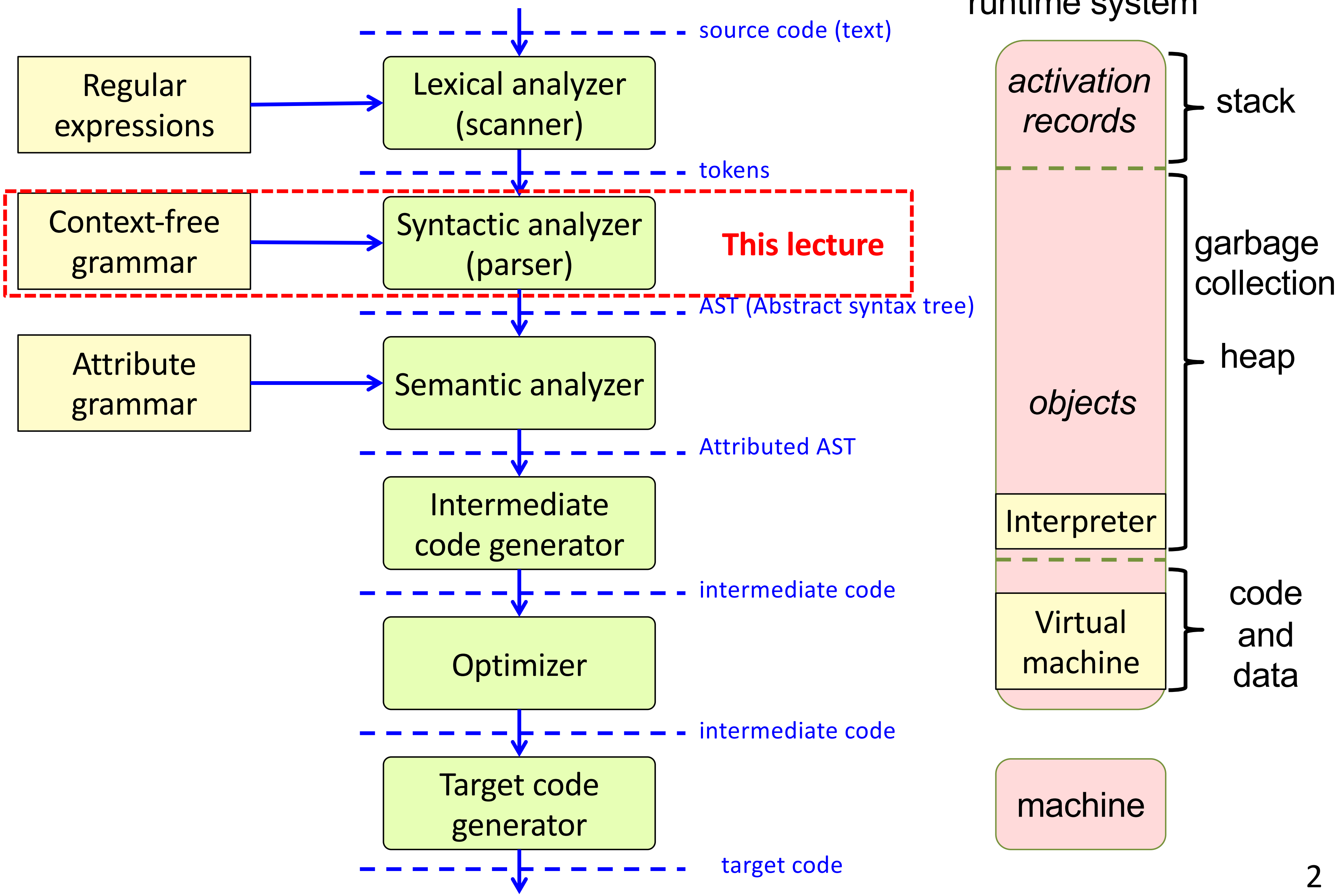
EDAN65: Compilers, Lecture 03

Context-free grammars, Introduction to parsing

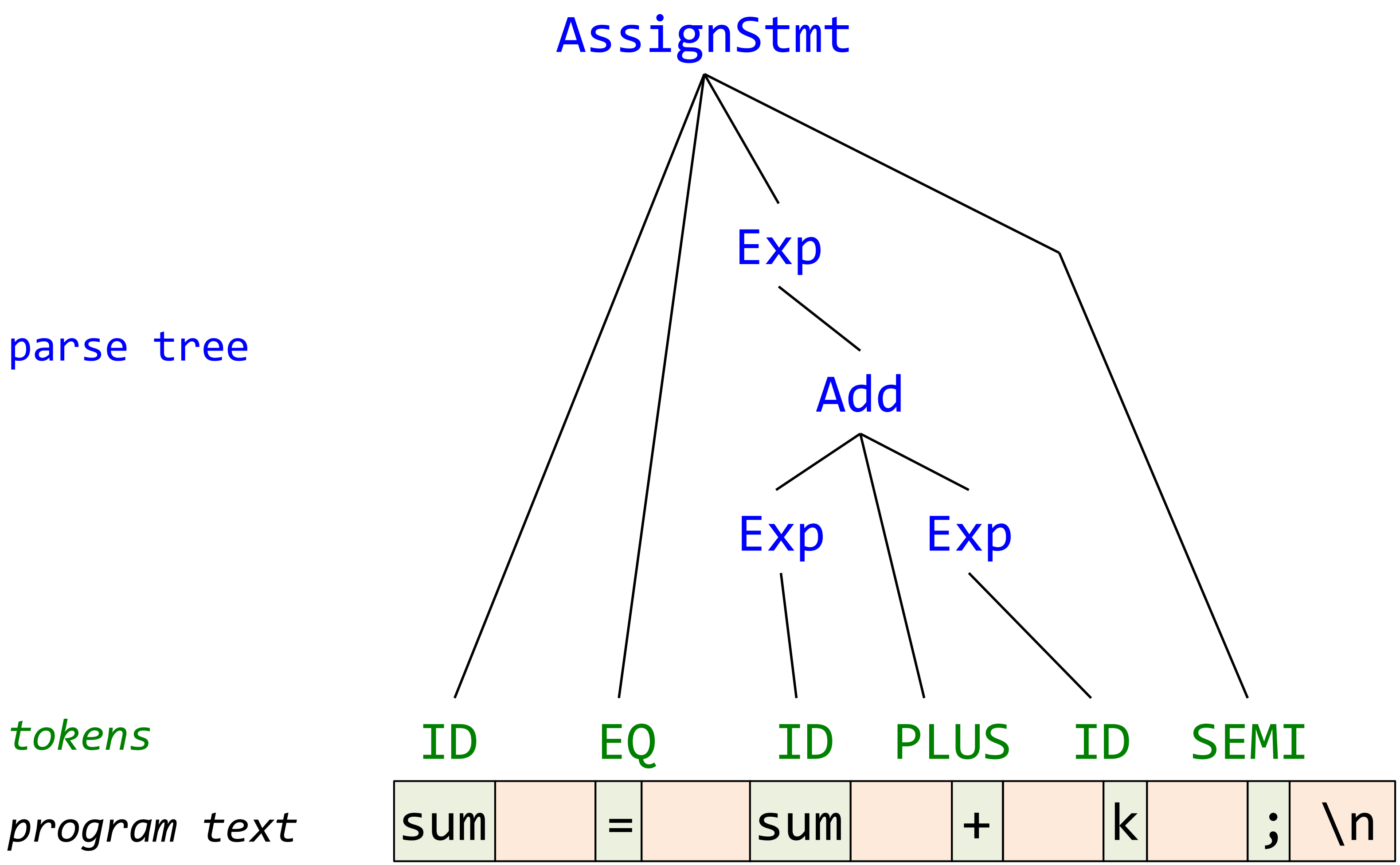
Görel Hedin

Revised: 2020-09-06

Course overview

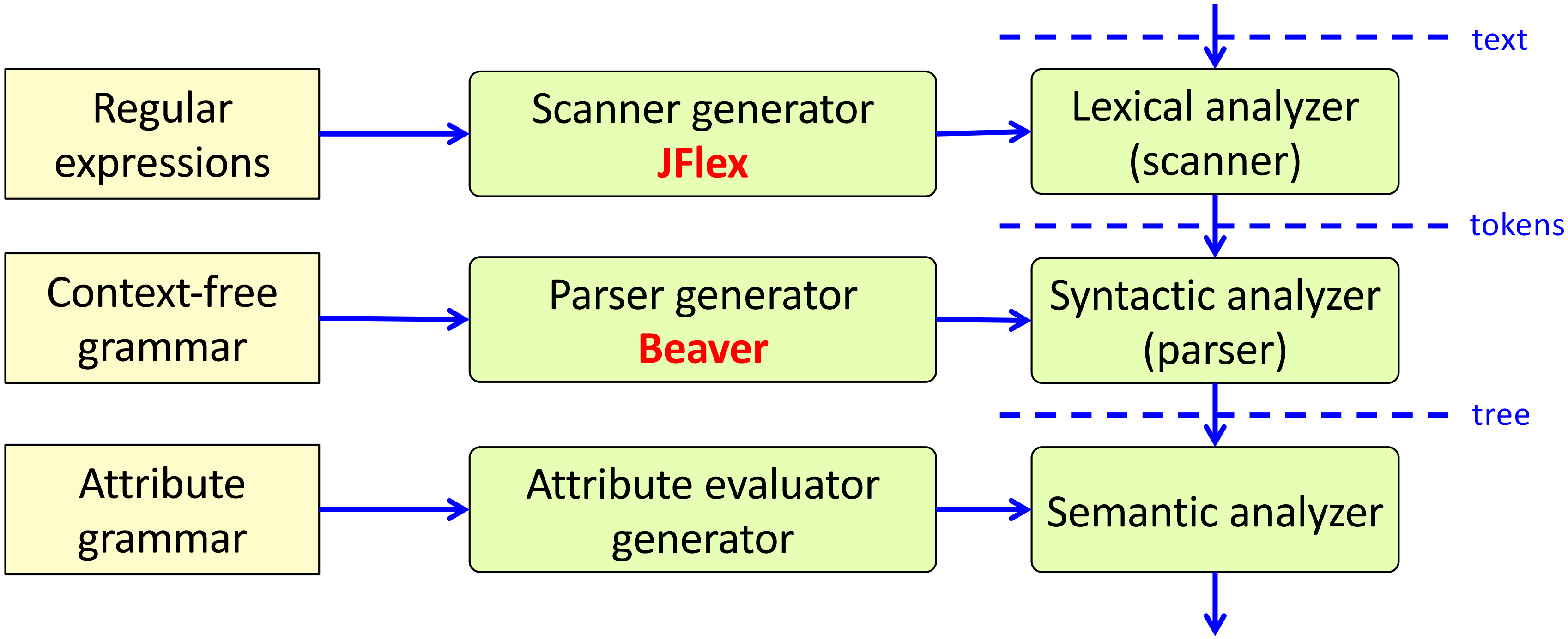


Analyzing program text



non-tokens (like white space) are discarded

Recall: Generating the compiler:



We will use a parser generator called **Beaver**

Context-Free Grammars

Regular Expressions vs Context-Free Grammars

Example REs:
```
WHILE = "while"
ID = [a-z][a-z0-9]*
LPAR = "("
RPAR = ")"
PLUS = "+"
...
```

Example CFG:
```
Stmt -> WhileStmt
Stmt -> AssignStmt
WhileStmt -> WHILE LPAR Exp RPAR Stmt
Exp -> ID
Exp -> Exp PLUS Exp
...
```

An RE can have *iteration*

A CFG can also have *recursion*

(it is possible to derive a symbol, e.g., `Stmt`, from itself)

Elements of a Context-Free Grammar

```
Example CFG:
Stmt -> WhileStmt
Stmt -> AssignStmt
WhileStmt -> WHILE LPAR Exp RPAR Stmt
AssignStmt -> ID EQ Exp SEMIC
…
```

Production rules:

$$X \text{ -> } s_1 \; s_2 \; \ldots \; s_n$$

where s_k is a *symbol* (terminal or nonterminal), n >= 0

Nonterminal symbols

Terminal symbols (tokens)

Start symbol

(one of the nonterminals, usually the left-hand side of the first production)

