Rasterization

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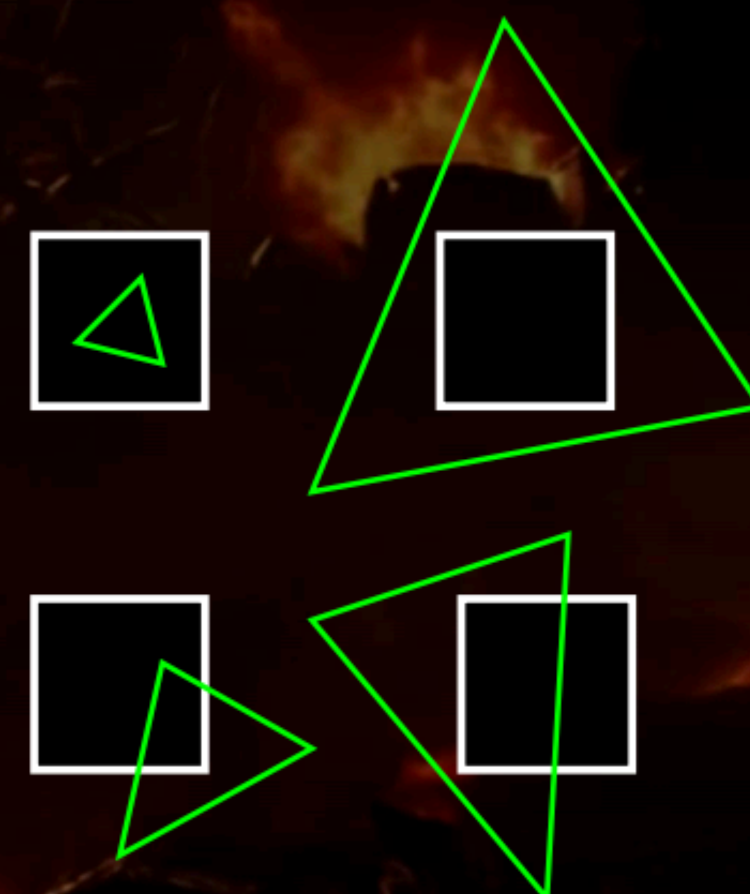
Rasterization algorithm is still in use

COMPUTE SHADER RASTERIZATION

- For simplicity we only bin hexahedra
- Low resolution
- Needs to be conservative
  - Many edge cases
  - Artists will find them all
- Reverse float depth for precision
- Binned rasterization [Abrash2009]
  1. Setup & Cull
  2. Coarse raster
  3. Fine raster
  4. Resolve bitfields to lists



Slide courtesy Jean Geffroy, Axel Gneiting, and Yixin Wang
from "Rendering the Hellscape of DOOM Eternal"
https://advances.realtimerendering.com/s2020/RenderingDoomEternal.pdf

