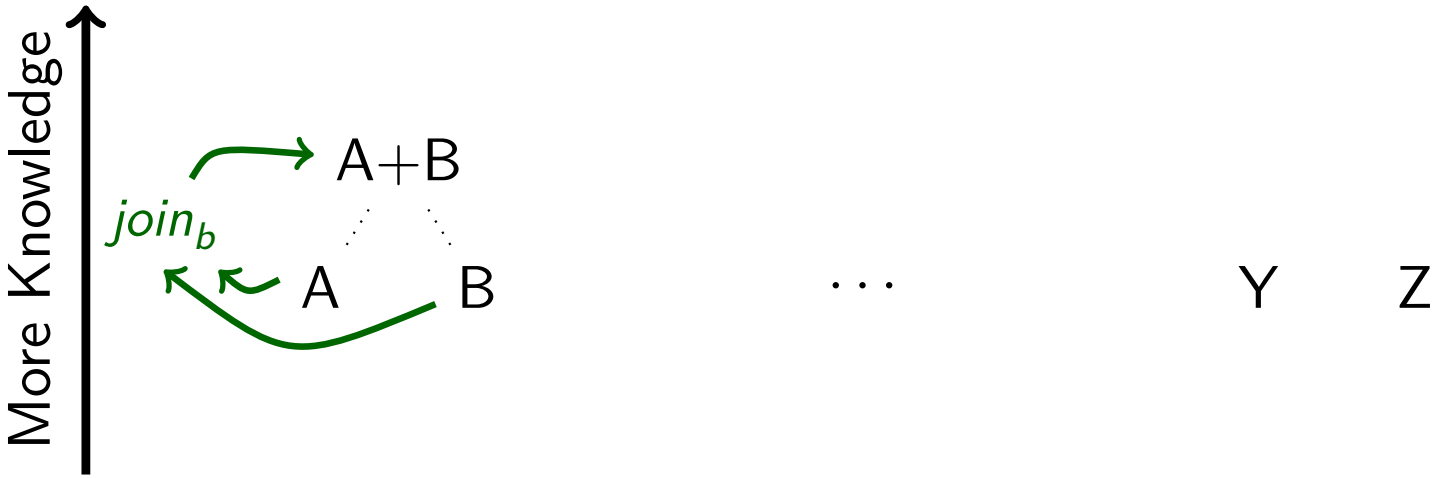

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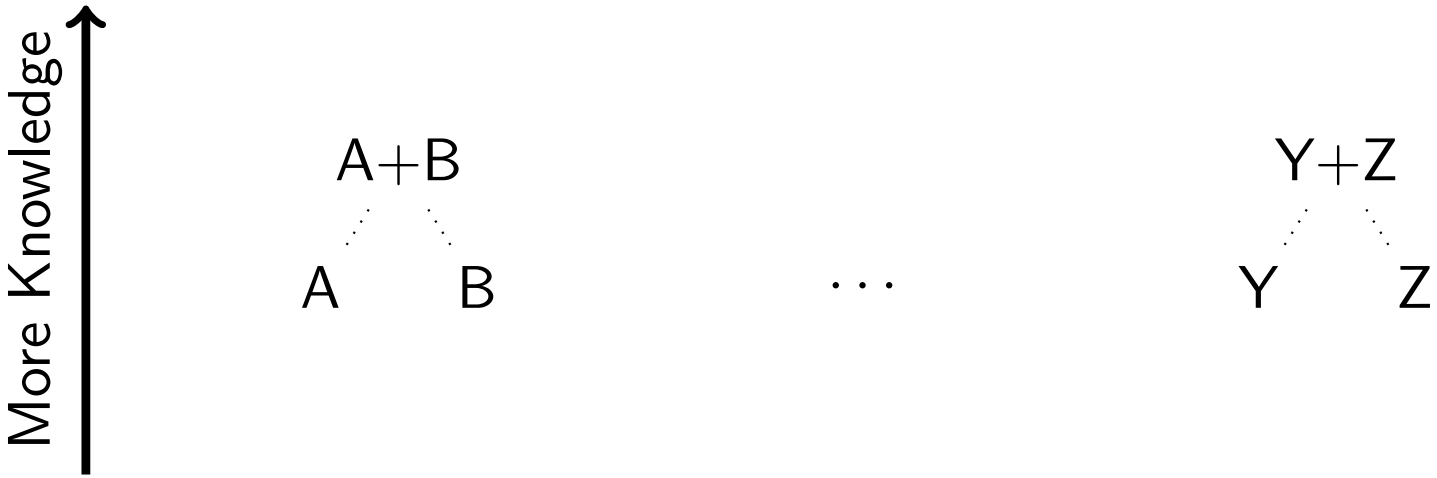

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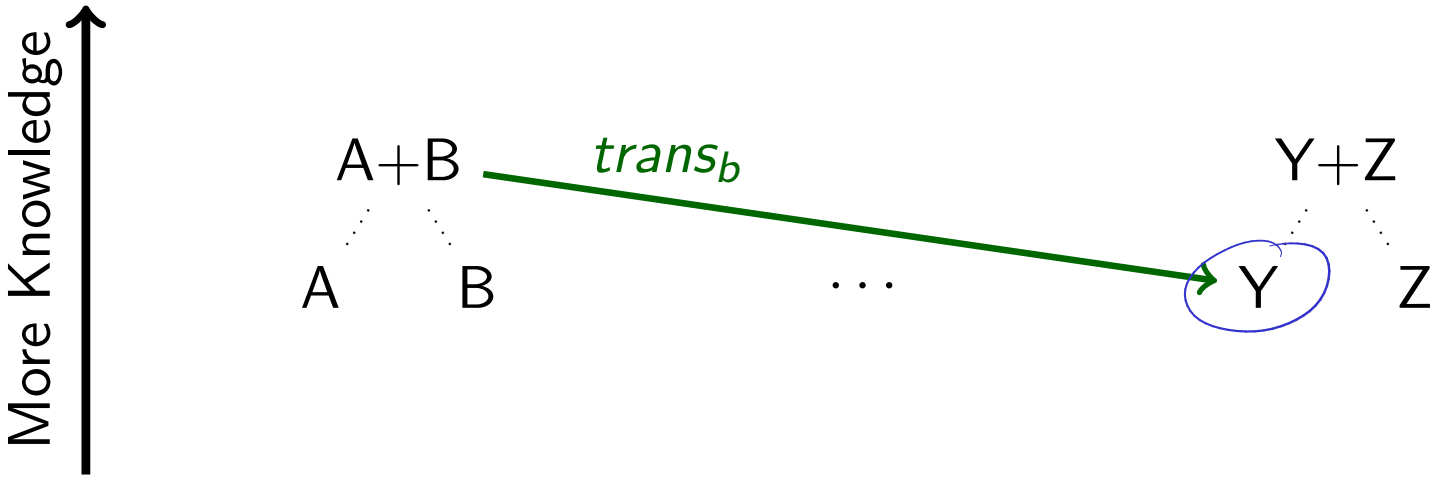

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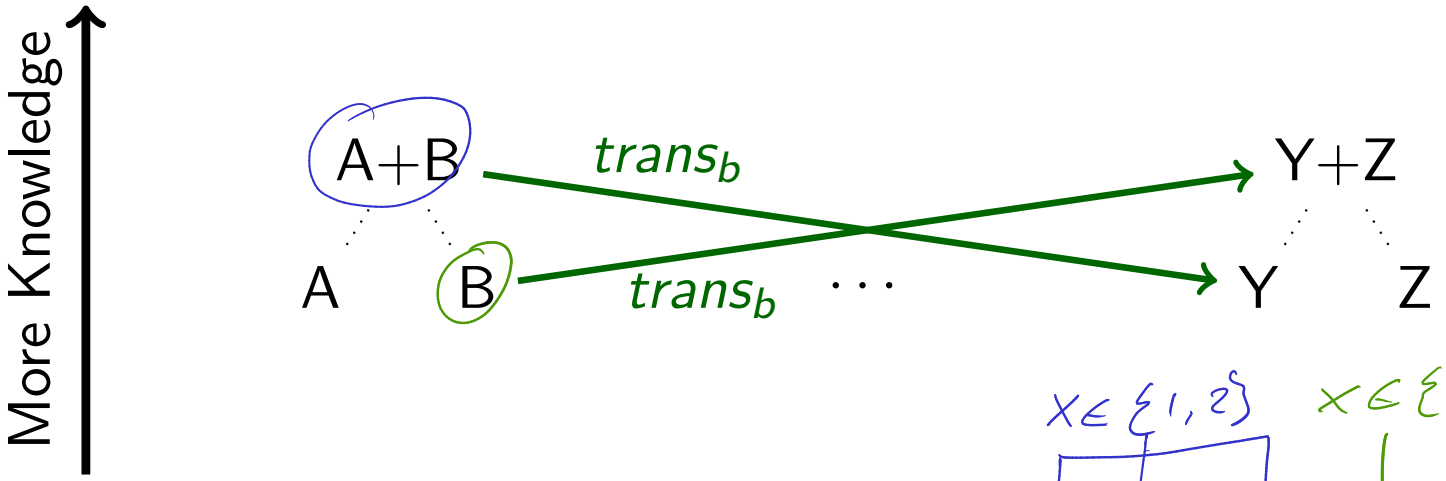

