

EDDAN40

examination

4 hp

3rd June 2019

14:00 - 19:00

WRITE ONLY ON ONE SIDE OF THE PAPER - the exams will be scanned in and only the front/odd pages will be read.

DO NOT WRITE WITH OTHER COLOUR THAN BLACK OR DARK BLUE - lightly coloured text may disappear during scanning

PUT YOUR ID AND PAGE NUMBER ON EACH PAGE YOU SUBMIT - make sure that the amount of pages is equal to the amount you note on the front information page

WRITE CLEARLY - if we cannot read you we cannot grade you.

PRELIMINARY AMOUNT OF POINTS: 6 (1+2+1+1+1)

```
{-A list of selected functions from the Haskell modules:
```

```
Prelude
Data.List
Data.Maybe
Data.Char -}
```

```
-----
-- standard type classes
```

```
class Show a where
  show :: a -> String
```

```
class Eq a where
  (==), (/=) :: a -> a -> Bool
```

```
class (Eq a) => Ord a where
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a
```

```
class (Eq a, Show a) => Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs, signum :: a -> a
  fromInteger :: Integer -> a
```

```
class (Num a, Ord a) => Real a where
  toRational :: a -> Rational
```

```
class (Real a, Enum a) => Integral a where
  quot, rem :: a -> a -> a
  div, mod :: a -> a -> a
  toInteger :: a -> Integer
```

```
class (Num a) => Fractional a where
  (/) :: a -> a -> a
  fromRational :: Rational -> a
```

```
class (Fractional a) => Floating a where
  exp, log, sqrt :: a -> a
  sin, cos, tan :: a -> a
```

```
class (Real a, Fractional a) => RealFrac a where
  truncate, round :: (Integral b) => a -> b
  ceiling, floor :: (Integral b) => a -> b
```

```
-----
-- numerical functions
```

```
even, odd :: (Integral a) => a -> Bool
even n = n `rem` 2 == 0
odd = not . even
```

```
-----
-- monadic functions
```

```
sequence :: Monad m => [m a] -> m [a]
sequence = foldr mcons (return [])
```

```
where mcons p q = do x <- p; xs <- q; return (x:xs)
```

```
sequence_ :: Monad m => [m a] -> m ()
sequence_ xs = do sequence xs; return ()
```

```
-----
-- functions on functions
```

```
id :: a -> a
id x = x
```

```
const :: a -> b -> a
const x _ = x
```

```
(.) :: (b -> c) -> (a -> b) -> a -> c
f . g = \x -> f (g x)
```

```
flip :: (a -> b -> c) -> b -> a -> c
flip f x y = f y x
```

```
($) :: (a -> b) -> a -> b
f $ x = f x
```

```
-----
-- functions on Booleans
```

```
data Bool = False | True
```

```
(&&), (||) :: Bool -> Bool -> Bool
```

```
True && x = x
```

```
False && _ = False
```

```
True || _ = True
```

```
False || x = x
```

```
not :: Bool -> Bool
```

```
not True = False
```

```
not False = True
```

```
-----
-- functions on Maybe
```

```
data Maybe a = Nothing | Just a
```

```
isJust :: Maybe a -> Bool
```

```
isJust (Just a) = True
```

```
isJust Nothing = False
```

```
isNothing :: Maybe a -> Bool
```

```
isNothing = not . isJust
```

```
fromJust :: Maybe a -> a
```

```
fromJust (Just a) = a
```

```
maybeToList :: Maybe a -> [a]
```

```
maybeToList Nothing = []
```

```
maybeToList (Just a) = [a]
```

```
listToMaybe      :: [a] -> Maybe a
listToMaybe []   = Nothing
listToMaybe (a:_) = Just a
```

-- a hidden goodie

```
instance Monad [] where
  return x = [x]
  xs >>= f = concat (map f xs)
```

-- functions on pairs

```
fst      :: (a, b) -> a
fst (x, y) = x
```

```
snd      :: (a, b) -> b
snd (x, y) = y
```

```
curry    :: ((a, b) -> c) -> a -> b -> c
curry f x y = f (x, y)
```

```
uncurry  :: (a -> b -> c) -> (a, b) -> c
uncurry f p = f (fst p) (snd p)
```