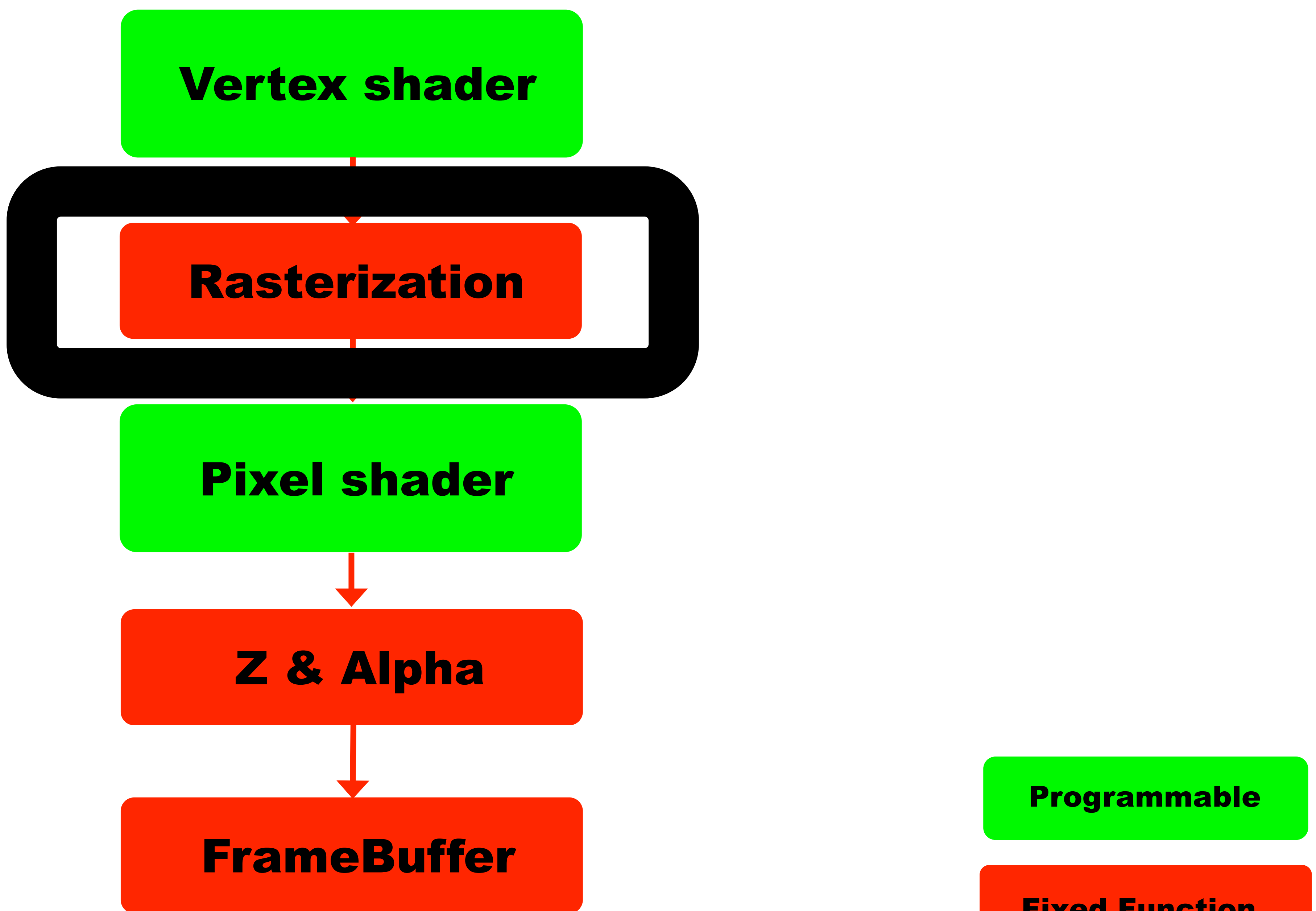
Unreal Engine 5 Nanite Virtualized Geometry

https://advances.realtimerendering.com/s2021/Karis_Nanite_SIGGRAPH_Advances_2021_final.pdf