Shorthand for alternatives

```
Stmt -> WhileStmt | AssignStmt
```

is equivalent to

```
Stmt -> WhileStmt
Stmt -> AssignStmt
```

Shorthand for repetition

```
Stmt*
```

is equivalent to

```
StmtList
```

where

```
StmtList -> ε | Stmt StmtList
```

Exercise

Construct a grammar covering this program and similar ones:

Example program:

```
while (k <= n) {sum = sum + k; k = k+1;}
```

Solution

Construct a grammar covering this program and similar ones:

Example program:

```
while (k <= n) {sum = sum + k; k = k+1;}
```

CFG:

```
Stmt -> WhileStmt | AssignStmt | Block
WhileStmt -> "while" "(" Exp ")" Stmt
AssignStmt -> ID "=" Exp ";"
Block -> "{" Stmt* "}"
Exp -> LessEq | Add | ID | INT
LessEq -> Exp "<=" Exp
Add -> Exp "+" Exp
```

(Often, simple tokens are written directly as text strings)

Parsing

Use the grammar to derive a tree for a program (top-down):

```
Stmt -> WhileStmt | AssignStmt | Block
WhileStmt -> "while" "(" Exp ")" Stmt
AssignStmt -> ID "=" Exp ";"
Block -> "{" Stmt* "}"
Exp -> LessEq | Add | ID | INT
LessEq -> Exp "<=" Exp
Add -> Exp "+" Exp
```

Start symbol ⟶ `Stmt`

```
sum = sum + k ;
```

Parsing

Use the grammar to derive a tree for a program (bottom-up):