-- functions on lists

```
map      :: (a -> b) -> [a] -> [b]
map f xs = [ f x | x <- xs ]
```

```
(++)     :: [a] -> [a] -> [a]
xs ++ ys = foldr (:) ys xs
```

```
filter   :: (a -> Bool) -> [a] -> [a]
filter p xs = [ x | x <- xs, p x ]
```

```
concat   :: [[a]] -> [a]
concat xss = foldr (++) [] xss
```

```
concatMap :: (a -> [b]) -> [a] -> [b]
concatMap f = concat . map f
```

```
head, last :: [a] -> a
head (x:_) = x
```

```
last [x] = x
last (_:xs) = last xs
```

```
tail, init :: [a] -> [a]
tail (_:xs) = xs
```

```
init [x] = []
init (x:xs) = x : init xs
```

```
null      :: [a] -> Bool
null []    = True
null (_:_) = False
```

```
length    :: [a] -> Int
length []  = 0
length (_:l) = 1 + length l
```

```
(!!)      :: [a] -> Int -> a
(x:_) !! 0 = x
(_:xs) !! n = xs !! (n-1)
```

```
foldr     :: (a -> b -> b) -> b -> [a] -> b
foldr f z [] = z
foldr f z (x:xs) = f x (foldr f z xs)
```

```
foldl     :: (a -> b -> a) -> a -> [b] -> a
foldl f z [] = z
foldl f z (x:xs) = foldl f (f z x) xs
```

```
iterate   :: (a -> a) -> a -> [a]
iterate f x = x : iterate f (f x)
```

```
repeat    :: a -> [a]
repeat x   = xs where xs = x:xs
```

```
replicate :: Int -> a -> [a]
replicate n x = take n (repeat x)
```

```
cycle     :: [a] -> [a]
cycle []   = error "Prelude.cycle: empty list"
cycle xs = xs' where xs' = xs++xs'
```

```
take, drop :: Int -> [a] -> [a]
take n _ | n <= 0 = []
take _ [] = []
take n (x:xs) = x : take (n-1) xs
```

```
drop n xs | n <= 0 = xs
drop _ [] = []
drop n (_:xs) = drop (n-1) xs
```

```
splitAt   :: Int -> [a] -> ([a],[a])
splitAt n xs = (take n xs, drop n xs)
```

```
takeWhile, dropWhile :: (a -> Bool) -> [a] -> [a]
takeWhile p [] = []
takeWhile p (x:xs)
  | p x = x : takeWhile p xs
  | otherwise = []
```

```
dropWhile p [] = []
dropWhile p xs@(x:xs')
  | p x = dropWhile p xs'
  | otherwise = xs
```

```

lines, words      :: String -> [String]
-- lines "apa\nbepa\ncepa\n" == ["apa","bepa","cepa"]
-- words "apa bepa\n cepa" == ["apa","bepa","cepa"]

unlines, unwords  :: [String] -> String
-- unlines ["apa","bepa","cepa"] == "apa\nbepa\ncepa"
-- unwords ["apa","bepa","cepa"] == "apa bepa cepa"

and, or           :: [Bool] -> Bool
and              = foldr (&&) True
or              = foldr (||) False

any, all         :: (a -> Bool) -> [a] -> Bool
any p           = or . map p
all p          = and . map p

elem, notElem   :: (Eq a) => a -> [a] -> Bool
elem x         = any (== x)
notElem x      = all (/= x)

lookup          :: (Eq a) => a -> [(a,b)] -> Maybe b
lookup key []  = Nothing
lookup key ((x,y):xys)
  | key == x   = Just y
  | otherwise  = lookup key xys

sum, product    :: (Num a) => [a] -> a
sum            = foldl (+) 0
product       = foldl (*) 1

maximum, minimum :: (Ord a) => [a] -> a
maximum []    = error "Prelude.maximum: empty list"
maximum xs   = foldl1 max xs

minimum []    = error "Prelude.minimum: empty list"
minimum xs   = foldl1 min xs

zip           :: [a] -> [b] -> [(a,b)]
zip          = zipWith (,)

zipWith      :: (a->b->c) -> [a]->[b]->[c]
zipWith z (a:as) (b:bs)
  = z a b : zipWith z as bs
zipWith _ _ _ = []

unzip       :: [(a,b)] -> ([a],[b])
unzip      = foldr (\(a,b) ~(as,bs) -> (a:as,b:bs)) ([],[])

nub         :: (Eq a) => [a] -> [a]
nub []     = []
nub (x:xs) = x : nub [ y | y <- xs, x /= y ]

delete     :: Eq a => a -> [a] -> [a]
delete y [] = []