Visibility Buffer

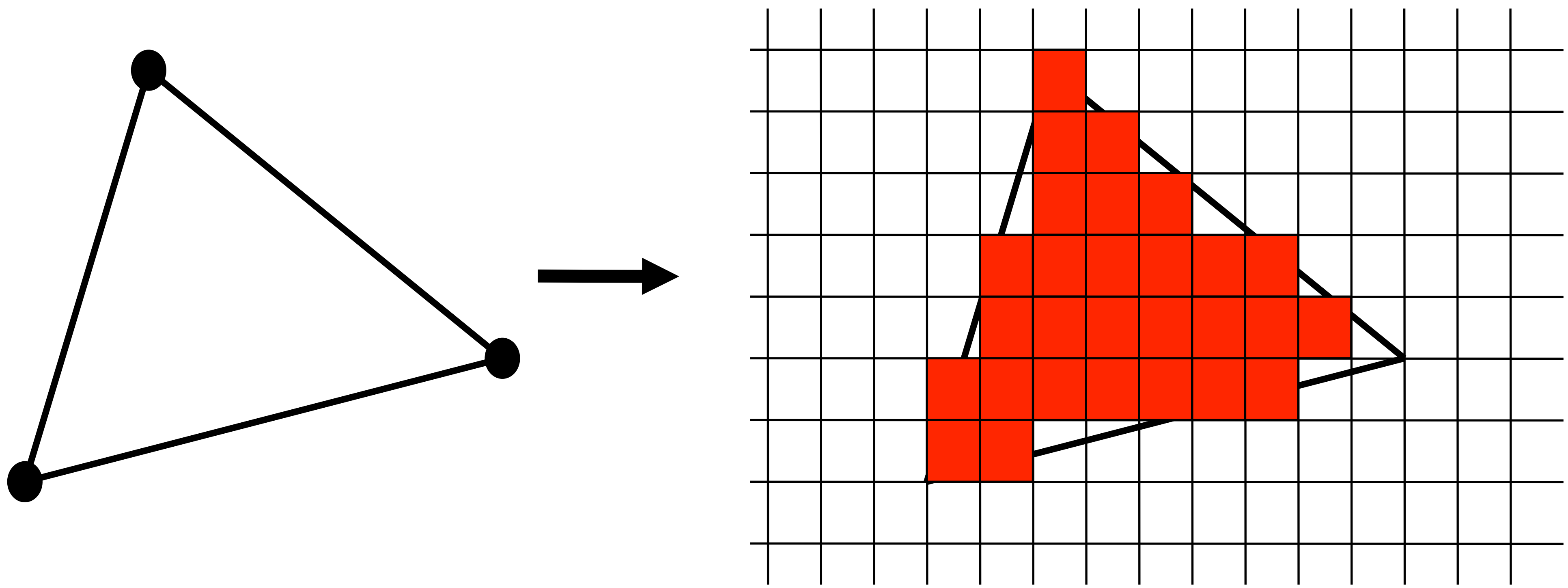
- Sounds crazy? Not as slow as it seems
  - Lots of cache hits
  - No overdraw or pixel quad inefficiencies
- Material pass writes GBuffer
  - Integrates with rest of our deferred shading renderer
- Now we can draw all opaque geometry with 1 draw
  - Completely GPU driven
  - Not just depth prepass
  - Rasterize triangles once per view

Today's stage of the Graphics Pipeline



How to rasterize a triangle?