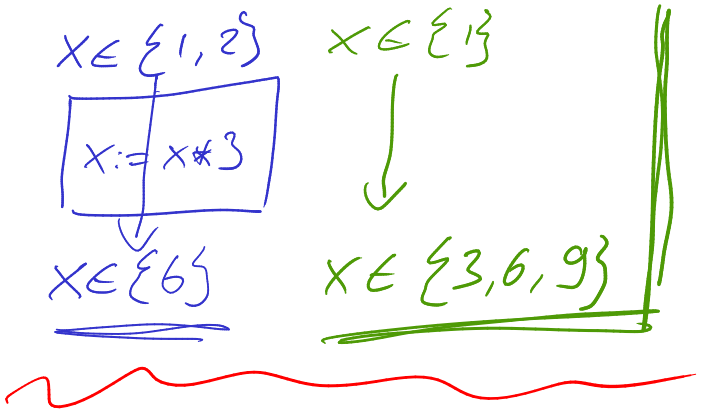

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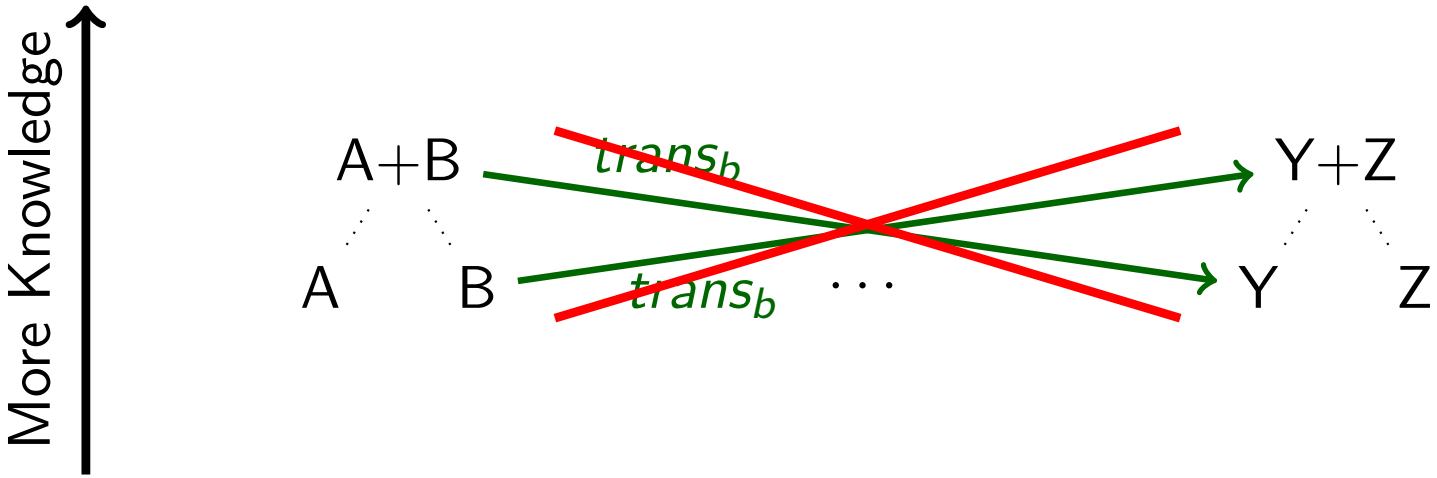

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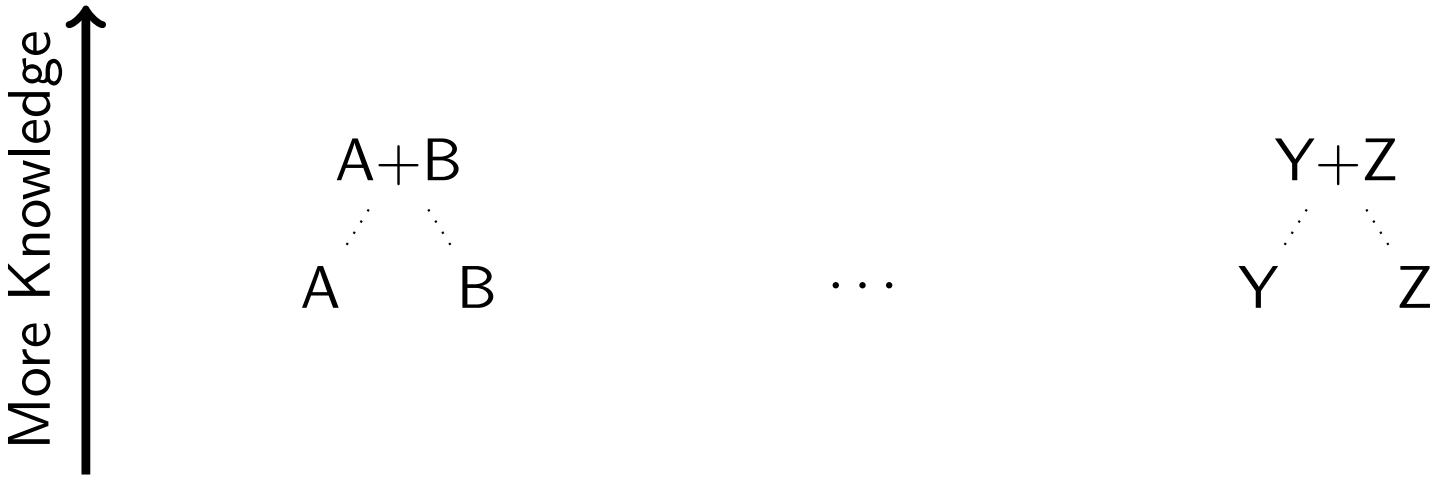

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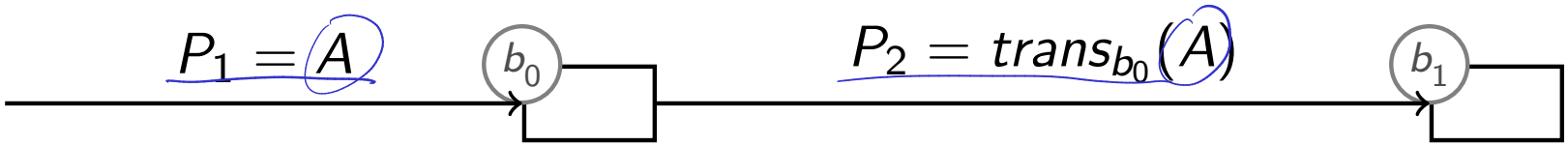
- $trans_b$ must be *monotonic* over amount of knowledge:

$$x \sqsupseteq y \implies trans_b(x) \sqsupseteq trans_b(y)$$

Aggregating Knowledge



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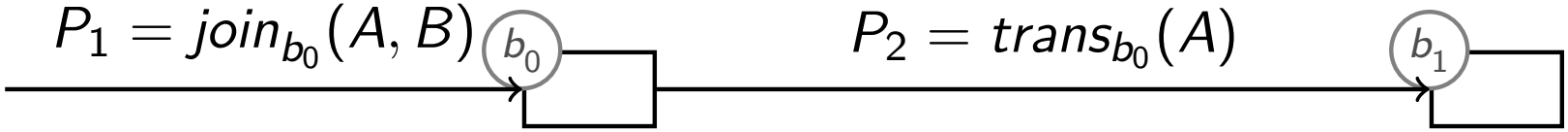