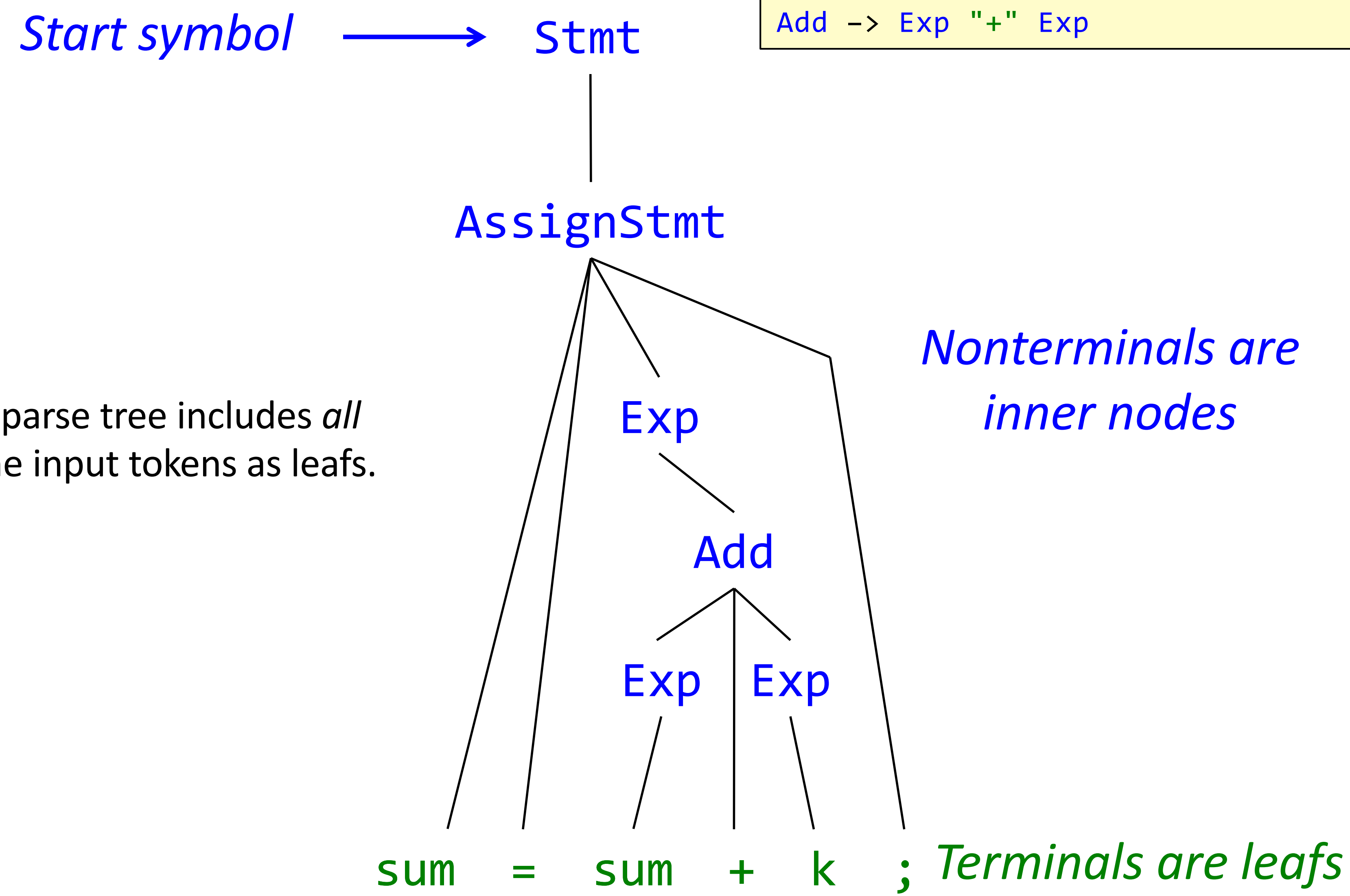
```
Stmt -> WhileStmt | AssignStmt | Block
WhileStmt -> "while" "(" Exp ")" Stmt
AssignStmt -> ID "=" Exp ";"
Block -> "{" Stmt* "}"
Exp -> LessEq | Add | ID | INT
LessEq -> Exp "<=" Exp
Add -> Exp "+" Exp
```

```
sum = sum + k ;
```

Parsing

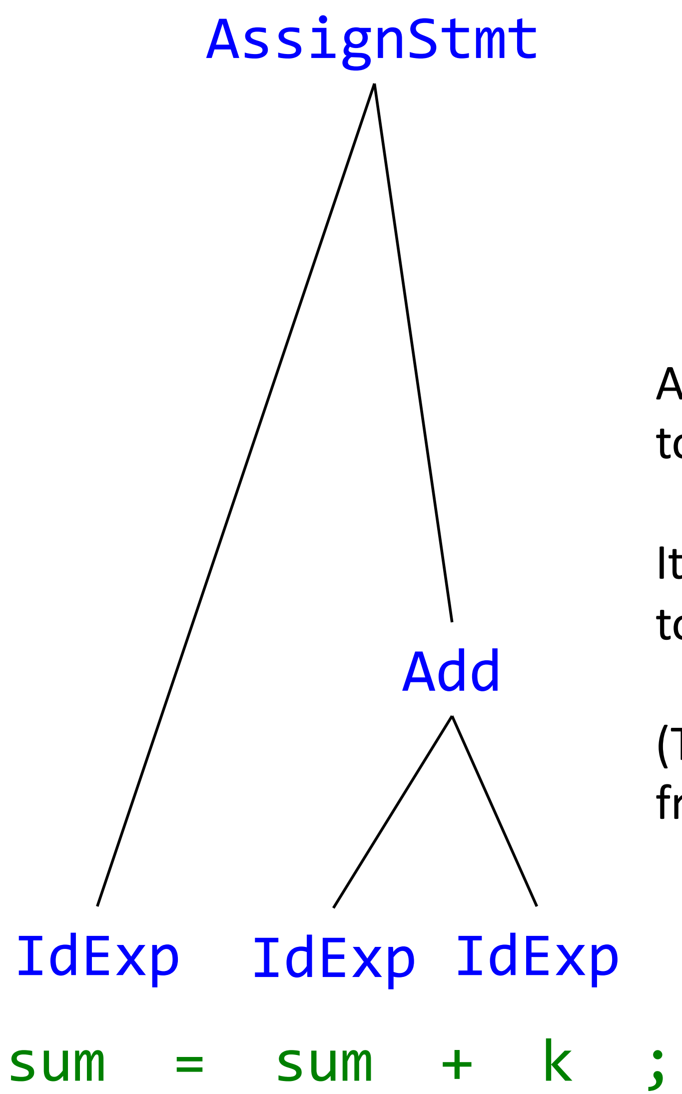
Use the grammar to derive a tree for a program:

```
Stmt -> WhileStmt | AssignStmt | Block
WhileStmt -> "while" "(" Exp ")" Stmt
AssignStmt -> ID "=" Exp ";"
Block -> "{" Stmt* "}"
Exp -> LessEq | Add | ID | INT
LessEq -> Exp "<=" Exp
Add -> Exp "+" Exp
```



A parse tree includes *all* the input tokens as leafs.

Corresponding abstract syntax tree
(will be discussed in later lecture)



An abstract syntax tree is similar to a parse tree, but simpler.

It includes only some of the tokens.

(Tokens that could be generated from the tree are excluded.)

EBNF vs Canonical Form

EBNF:

```
Stmt -> AssignStmt | Block
AssignStmt -> ID "=" Exp ";"
Block -> "{" Stmt* "}"
Exp -> Add | ID
Add -> Exp "+" Exp
```

(Extended) Backus-Naur Form:

- Compact, easy to read and write
- EBNF has alternatives, repetition, optionals, parentheses (like REs)
- Common notation for practical use

Canonical form:

```
Stmt -> ID "=" Exp ";"
Stmt -> "{" Stmts "}"
Stmts -> ε
Stmts -> Stmt Stmts
Exp -> Exp "+" Exp
Exp -> ID
```

Canonical form:

- Core formalism for CFGs
- Useful for proving properties and explaining algorithms

Real world example: The Java Language Specification

```
OrdinaryCompilationUnit:
  [PackageDeclaration] {ImportDeclaration} {TypeDeclaration}

PackageDeclaration:
  {PackageModifier} package Identifier {. Identifier} ;

PackageModifier:
  Annotation

…
```

See `http://docs.oracle.com/javase/specs/jls/se11/html`

- See Chapter 2 about the Java grammar notation.
- See Chapter 19 for the full syntax

Formal definition of CFGs

Formal definition of CFGs (canonical form)

A context-free grammar $G = (N, T, P, S)$, where

N – the set of nonterminal symbols

T – the set of terminal symbols

P – the set of production rules, each with the form

$$X \rightarrow Y_1\ Y_2\ \ldots\ Y_n$$

where $X \in N$, $n \geq 0$, and $Y_k \in N \cup T$

S – the start symbol (one of the nonterminals). I.e., $S \in N$

Formal definition of CFGs (canonical form)

A context-free grammar $G = (N, T, P, S)$, where

N – the set of nonterminal symbols

T – the set of terminal symbols

P – the set of production rules, each with the form

$$X \rightarrow Y_1\ Y_2\ \ldots\ Y_n$$

where $X \in N$, $n \geq 0$, and $Y_k \in N \cup T$

S – the start symbol (one of the nonterminals). I.e., $S \in N$

So, the *left-hand side* X of a rule is a nonterminal.