```

```

delete y (x:xs) = if x == y then xs else x : delete y xs

(\\)          :: Eq a => [a] -> [a]-> [a]
(\\)         = foldl (flip delete)

union         :: Eq a => [a] -> [a] -> [a]
union xs ys  = xs ++ ( ys \\ xs )

intersect     :: Eq a => [a] -> [a]-> [a]
intersect xs ys = [ x | x <- xs, x `elem` ys ]

intersperse  :: a -> [a] -> [a]
-- intersperse 0 [1,2,3,4] == [1,0,2,0,3,0,4]

transpose    :: [[a]] -> [[a]]
-- transpose [[1,2,3],[4,5,6]] == [[1,4],[2,5],[3,6]]

partition    :: (a -> Bool) -> [a] -> ([a],[a])
partition p xs = (filter p xs, filter (not . p) xs)

group        :: Eq a => [a] -> [[a]]
-- group "aapaabbbbee" == ["aa","p","aa","bbb","eee"]

isPrefixOf, isSuffixOf :: Eq a => [a] -> [a] -> Bool
isPrefixOf [] _         = True
isPrefixOf _ []        = False
isPrefixOf (x:xs) (y:ys) = x == y && isPrefixOf xs ys

isSuffixOf x y         = reverse x `isPrefixOf` reverse y

sort           :: (Ord a) => [a] -> [a]
sort          = foldr insert []

insert        :: (Ord a) => a -> [a] -> [a]
insert x []   = [x]
insert x (y:xs) = if x <= y then x:y:xs else y:insert x xs

-----
-- functions on Char

type String = [Char]

toUpper, toLower :: Char -> Char
-- toUpper 'a'   == 'A'
-- toLower 'Z'   == 'z'

digitToInt      :: Char -> Int
-- digitToInt '8' == 8

intToDigit      :: Int -> Char
-- intToDigit 3  == '3'

ord             :: Char -> Int
chr            :: Int -> Char

```

Exam

1. Proving program properties (1p)

The `Functor` class is defined as follows:

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

It is mandatory that all instances of `Functor` should obey:

```
fmap id      = id
fmap (p . q) = (fmap p) . (fmap q)
```

Assume the following definition of lists as a functor instance:

```
instance Functor [] where
  fmap g []      = []
  fmap g (x:xs) = [g x] ++ (fmap g xs)
```

Is this a correct definition of a functor instance? Why or why not? **Prove your claim.**

2. Types and type classes (2p)

- (0,3p) Define a tree data structure so that the trees are ternary (i.e., each node has either exactly three children or is a leaf) and store strings in each node.
- (0,3p) Generalize your definition so that your ternary trees can contain objects of an arbitrary predetermined type in a node.
- (0,7p) Assuming your polymorphic trees type is denoted by `Tree3 a`, write all necessary code so that the following function is correct:

```
myLength :: Tree3 String -> Tree3 Integer
myLength = fmap length
```

and yields an (obviously ternary) tree with nodes containing lengths of the strings placed in the respective nodes of the argument tree. Your solution must contain the word **instance** to get full credit.

- (0,7p) write all necessary code so that you can compare two ternary trees for equality using the `==` operator.

3. Point-free notation (1p)

Rewrite the following two definitions into a point-free form (i.e., `f = ...`, `g = ...`), using neither lambda-expressions nor list comprehensions nor enumeration nor `where` clause nor `let` clause:

```
f x y = (3 - y) * x
g x y = map x $ filter (<3) y
```

Hint: you may find `flip` useful.

4. Sparks (1p)

Consider the following code parallelizing the quicksort algorithm.

```
-- file: ch24/Sorting.hs
import Control.Parallel (par, pseq)
parSort (x:xs) = greater `par` (lesser `pseq`
                               (lesser ++ x:greater))
  where lesser = parSort [y | y <- xs, y < x]
        greater = parSort [y | y <- xs, y >= x]
parSort _ = []
```

It will be almost as (in)efficient as the sequential version:

```
-- file: ch24/Sorting.hs
sort :: (Ord a) => [a] -> [a]
sort (x:xs) = lesser ++ x:greater
  where lesser = sort [y | y <- xs, y < x]
        greater = sort [y | y <- xs, y >= x]
sort _ = []
```

Why?

5. Parsing (1p)

This is an excerpt from the assignment N3 code:

```
assignment = word #- accept ":@" # Expr.parse
             #- require ";" >-> buildAss
buildAss (v, e) = Assignment v e
```

- (0,5p) Provide types for all functions and operators named here.
- (0,5p) Rewrite `assignment` using `do`-notation.

Good Luck!