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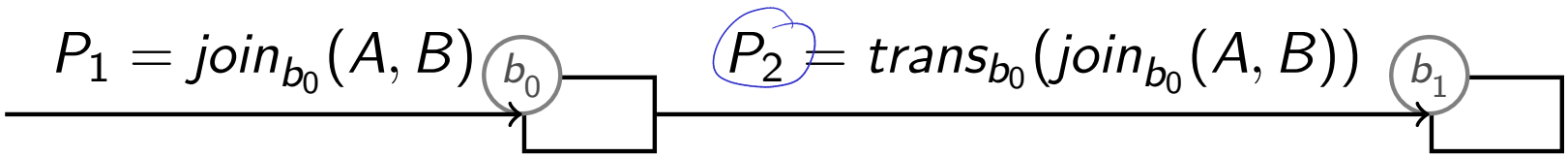

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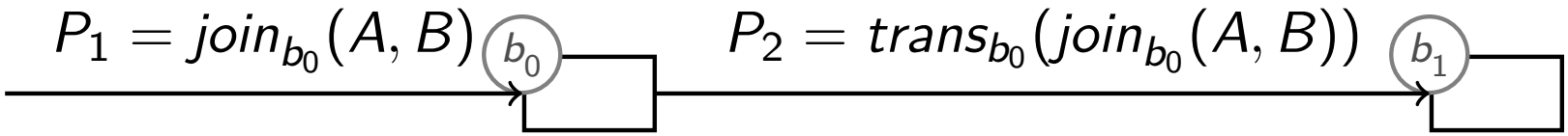

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$\Rightarrow$ At each node, we either stay equal or grow

Now we must only set a growth limit...

Ascending Chains

- A (possibly infinite) sequence $a_0, a_1, a_2, \ldots$ is an *ascending chain* iff:

$$a_i \sqsubseteq a_{i+1} \text{ (for all } i \geq 0)$$

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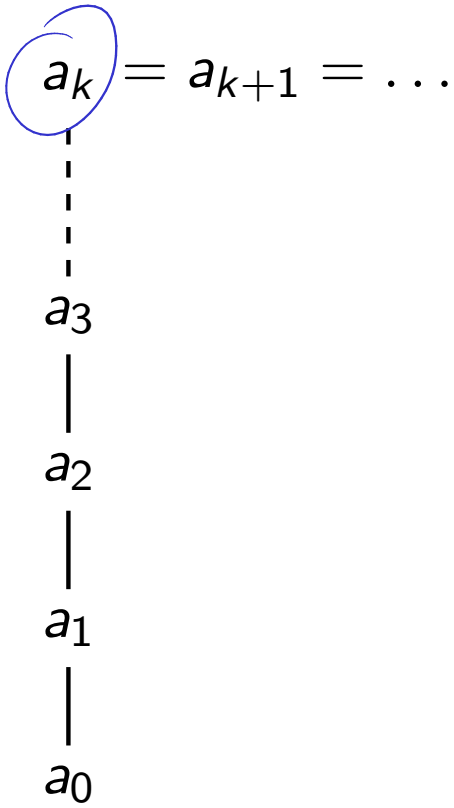

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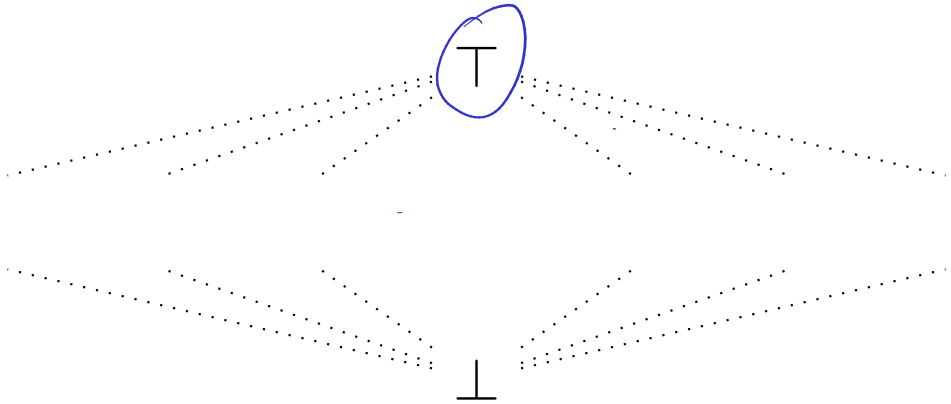
- *Ascending Chain Condition*:
  - For *every* ~~descending~~ ascending chain $a_0, a_1, a_2, \ldots$ in abstract domain L:
  - there exists $k \geq 0$ such that:

$$a_k = a_{k+n} \text{ for any } n \geq 0$$

ACC is formalisation of growth limit

Top and Bottom



- *Convention*: We introduce two distinguished elements:
  - **Top**: $\top$: $A \sqsubseteq \top$ for all A
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- Intuition:
  - $\top$: means 'contradictory / too much information'
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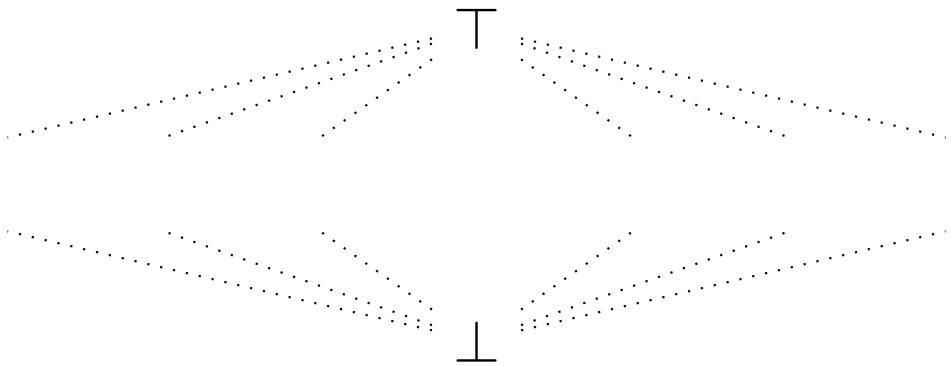

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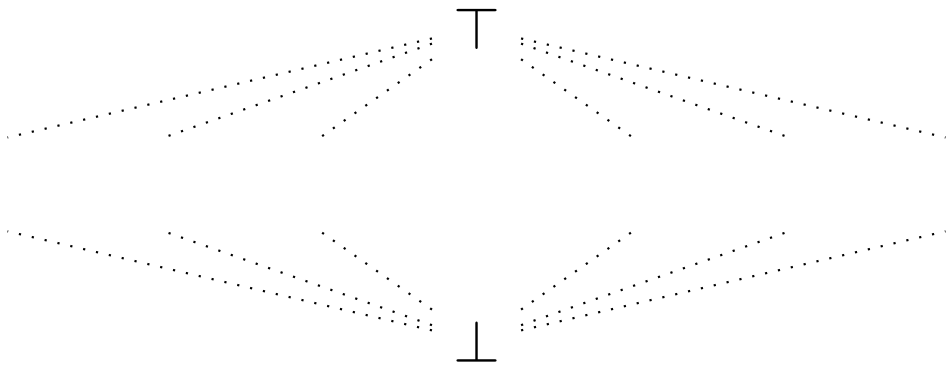