And the *right-hand side* $Y_1\ Y_2\ \ldots\ Y_n$ is a sequence of nonterminals and terminals.

If the rhs for a production is empty, i.e., $n = 0$, we write

$$X \rightarrow \varepsilon$$

A grammar G defines a language $L(G)$

~~A~~ context-free grammar $G = (N, T, P, S)$, where

N – the set of nonterminal symbols

T – the set of terminal symbols

P – the set of production rules, each with the form

$$X \rightarrow Y_1\ Y_2\ \ldots\ Y_n$$

where $X \in N$, $n \geq 0$, and $Y_k \in N \cup T$

S – the start symbol (one of the nonterminals). I.e., $S \in N$

A grammar G defines a language $L(G)$

A context-free grammar $G = (N, T, P, S)$, where

N – the set of nonterminal symbols

T – the set of terminal symbols

P – the set of production rules, each with the form

$$X \to Y_1\ Y_2\ \dots\ Y_n$$

where $X \in N$, $n \geq 0$, and $Y_k \in N \cup T$

S – the start symbol (one of the nonterminals). I.e., $S \in N$

G defines a *language* $L(G)$ over the alphabet T

T^* is the set of all possible sequences of T symbols.

$L(G)$ is the subset of T^* that can be derived from the start symbol S, by following the production rules P.

Exercise

```
G = (N, T, P, S)

P = {
  Stmt -> ID "=" Exp ";",
  Stmt -> "{" Stmts "}" ,
  Stmts -> ε ,
  Stmts -> Stmt Stmts ,
  Exp -> Exp "+" Exp ,
  Exp -> ID
}

N =

T =

S =
```

```
L(G) =
```

Solution

```
G = (N, T, P, S)

P = {
  Stmt -> ID "=" Exp ";",
  Stmt -> "{" Stmts "}" ,
  Stmts -> ε ,
  Stmts -> Stmt Stmts ,
  Exp -> Exp "+" Exp ,
  Exp -> ID
}

N = {Stmt, Exp, Stmts}

T = {ID, "=", "{", "}", ";", "+"}

S = Stmt
```

```
L(G) = {
 "{" "}",
 "{" "{" "}" "}",
 ID "=" ID ";",
 "{" ID "=" ID ";" "}",
 ID "=" ID "+" ID ";",
 "{" "{" "}" "{" "}" "}",
 "{" "{" "{" "}" "}" "}",
 "{" ID "=" ID "+" ID ";" "}",
 ID "=" ID "+" ID "+" ID ";",
 ...
}
```

The sequences in L(G) are usually called *sentences* or *strings*

Derivations

Derivation step

If we have a sequence of terminals and nonterminals, e.g.,

X a Y Y b

we can replace one of the nonterminals, applying a production rule. This is called a *derivation step*.
(Swedish: *Härledningssteg*)

Derivation step

If we have a sequence of terminals and nonterminals, e.g.,

X a Y Y b

we can replace one of the nonterminals, applying a production rule. This is called a *derivation step*.
(Swedish: *Härledningssteg*)

Suppose there is a production

Y –> X a

and we apply it for the first Y in the sequence. We write the derivation step as follows:

X a Y Y b => X a X a Y b

Derivation

A *derivation*, is simply a sequence of derivation steps, e.g.:

$\gamma_0 \Rightarrow \gamma_1 \Rightarrow \ldots \Rightarrow \gamma_n \quad (n \geq 0)$

where each γ_i is a sequence of terminals and nonterminals

If there is a derivation from γ_0 to γ_n, we can write this as

$\gamma_0 \Rightarrow^* \gamma_n$

So this means it is possible to get from the sequence γ_0 to the sequence γ_n by applying production rules.

Definition of the language L(G)

Recall that:

G = (N, T, P, S)

T* is the set of all possible sequences of T symbols.

L(G) is the subset of T* that can be derived from the start symbol S, by applying production rules in P.

Definition of the language L(G)

Recall that:

$G = (N, T, P, S)$

T^* is the set of all possible sequences of T symbols.

$L(G)$ is the subset of T^* that can be derived from the start symbol S, by applying production rules in P.

Using the concept of derivations, we can formally define $L(G)$ as follows:

$$L(G) = \{ w \in T^* \mid S \Rightarrow^* w \}$$

Exercise:

Prove that a sentence belongs to a language

Prove that

INT + INT * INT

belongs to the language of the following grammar:

p_1: Exp –> Exp "+" Exp
p_2: Exp –> Exp "*" Exp
p_3: Exp –> INT

Proof:

Solution:

Prove that a sentence belongs to a language

Prove that

INT + INT * INT

belongs to the language of the following grammar:

p_1: Exp –> Exp "+" Exp
p_2: Exp –> Exp "*" Exp
p_3: Exp –> INT

Proof:
(by showing all the derivation steps from the start symbol Exp)

Exp
$=>^{p1}$ Exp "+" Exp
$=>^{p3}$ INT "+" Exp
$=>^{p2}$ INT "+" Exp "*" Exp
$=>^{p3}$ INT "+" INT "*" Exp
$=>^{p3}$ INT "+" INT "*" INT

Leftmost and rightmost derivations

p_1: Exp –> Exp "+" Exp
p_2: Exp –> Exp "*" Exp
p_3: Exp –> INT

In a *leftmost* derivation, the leftmost nonterminal is replaced in each derivation step, e.g.,:

Exp =>
Exp "+" Exp =>
INT "+" Exp =>
INT "+" Exp "*" Exp =>
INT "+" INT "*" Exp =>
INT "+" INT "*" INT

Leftmost and rightmost derivations

p_1: Exp –> Exp "+" Exp
p_2: Exp –> Exp "*" Exp
p_3: Exp –> INT

In a *leftmost* derivation, the leftmost nonterminal is replaced in each derivation step, e.g.,:

Exp =>
Exp "+" Exp =>
INT "+" Exp =>
INT "+" Exp "*" Exp =>
INT "+" INT "*" Exp =>
INT "+" INT "*" INT

In a *rightmost* derivation, the rightmost nonterminal is replaced in each derivation step, e.g.,:

Exp =>
Exp "+" Exp =>
Exp "+" Exp "*" Exp =>
Exp "+" Exp "*" INT =>
Exp "+" INT "*" INT =>
INT "+" INT "*" INT

LL parsing algorithms use leftmost derivation.
LR parsing algorithms use rightmost derivation.
Will be discussed in later lectures.