Edge functions



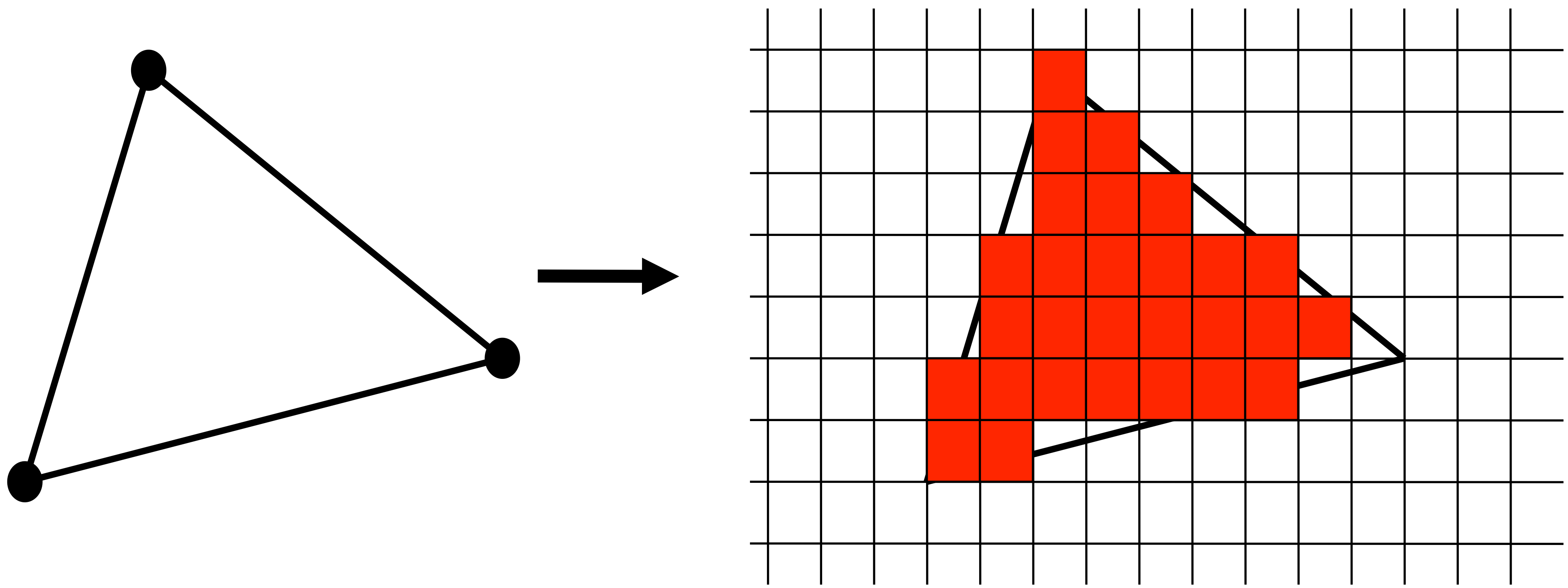
Vertex positioning

Traversal

Interpolation

Edge functions

Vertex positioning

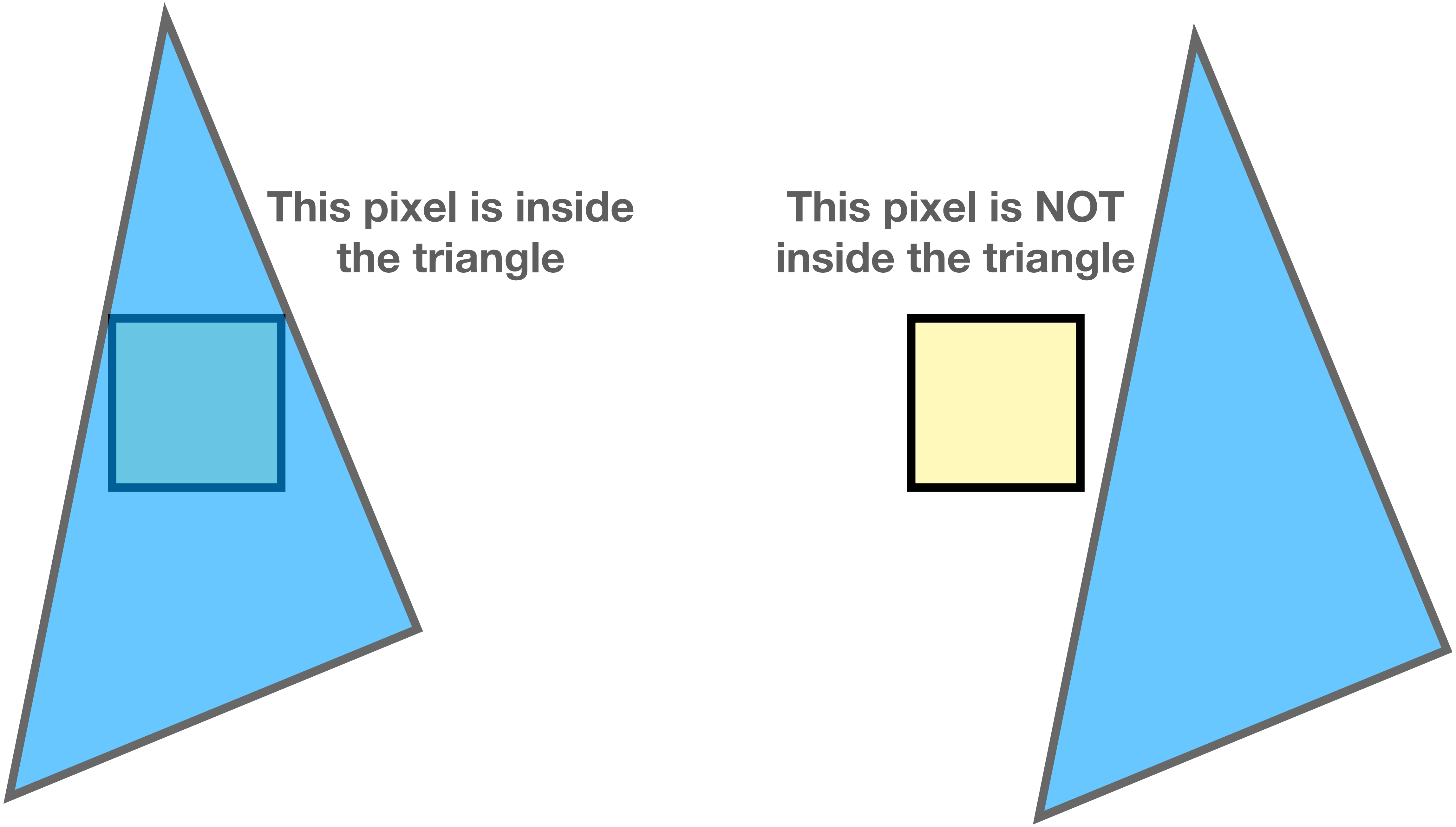