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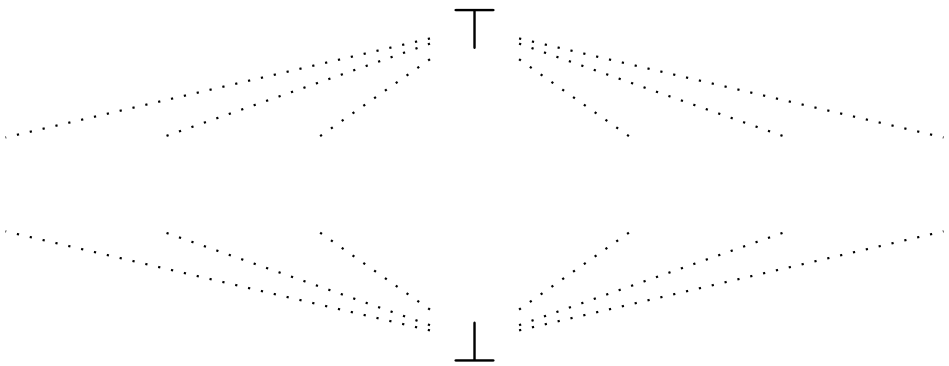

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  - $join_b(\bot, A) \sqsupseteq A \sqsupseteq \bot$
    - In practice, it's safe and simple to set: $join_b(\bot, A) = A = join_b(A, \bot)$
- Intuition:
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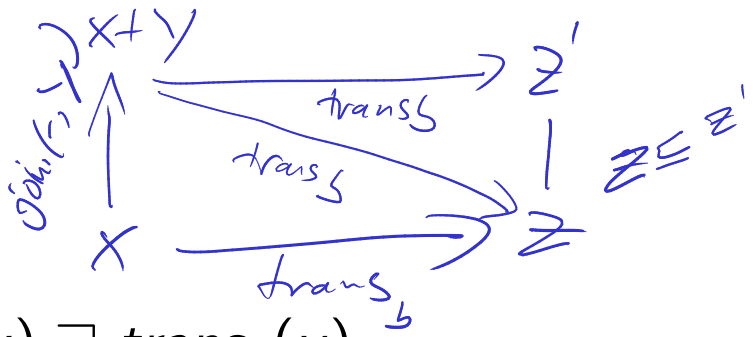
Summary

- Designing a *Forward* or *backward* analysis:
- Pick **Abstract Domain** L
  - Must be **partially ordered** with $(\sqsupseteq) \subseteq L \times L$:
    $A \sqsupseteq B$ iff A 'knows' at least as much as B
  - Unique top element $\top$
  - Unique bottom element $\perp$
- $trans_b : L \to L$
  - Must be *monotonic*:



$$x \sqsupseteq y \implies trans_b(x) \sqsupseteq trans_b(y)$$

- $join_b : L \times L \to L$ must produce an *upper bound* for its parameters:
  - $join_b(A, B) \sqsupseteq A$
  - $join_b(A, B) \sqsupseteq B$
- Satisfy **Ascending Chain Condition** to ensure termination
  - Easiest solution: make L finite

Abstract Domains Revisited

$T = D^?$

D^+

$$\overbrace{\cdots \quad -3 \quad -2 \quad -1}^{} \quad 0 \quad \overbrace{1 \quad 2 \quad 3 \quad \cdots}^{}$$

D^- D^0 D^+

$D^?$

$\ominus$ *is compatible with* neg

$T \sqsupseteq$ every thing $\quad \bot \sqsubseteq$ everything

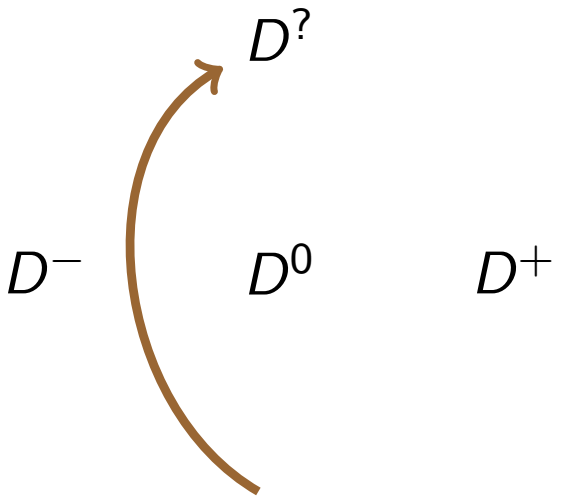
$$\begin{aligned}
\ominus D^0 &= D^0 \\
\ominus D^+ &= D^- \\
\ominus D^- &= D^+ \\
\ominus D^? &= D^?
\end{aligned}$$

TOP: ✓

BOTTOM: ✗

$D^+ \sqsubseteq \underline{\text{join}(D^-, D^+)} \sqsupseteq D^-$

Abstract Domains Revisited



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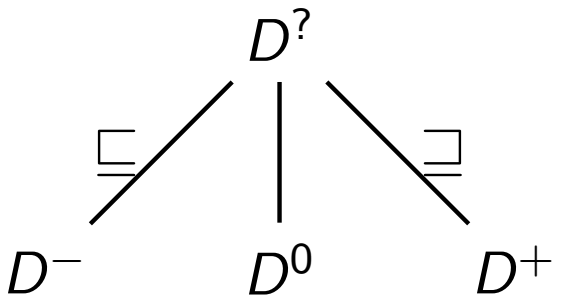
$$

Abstract Domains Revisited



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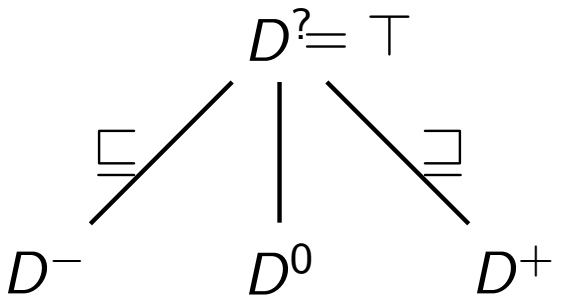
\end{array}
$$

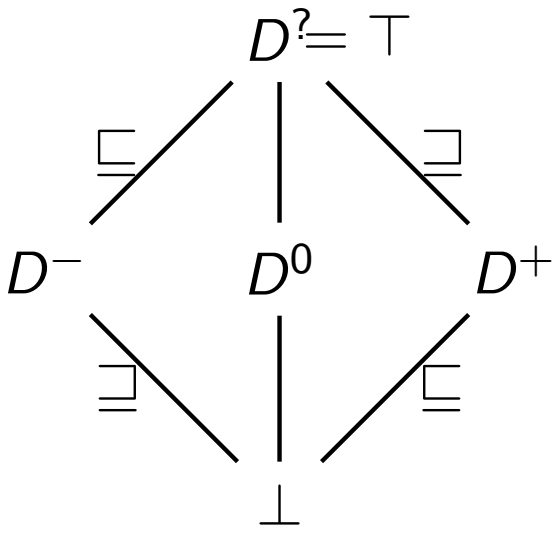
Abstract Domains Revisited



$\ominus$ *is compatible with* neg

$$
\begin{array}{lcl}
\ominus D^0 & = & D^0 \\
\ominus D^+ & = & D^- \\
\ominus D^- & = & D^+ \\
\ominus D^? & = & D^?
\end{array}
$$

Abstract Domains Revisited



$\ominus$ *is compatible with* neg

$$
\begin{array}{lcl}
\ominus \bot & = & \bot \\
\ominus D^0 & = & D^0 \\
\ominus D^+ & = & D^- \\
\ominus D^- & = & D^+ \\
\ominus D^? & = & D^?
\end{array}
$$

$\ominus$ is monotonic (and $\oplus$ extended with $\bot$ is, too)