A derivation corresponds to building a parse tree

Grammar:
Exp –> Exp "+" Exp
Exp –> Exp "*" Exp
Exp –> INT

Example derivation:

Exp =>
Exp "+" Exp =>
INT "+" Exp =>
INT "+" Exp "*" Exp =>
INT "+" INT "*" Exp =>
INT "+" INT "*" INT

Exercise: draw the parse tree (also called derivation tree).

A derivation corresponds to building a parse tree

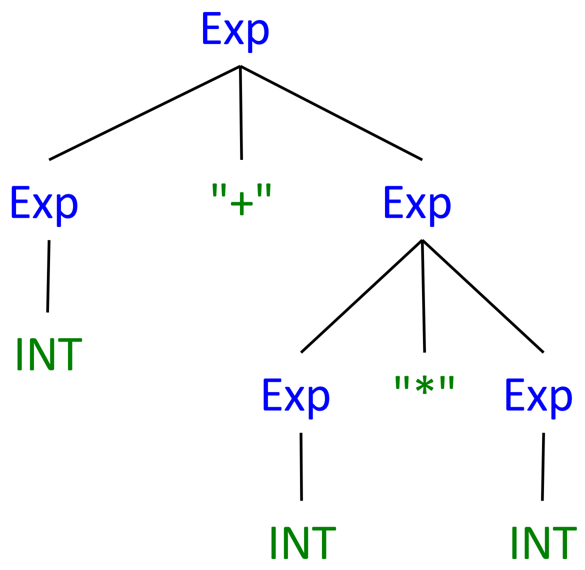
Grammar:

Exp –> Exp "+" Exp
Exp –> Exp "*" Exp
Exp –> INT

Example derivation:

Exp =>
Exp "+" Exp =>
INT "+" Exp =>
INT "+" Exp "*" Exp =>
INT "+" INT "*" Exp =>
INT "+" INT "*" INT

Parse tree (derivation tree):



Ambiguities

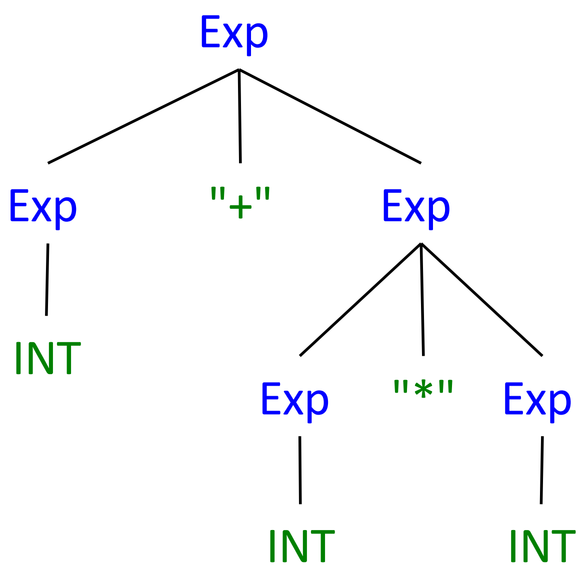
Exercise:

Can we do another derivation of the same sentence, that gives a different parse tree?

Exp –> Exp "+" Exp
Exp –> Exp "*" Exp
Exp –> INT

One derivation and parse tree

Exp =>
Exp "+" Exp =>
INT "+" Exp =>
INT "+" Exp "*" Exp =>
INT "+" INT "*" Exp =>
INT "+" INT "*" INT



Other derivation that gives *different* parse tree

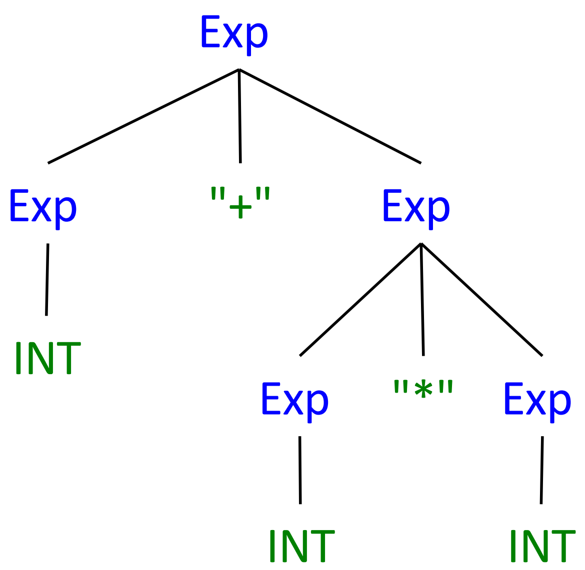
Solution:

Can we do another derivation of the same sentence, that gives a different parse tree?

```
Exp –> Exp "+" Exp
Exp –> Exp "*" Exp
Exp –> INT
```

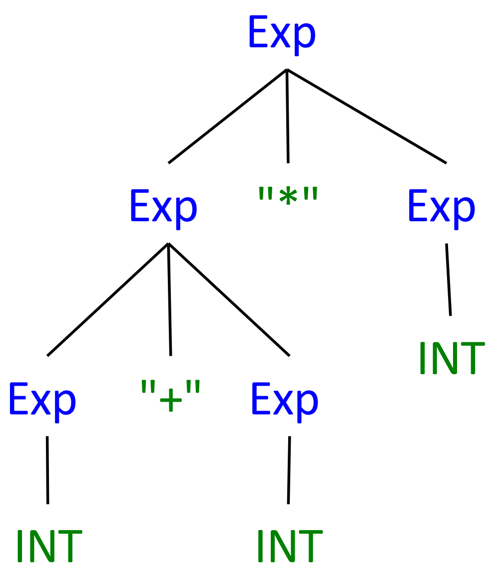
One derivation and parse tree

```
Exp =>
Exp "+" Exp =>
INT "+" Exp =>
INT "+" Exp "*" Exp =>
INT "+" INT "*" Exp =>
INT "+" INT "*" INT
```



Other derivation that gives *different* parse tree

```
Exp =>
Exp "*" Exp =>
Exp "+" Exp "*" Exp =>
INT "+" Exp "*" Exp =>
INT "+" INT "*" Exp =>
INT "+" INT "*" INT
```



Which parse tree would we prefer?

Ambiguous context-free grammars

A CFG is *ambiguous* if a sentence in the language can be derived by two (or more) *different* parse trees.

A CFG is *unambiguous* if each sentence in the language can be derived by only *one* parse tree.

(Swedish: *tvetydig, otvetydig*)

Note! There can be many different derivations that give the same parse tree.

How can we know if a CFG is ambiguous?

How can we know if a CFG is ambiguous?

If we find an example of an ambiguity, we know the grammar is ambiguous.

There are algorithms for deciding if a CFG belongs to certain subsets of CFGs, e.g. LL, LR, etc. (See later lectures.) These grammars are unambiguous.