Traversal

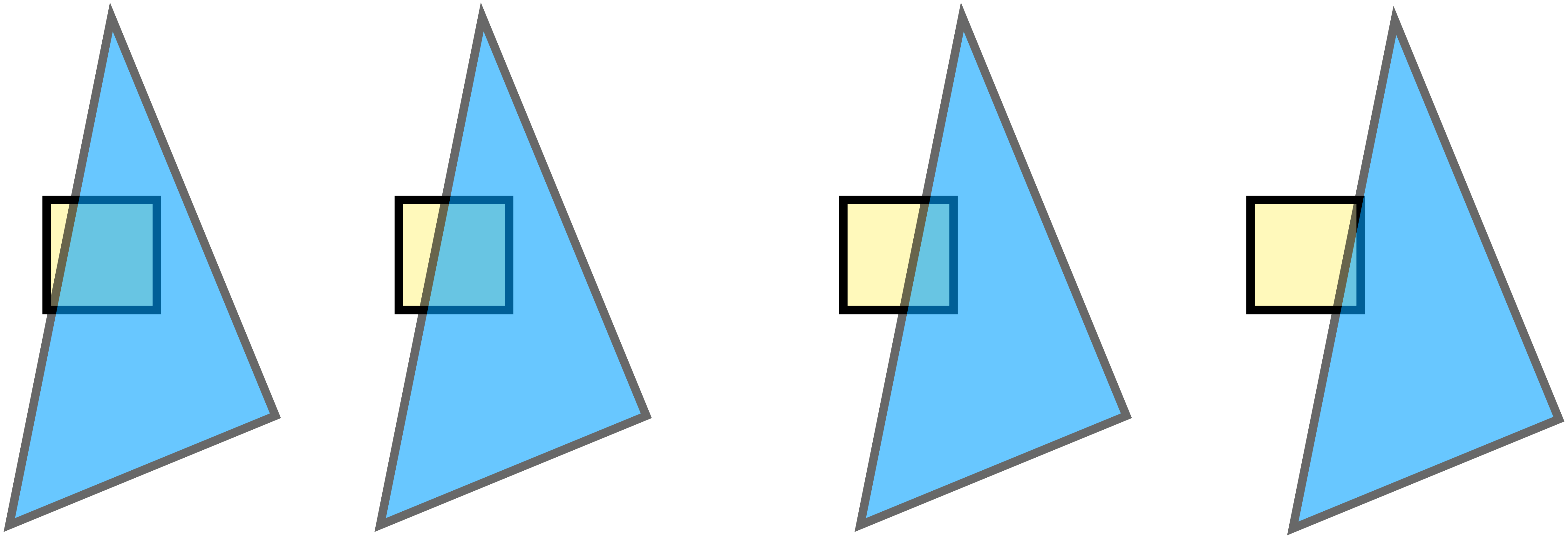
Interpolation

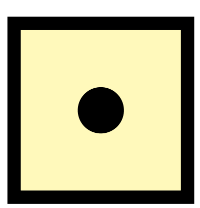
Which pixel is inside a triangle?