Summary

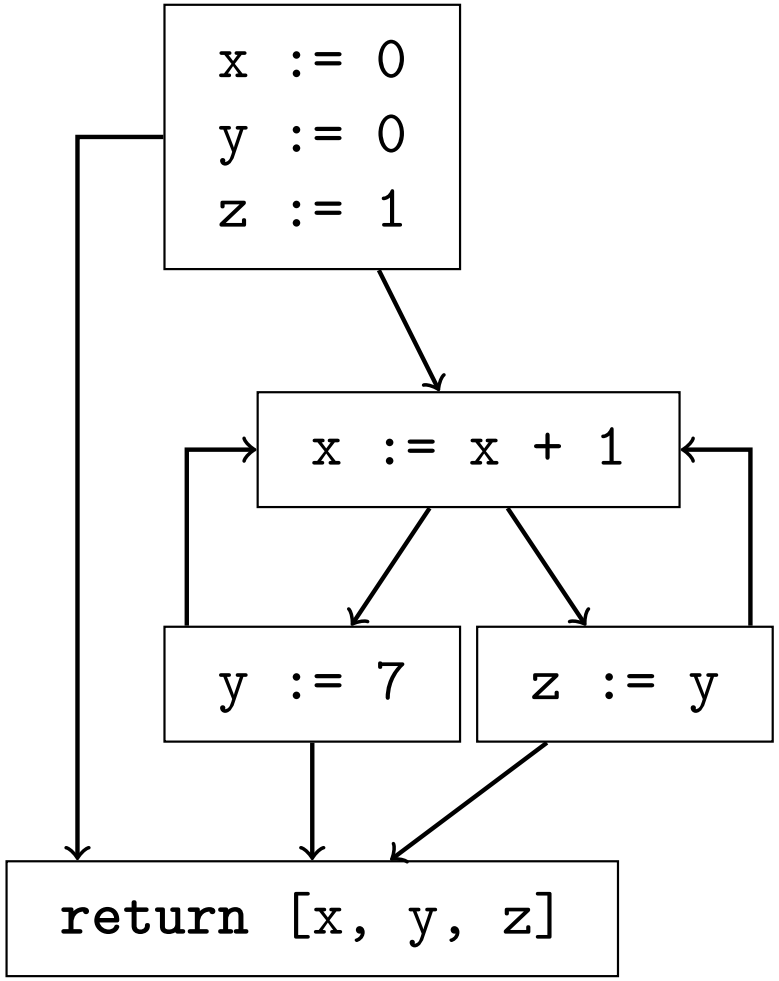
- We could extend $\{D^+, D^-, D^0, D^?\}$ to an Abstract Domain by adding $\perp$

$$L_D = \{D^+, D^-, D^0, D^?, \perp\}$$

- L_D is finite, so the DCC holds trivially
- Our *Transfer Functions* $\ominus, \oplus$ are monotonic

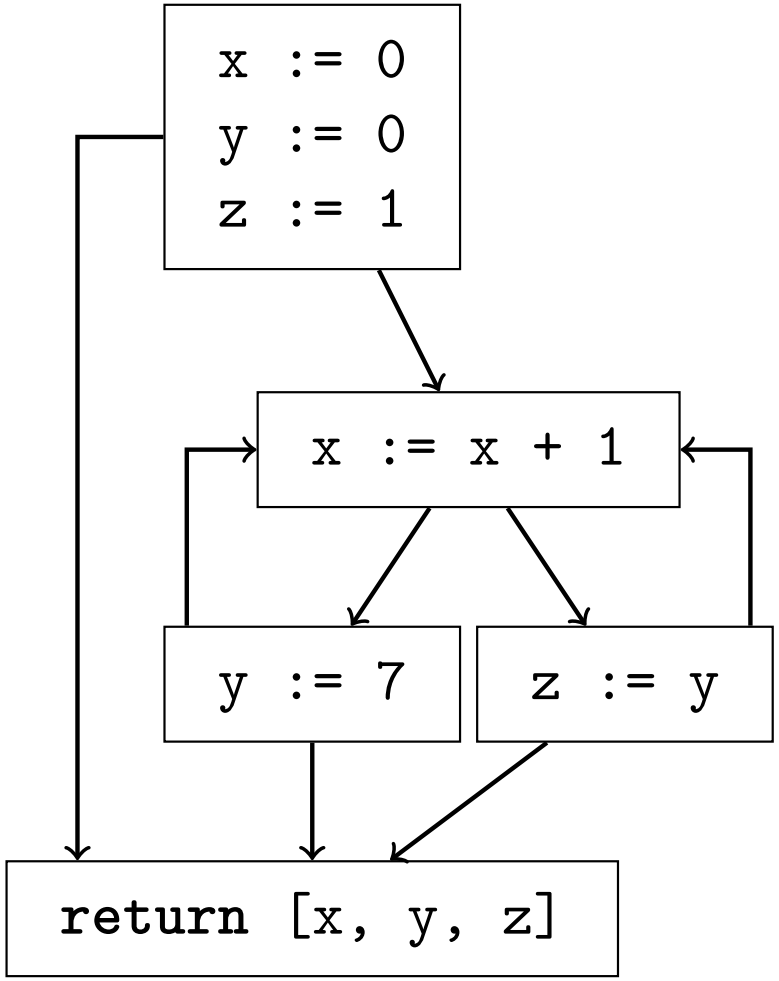
Example: Reaching Definitions

```
var x := 0;
var y := 0;
var z := 1;

while x < 5 {
  x := x + 1;
  if x >= 2 {
    y := 7;
  } else {
    z := y;
} }

return [x, y, z];
```



Example: Reaching Definitions

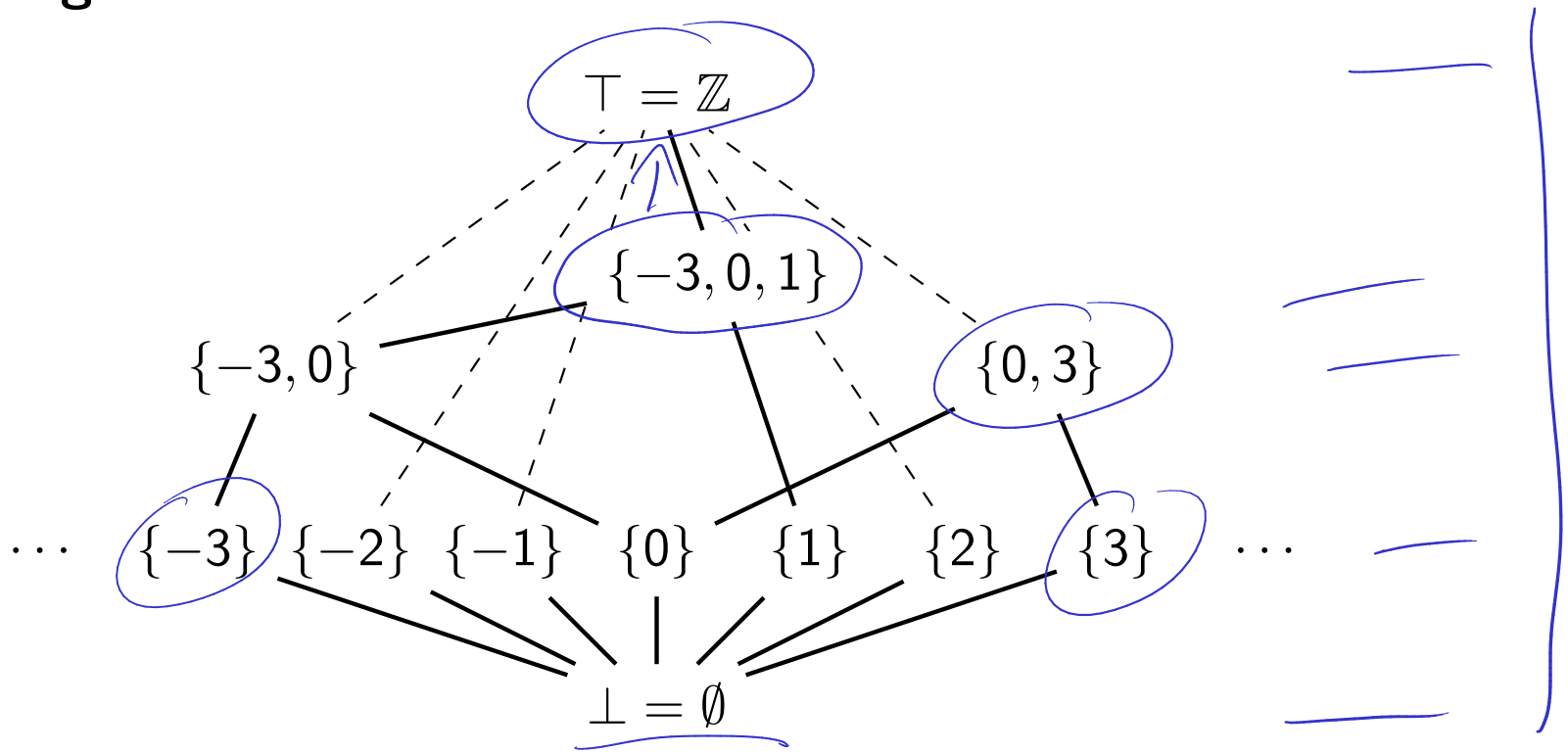
```
var x := 0;
var y := 0;
var z := 1;

while x < 5 {
  x := x + 1;
  if x >= 2 {
    y := 7;
  } else {
    z := y;
} }

return [x, y, z];
```



Reaching Definitions: What values are possible?