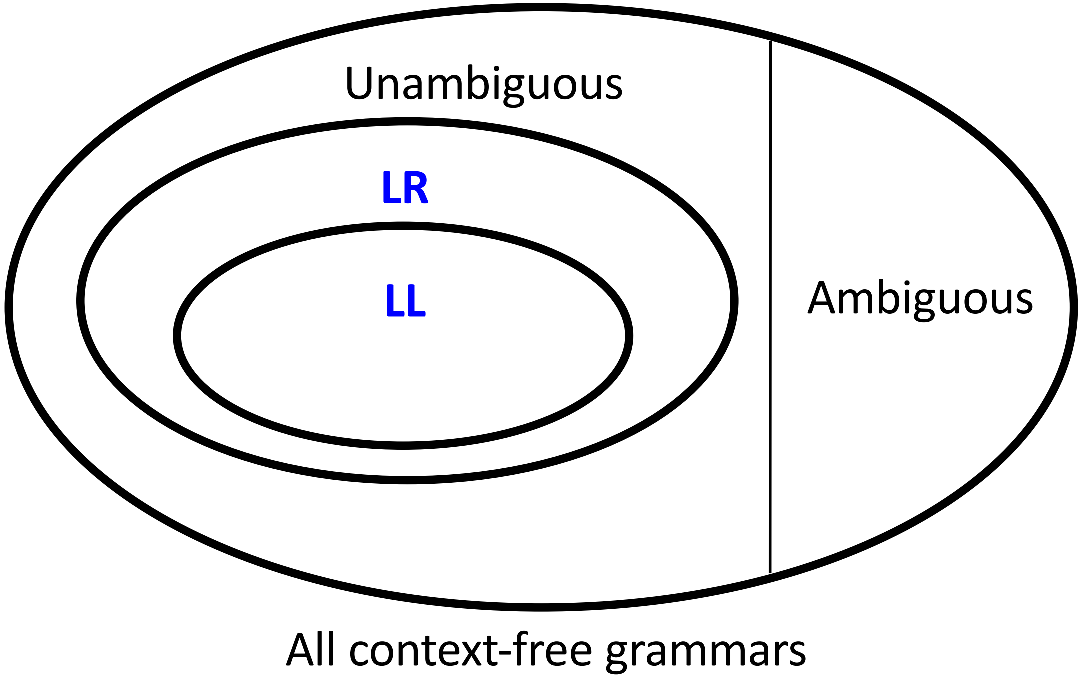
But in the general case, the problem is *undecidable*: it is not possible to construct a general algorithm that decides ambiguity for an arbitrary CFG.

Strategies for eliminating ambiguities, next lecture.

Parsing

Different parsing algorithms

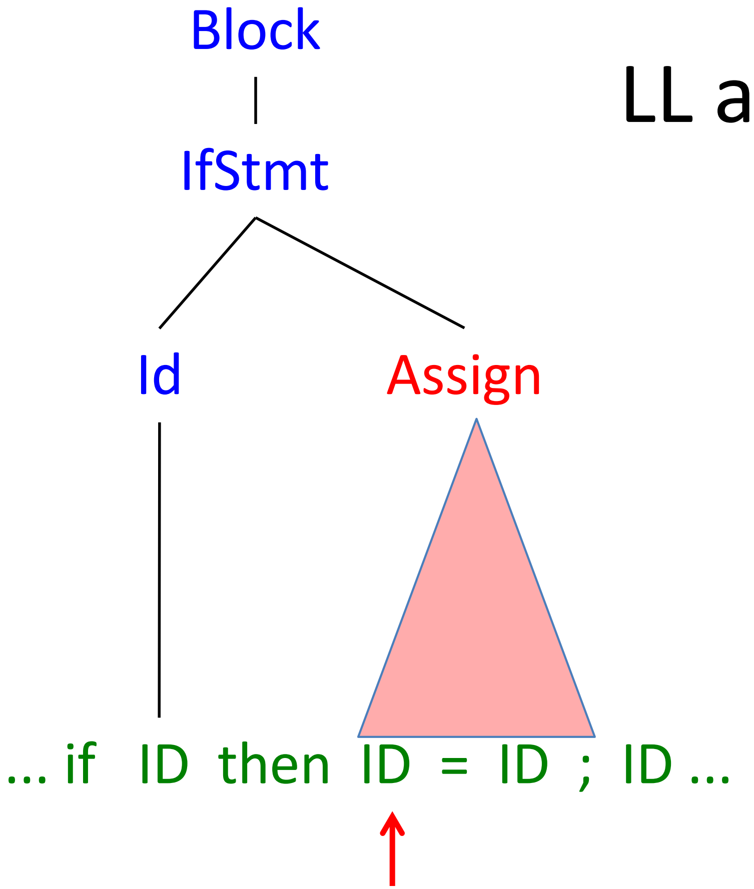
Different parsing algorithms



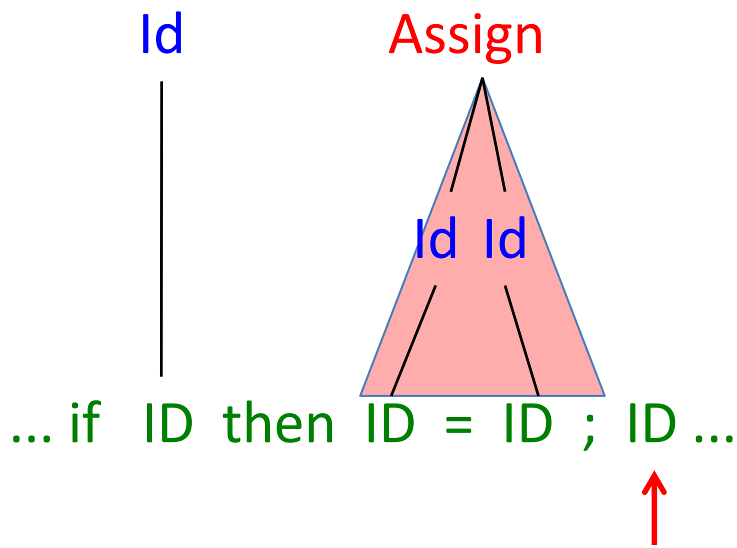
LL:
Left-to-right scan
Leftmost derivation
Builds tree top-down
Simple to understand