Example: Reaching Definitions

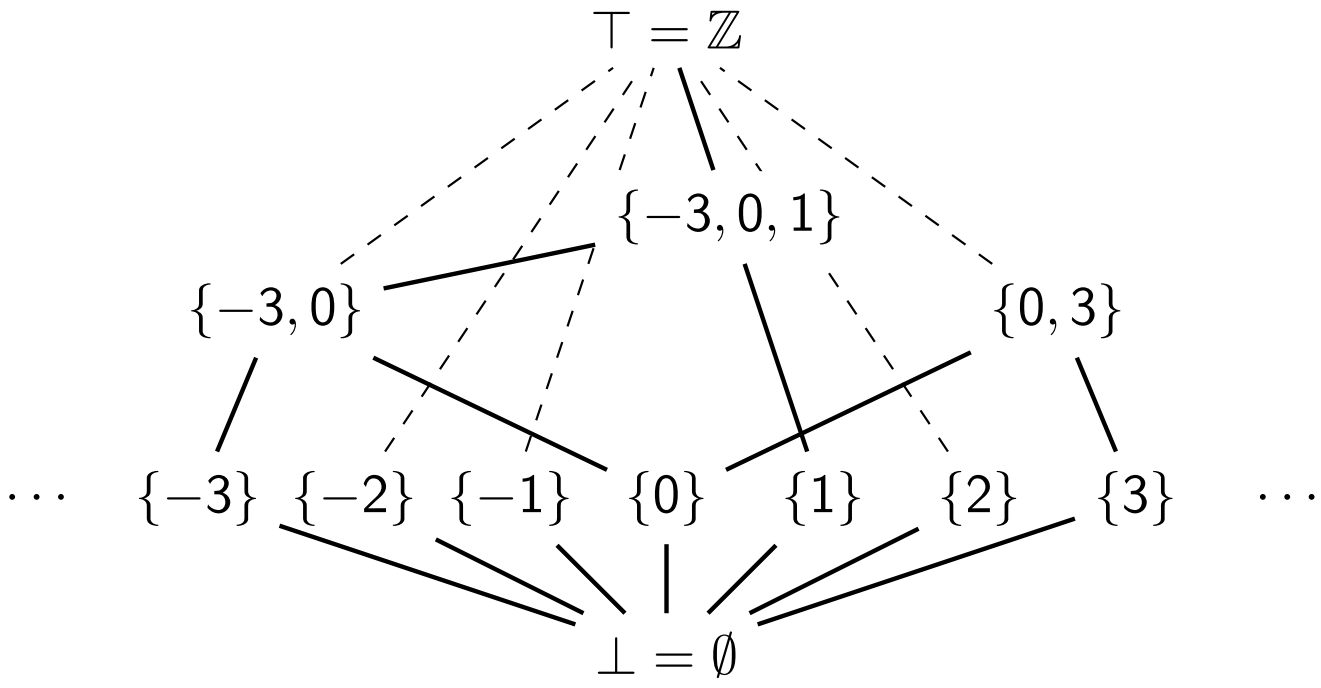
Designing our abstract domain:



- Capture sets of up to 3 possible numbers
- $\top$: More than 3 possible numbers
- $\bot$: $\emptyset$ (no possible numbers seen yet)

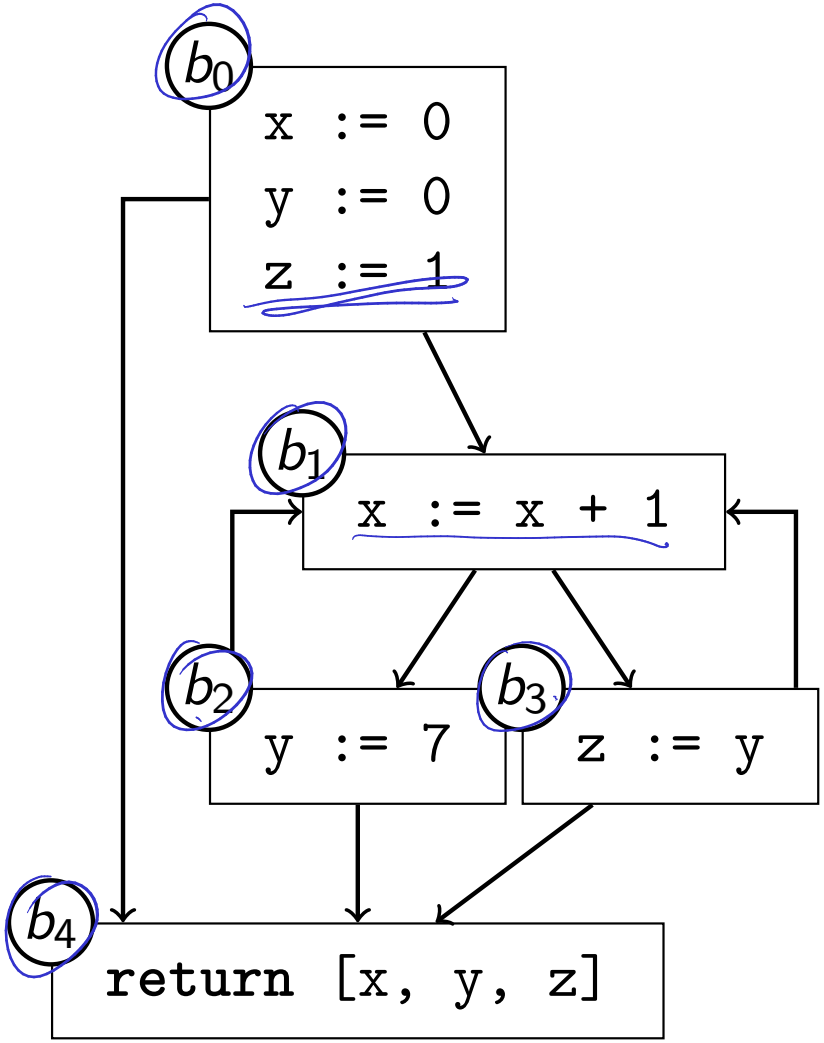
Example: Reaching Definitions

Designing our abstract domain:



- Capture sets of up to 3 possible numbers
- $\top$: More than 3 possible numbers
- $\bot$: $\emptyset$ (no possible numbers seen yet)
- Infinitely many elements, but finite height!