LR:
Left-to-right scan
Rightmost derivation
Builds tree bottom-up
More powerful

LL and LR parsers: main idea

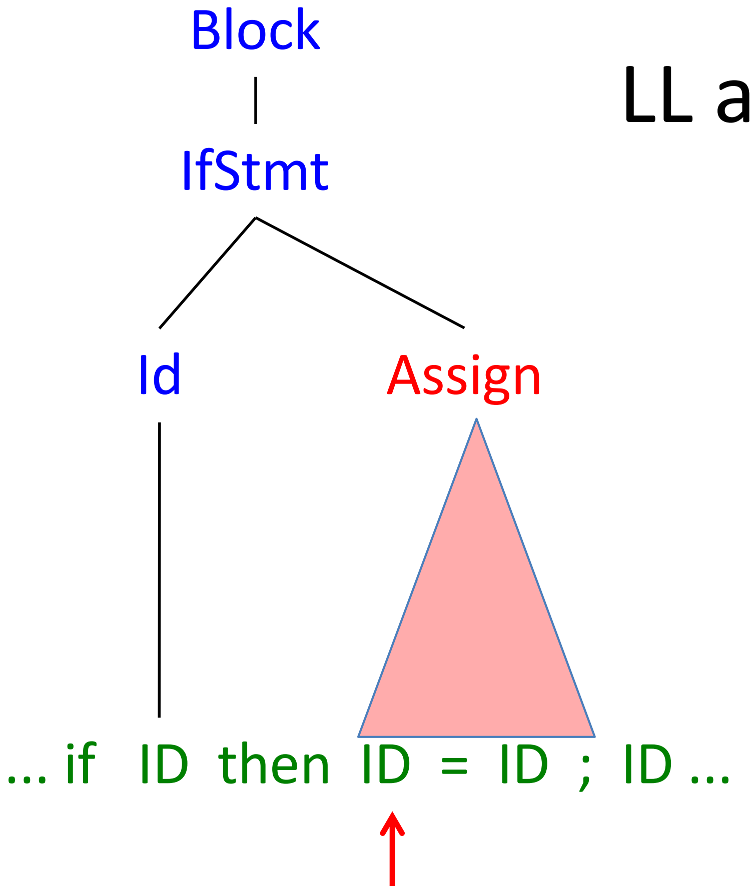


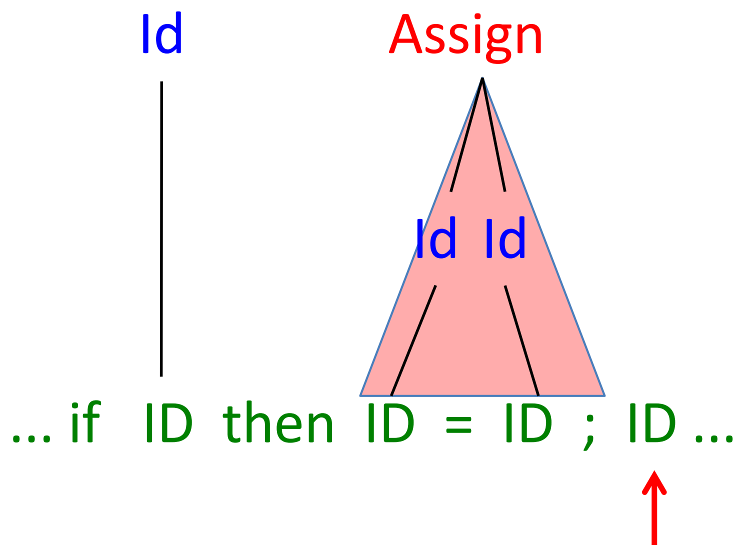
LL(1)



LR(1)

LL and LR parsers: main idea





LL(1): decides to build Assign after seeing the first token of its subtree. The tree is built top down.

LR(1): decides to build Assign after seeing the first token following its subtree. The tree is built bottom up.

The token is called lookahead.
LL(*k*) and LR(*k*) use *k* lookahead tokens.

In practice, k=1 is usually used

Recursive-descent parsing

A way of programming an LL(1) parser by recursive method calls

```
A –> B | C | D
B –> e C f D
C –> ...
D –> ...
```

Recursive-descent parsing

A way of programming an LL(1) parser by recursive method calls

A –> B | C | D
B –> e C f D
C –> ...
D –> ...

Assume a BNF grammar with exactly *one* production rule for each nonterminal. (Can easily be generalized to EBNF.)

Each production rule RHS is either

1. a sequence of token/nonterminal symbols, or
2. a set of nonterminal symbol alternatives

For each nonterminal, a method is constructed. The method

1. matches tokens and calls nonterminal methods, or
2. calls one of the nonterminal methods – which one depends on the lookahead token.

If the lookahead token does not match, a parsing error is reported.

Example Java implementation: overview

```
statement –> assignment | block
assignment –> ID ASSIGN expr SEMICOLON
block –> LBRACE statement* RBRACE
...
```

```
class Parser {
  private int token;                  // current lookahead token
  void accept(int t) {...}            // accept t and read in next token
  void error(String str) {...}        // generate error message
  void statement() {...}
  void assignment() {...}
  void block() {...}
  ...

}
```

Example: recursive descent methods

```
statement –> assignment | block
assignment–> ID ASSIGN expr SEMICOLON
block –> LBRACE statement* RBRACE
```

```
class Parser {
 void statement() {
  switch(token) {
   case ID: assignment(); break;
   case LBRACE: block(); break;
   default: error("Expecting statement, found: " + token);
  }
 }
 void assignment() {
  accept(ID); accept(ASSIGN); expr(); accept(SEMICOLON);
 }
 void block() {
  accept(LBRACE);
  while (token!=RBRACE) { statement(); }
  accept(RBRACE);
 }
 ...
}
```

Example: Parser skeleton details

```
statement –> assignment | block
assignment–> ID ASSIGN expr SEMICOLON
block –> LBRACE statement* RBRACE
expr –> ...
```

```
class Parser {
  final static int ID=1, WHILE=2, DO=3, ASSIGN=4, ...;
  private int token;                  // current lookahead token
  void accept(int t) {                // accept t and read in next token
    if (token==t) {
      token = nextToken();
    } else {
      error("Expected " + t + " , but found " + token);
    }
  }
  void error(String str) {...}        // generate error message
  private int nextToken() {...}       // read next token from scanner
  void statement() ...
  ...

}
```

Is this grammar LL(1)?

expr –> name params | name

What would happen in a recursive-descent parser?

Could the grammar be LL(2)? LL(k)?

Is this grammar LL(1)?

expr –> name params | name

This is called *common prefix*

What would happen in a recursive-descent parser?
Answer: The expr method would not know which alternative to call

Could the grammar be LL(2)? LL(k)?
Answer: This depends on the definition of *name*

Is this grammar LL(1)?

```
expr –> expr "+" term
```

What would happen in a recursive-descent parser?

Could the grammar be LL(2)? LL(k)?

Is this grammar LL(1)?

```
expr –> expr "+" term
```

This is called *left recursion*

What would happen in a recursive-descent parser?
Answer: The expr method would call expr recursively without reading any token, resulting in an endless recursion.

Could the grammar be LL(2)? LL(k)?
Answer: No.

Dealing with common prefix of limited length:

Local lookahead

LL(2) grammar:

statement –> assignment | block | callStmt
assignment–> ID ASSIGN expr SEMICOLON
block –> LBRACE statement* RBRACE
callStmt –> ID LPAR expr RPAR SEMICOLON

```
void statement() ...
```

Dealing with common prefix of limited length:

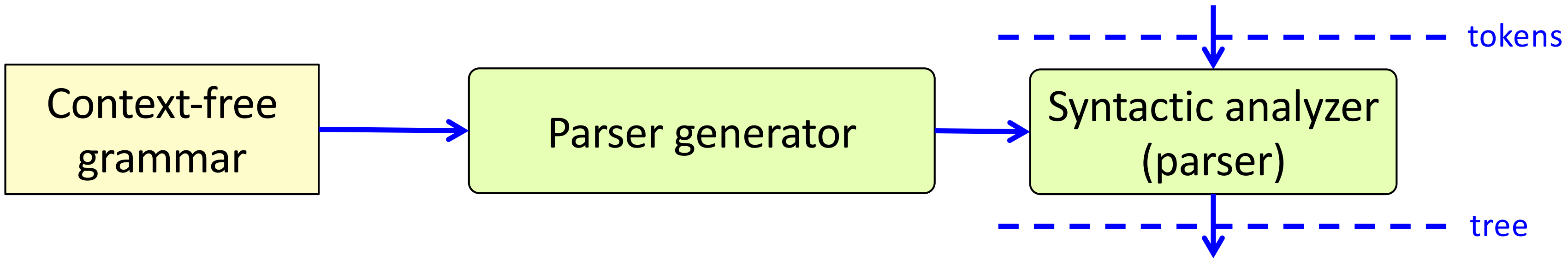
Local lookahead

LL(2) grammar:

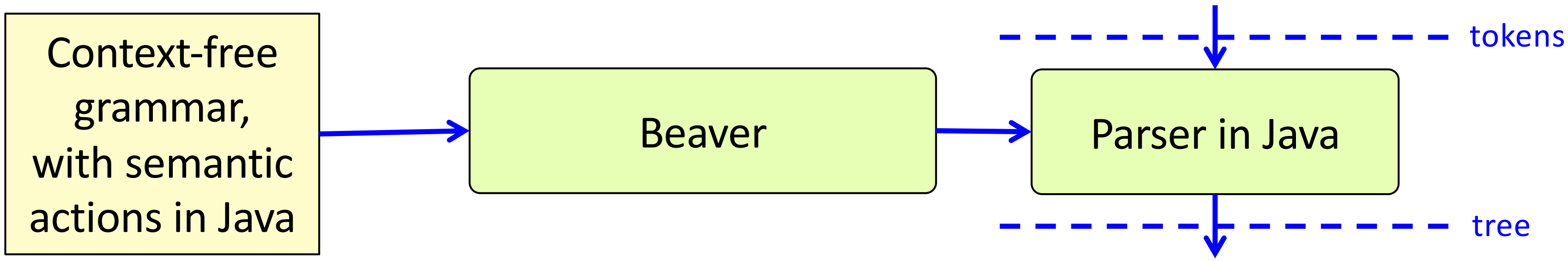
statement –> assignment | block | callStmt
assignment–> ID ASSIGN expr SEMICOLON
block –> LBRACE statement* RBRACE
callStmt –> ID LPAR expr RPAR SEMICOLON

```
void statement() {
 switch(token) {
  case ID:
   if (lookahead(2) == ASSIGN) {
    assignment();
   } else {
    callStmt();
   }
   break;
  case LBRACE: block(); break;
  default: error("Expecting statement, found: " + token);
 }
}
```

Generating the parser:



Beaver: an LR-based parser generator



Example beaver specification

```
%class "LangParser";
%package "lang";
...
%terminals LET, IN, END, ASSIGN, MUL, ID, NUMERAL;

%goal program; // The start symbol

// Context-free grammar
program = exp;
exp = factor | exp MUL factor;
factor = let | numeral | id;
let = LET id ASSIGN exp IN exp END;
numeral = NUMERAL;
id = ID;
```

Later on, we will extend this specification with semantic actions to build the syntax tree.

Regular Expressions vs Context-Free Grammars

	RE	**CFG**
Typical Alphabet	characters	terminal symbols (tokens)
Language is a set of ...	strings (char sequences)	sentences (token sequences)
Used for...	tokens	parse trees
Power	iteration	recursion
Recognizer	DFA	DFA with stack

The Chomsky hierarchy of formal grammars

Grammar	Rule patterns	Type
regular	$X \rightarrow aY$ or $X \rightarrow a$ or $X \rightarrow \varepsilon$	3
context free	$X \rightarrow \gamma$	2
context sensitive	$\alpha X \beta \rightarrow \alpha \gamma \beta$	1
arbitrary	$\gamma \rightarrow \delta$	0

a – terminal symbol
$\alpha, \beta, \gamma, \delta$ – *sequences* of (terminal or nonterminal) symbols

Type(3) $\subset$ Type (2) $\subset$ Type(1) $\subset$ Type(0)

The Chomsky hierarchy of formal grammars

Grammar	**Rule patterns**	**Type**
regular	$X \rightarrow aY$ or $X \rightarrow a$ or $X \rightarrow \varepsilon$	3
context free	$X \rightarrow \gamma$	2
context sensitive	$\alpha X \beta \rightarrow \alpha \gamma \beta$	1
arbitrary	$\gamma \rightarrow \delta$	0

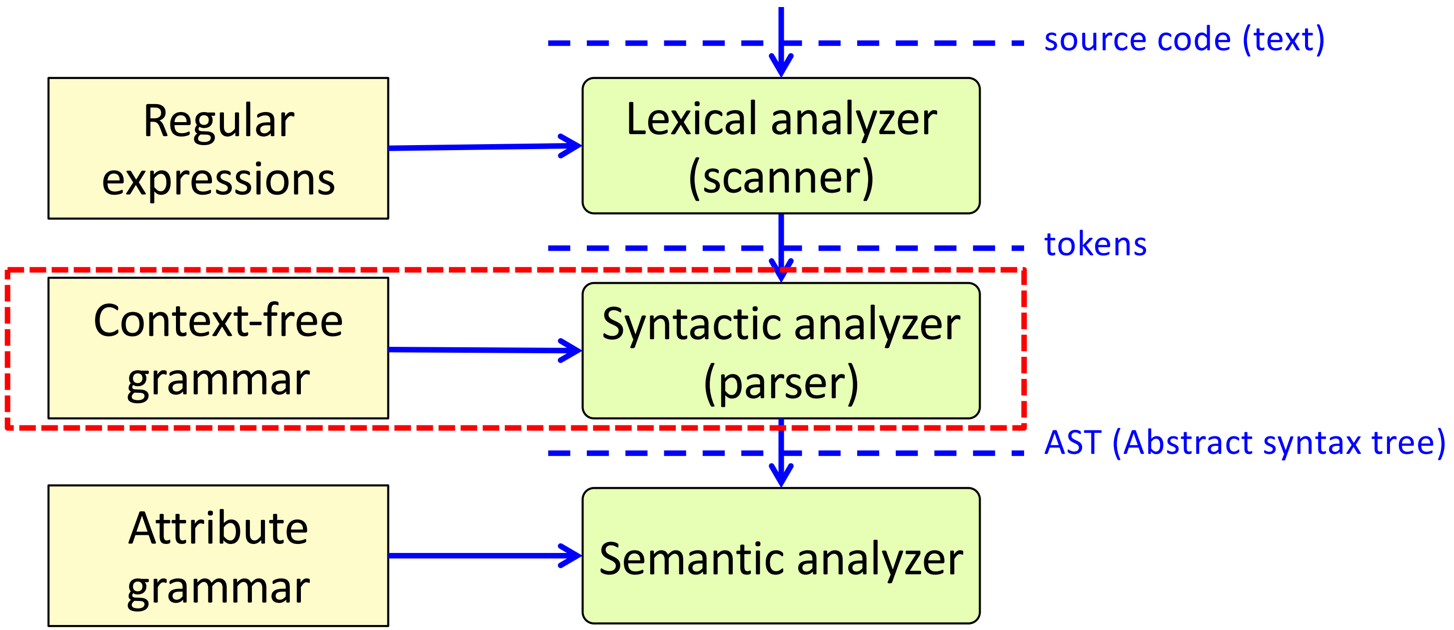
a – terminal symbol
$\alpha, \beta, \gamma, \delta$ – *sequences* of (terminal or nonterminal) symbols

Type(3) $\subset$ Type (2) $\subset$ Type(1) $\subset$ Type(0)

Regular grammars have the same power as regular expressions
(tail recursion = iteration).

Type 2 and 3 are of practical use in compiler construction.
Type 0 and 1 are only of theoretical interest.

Course overview



What we have covered:

- **Context-free grammars, derivations, parse trees**
- **Ambiguous grammars**
- **Introduction to parsing, recursive-descent**

You can now finish assignment 1

Summary questions

- Construct a CFG for a simple part of a programming language.
- What is a nonterminal symbol? A terminal symbol? A production? A start symbol? A parse tree?
- What is a left-hand side of a production? A right-hand side?
- Given a grammar G, what is meant by the language L(G)?
- What is a derivation step? A derivation? A leftmost derivation? A righmost derivation?
- How does a derivation correspond to a parse tree?
- What does it mean for a grammar to be ambiguous? Unambiguous?
- Give an example an ambiguous CFG.
- What is the difference between an LL and an LR parser?
- What is the difference between LL(1) and LL(2)? Or between LR(1) and LR(2)?
- Construct a recursive descent parser for a simple language.
- Give typical examples of grammars that cannot be handled by a recursive-descent parser.
- Explain why context-free grammars are more powerful than regular expressions.
- In what sense are context-free grammars "context-free"?