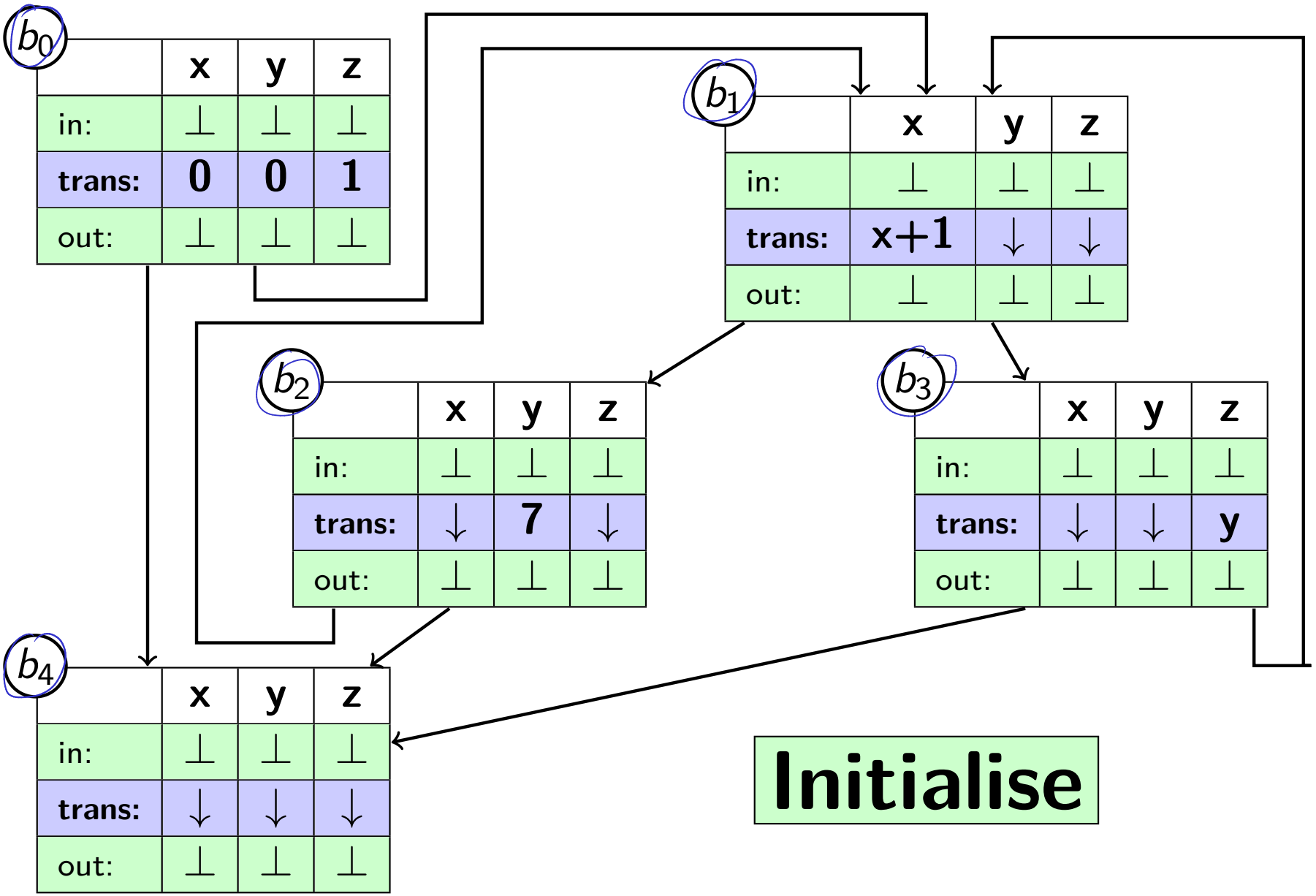
Example: Control-Flow Graph



b	inputs	$trans_b$		
		x	y	z
b_0	$\emptyset$	0	0	1
b_1	$\{b_0, b_2, b_3\}$	$x+1$	y	z
b_2	$\{b_1\}$	x	7	z
b_3	$\{b_1\}$	x	y	y
b_4	$\{b_0, b_2, b_3\}$	x	y	z

$$join_b = \text{let} \quad j = \bigcup_{s \in \text{inputs}_b} s \quad \text{in} \quad \begin{cases} j & \iff \#j \leq 3 \\ \top & \iff \#j > 3 \end{cases}$$

Example: Computing the Fixpoint



Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	$\bot$	$\bot$	$\bot$

b_2

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	$\downarrow$	**7**	$\downarrow$
out:	$\bot$	7	$\bot$

b_3

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	$\downarrow$	$\downarrow$	**y**
out:	$\bot$	$\bot$	$\bot$

b_4

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	$\bot$	$\bot$	$\bot$

Transfer (1st)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0	0,7	1
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	$\bot$	$\bot$	$\bot$

b_2

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	$\downarrow$	**7**	$\downarrow$
out:	$\bot$	7	$\bot$

b_3

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	$\downarrow$	$\downarrow$	**y**
out:	$\bot$	$\bot$	$\bot$

b_4

	x	y	z
in:	0	0,7	1
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	$\bot$	$\bot$	$\bot$

Join (1st)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0	0,7	1
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1	0,7	1

b_2

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	$\downarrow$	**7**	$\downarrow$
out:	$\perp$	7	$\perp$

b_3

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	$\downarrow$	$\downarrow$	**y**
out:	$\perp$	$\perp$	$\perp$

b_4

	x	y	z
in:	0	0,7	1
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0	0,7	1

Transfer (2nd)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0	0,7	1
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1	0,7	1

b_2

	x	y	z
in:	1	0,7	1
trans:	$\downarrow$	**7**	$\downarrow$
out:	$\bot$	7	$\bot$

b_3

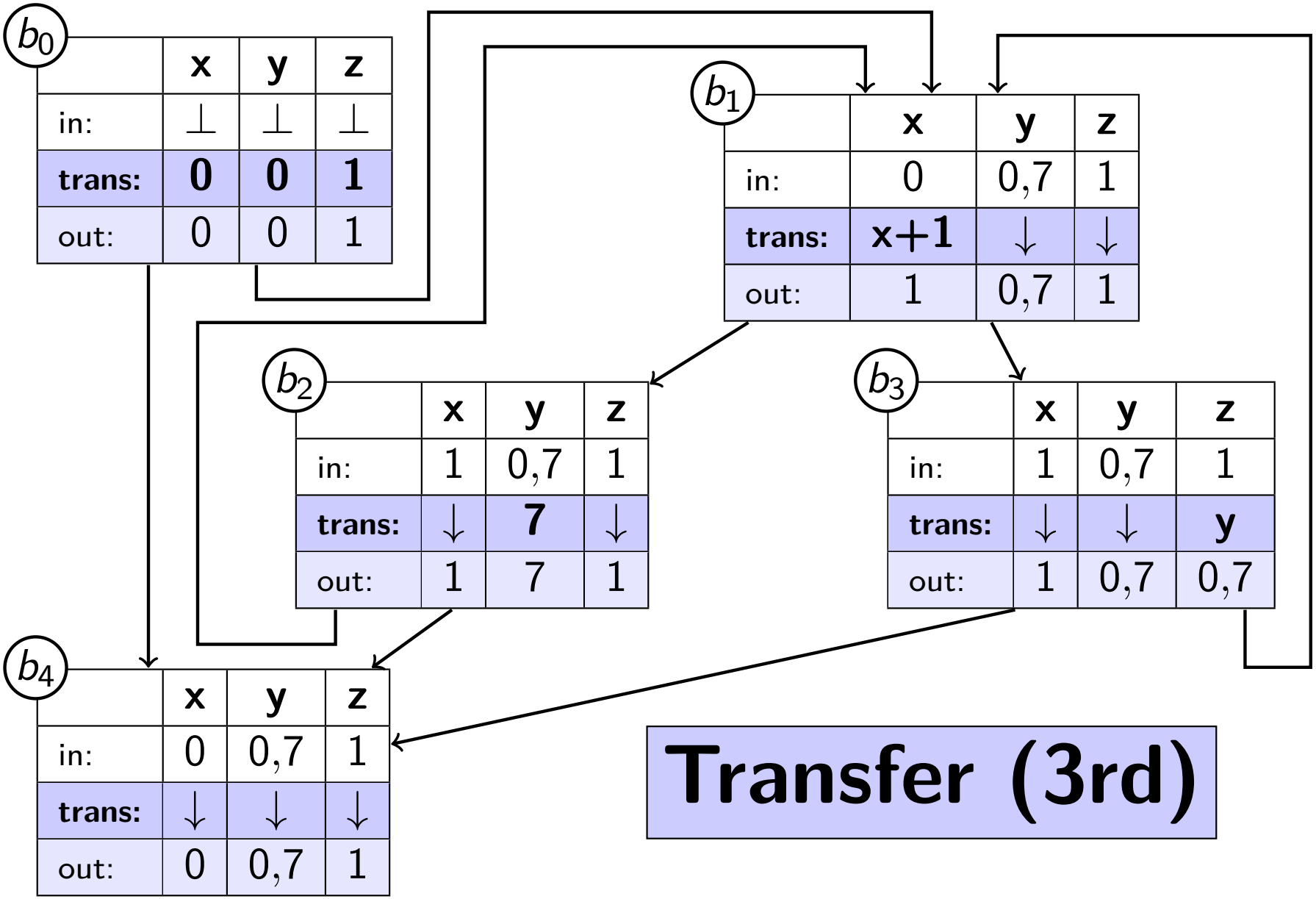
	x	y	z
in:	1	0,7	1
trans:	$\downarrow$	$\downarrow$	**y**
out:	$\bot$	$\bot$	$\bot$

b_4

	x	y	z
in:	0	0,7	1
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0	0,7	1

Join (2nd)

Example: Computing the Fixpoint



Transfer (3rd)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1	0,7	1

b_2

	x	y	z
in:	1	0,7	1
trans:	$\downarrow$	**7**	$\downarrow$
out:	1	7	1

b_3

	x	y	z
in:	1	0,7	1
trans:	$\downarrow$	$\downarrow$	**y**
out:	1	0,7	0,7

b_4

	x	y	z
in:	0,1	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0	0,7	1

Join (3rd)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2	0,7	0,1,7

b_2

	x	y	z
in:	1	0,7	1
trans:	$\downarrow$	**7**	$\downarrow$
out:	1	7	1

b_3

	x	y	z
in:	1	0,7	1
trans:	$\downarrow$	$\downarrow$	**y**
out:	1	0,7	0,7

b_4

	x	y	z
in:	0,1	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1	0,7	0,1,7

Transfer (4th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2	0,7	0,1,7

b_2

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1	7	1

b_3

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1	0,7	0,7

b_4

	x	y	z
in:	0,1	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1	0,7	0,1,7

Join (4th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2	0,7	0,1,7

b_2

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2	7	0,1,7

b_3

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2	0,7	0,7

b_4

	x	y	z
in:	0,1	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1	0,7	0,1,7

Transfer (5th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2	0,7	0,1,7

b_2

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2	7	0,1,7

b_3

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2	0,7	0,7

b_4

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1	0,7	0,1,7

Join (5th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2,3	0,7	0,1,7

b_2

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2	7	0,1,7

b_3

	x	y	z
in:	1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2	0,7	0,7

b_4

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1,2	0,7	0,1,7

Transfer (6th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2,3	0,7	0,1,7

b_2

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2	7	0,1,7

b_3

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2	0,7	0,7

b_4

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1,2	0,7	0,1,7

Join (6th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\perp$	$\perp$	$\perp$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2,3	0,7	0,1,7

b_2

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2,3	7	0,1,7

b_3

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2,3	0,7	0,7

b_4

	x	y	z
in:	0,1,2	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1,2	0,7	0,1,7

Transfer (7th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	1,2,3	0,7	0,1,7

b_2

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2,3	7	0,1,7

b_3

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2,3	0,7	0,7

b_4

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	0,1,2	0,7	0,1,7

Join (7th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	$\top$	0,7	0,1,7

b_2

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2,3	7	0,1,7

b_3

	x	y	z
in:	1,2,3	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2,3	0,7	0,7

b_4

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	$\top$	0,7	0,1,7

Transfer (8th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	$\top$	0,7	0,1,7

b_2

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	$\downarrow$	**7**	$\downarrow$
out:	1,2,3	7	0,1,7

b_3

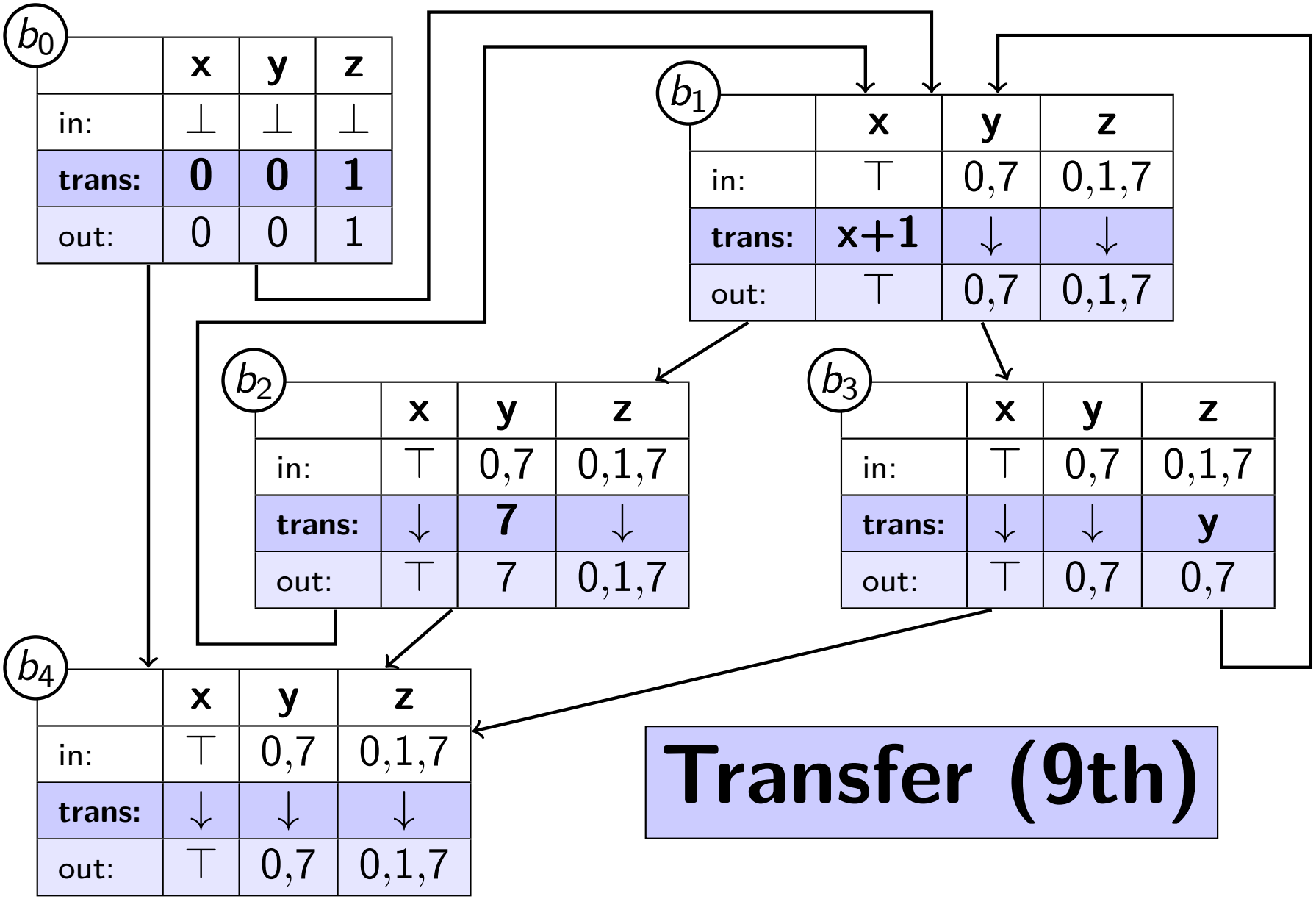
	x	y	z
in:	$\top$	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	**y**
out:	1,2,3	0,7	0,7

b_4

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	$\top$	0,7	0,1,7

Join (8th)

Example: Computing the Fixpoint



Transfer (9th)

Example: Computing the Fixpoint

b_0

	x	y	z
in:	$\bot$	$\bot$	$\bot$
trans:	**0**	**0**	**1**
out:	0	0	1

b_1

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	**x+1**	$\downarrow$	$\downarrow$
out:	$\top$	0,7	0,1,7

b_2

	x	y	z
out:	$\top$	7	0,1,7

b_3

	x	y	z
	$\top$	0,7	0,1,7
	$\downarrow$	$\downarrow$	**y**
out:	$\top$	0,7	0,7

Fixpoint reached

b_4

	x	y	z
in:	$\top$	0,7	0,1,7
trans:	$\downarrow$	$\downarrow$	$\downarrow$
out:	$\top$	0,7	0,1,7

Join (9th)

Example: Conclusion

```
var x := 0;
var y := 0;
var z := 1;

while x < 5 {
  x := x + 1;
  if x >= 2 {
    y := 7;
  } else {
    z := y;
} }

return [x, y, z];
```

- Applied abstract domain to three variables
- Reached fixpoint after 9 iterations
- Return values:

  x : $\top$ (unknown/any)
  y : 0 or 7
  z : 0 or 1 or 7
- Conservative approximation of reality
- Once x reached more than 3 values, algorithm gave up and went to $\top$
- *This is only one possible design for this analysis*