- **Triangle traversal**



Which pixel is inside a triangle?

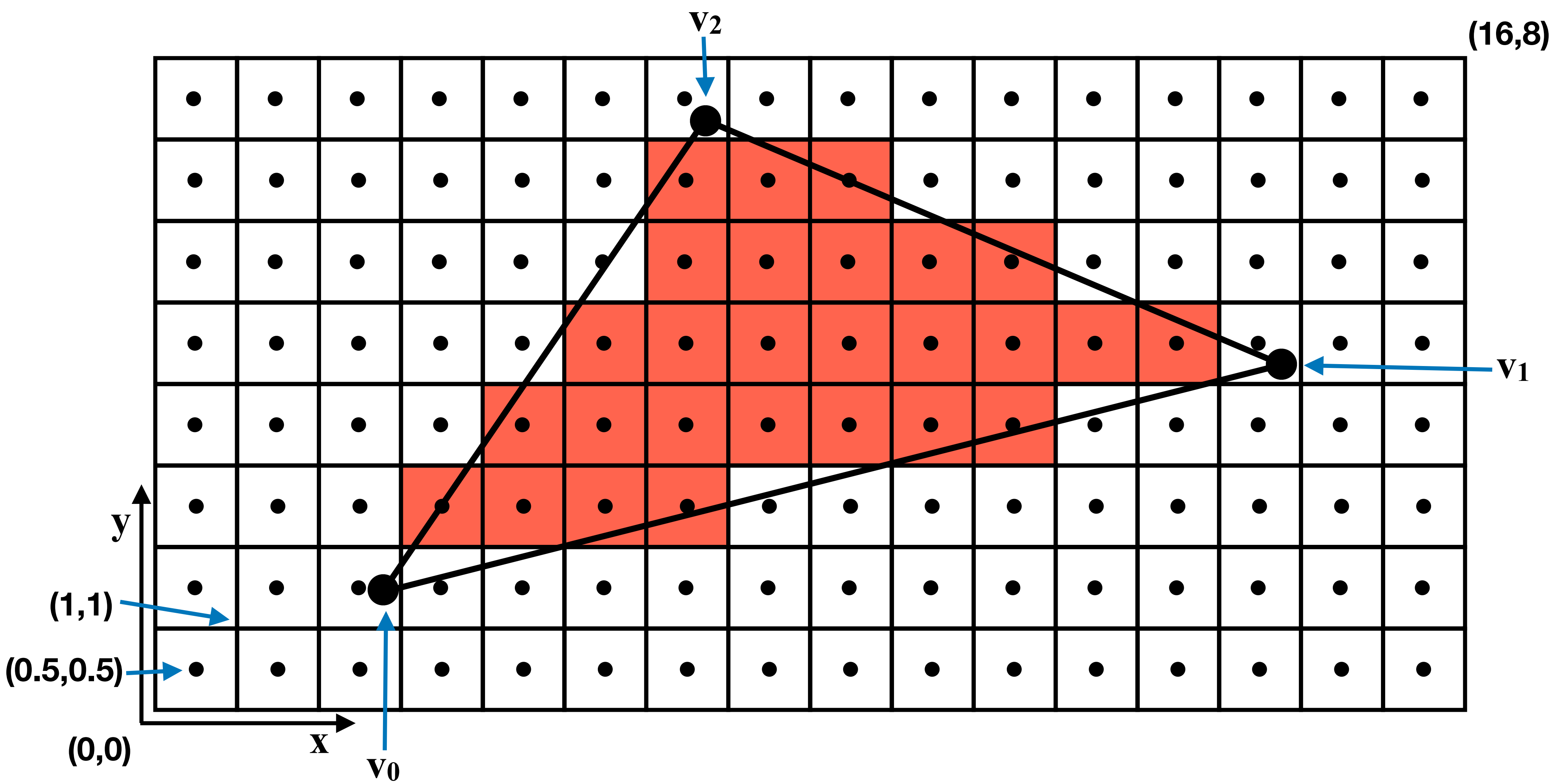




- Sample pixel at the center
- Later we will talk about how more than one sample per pixel can improve quality

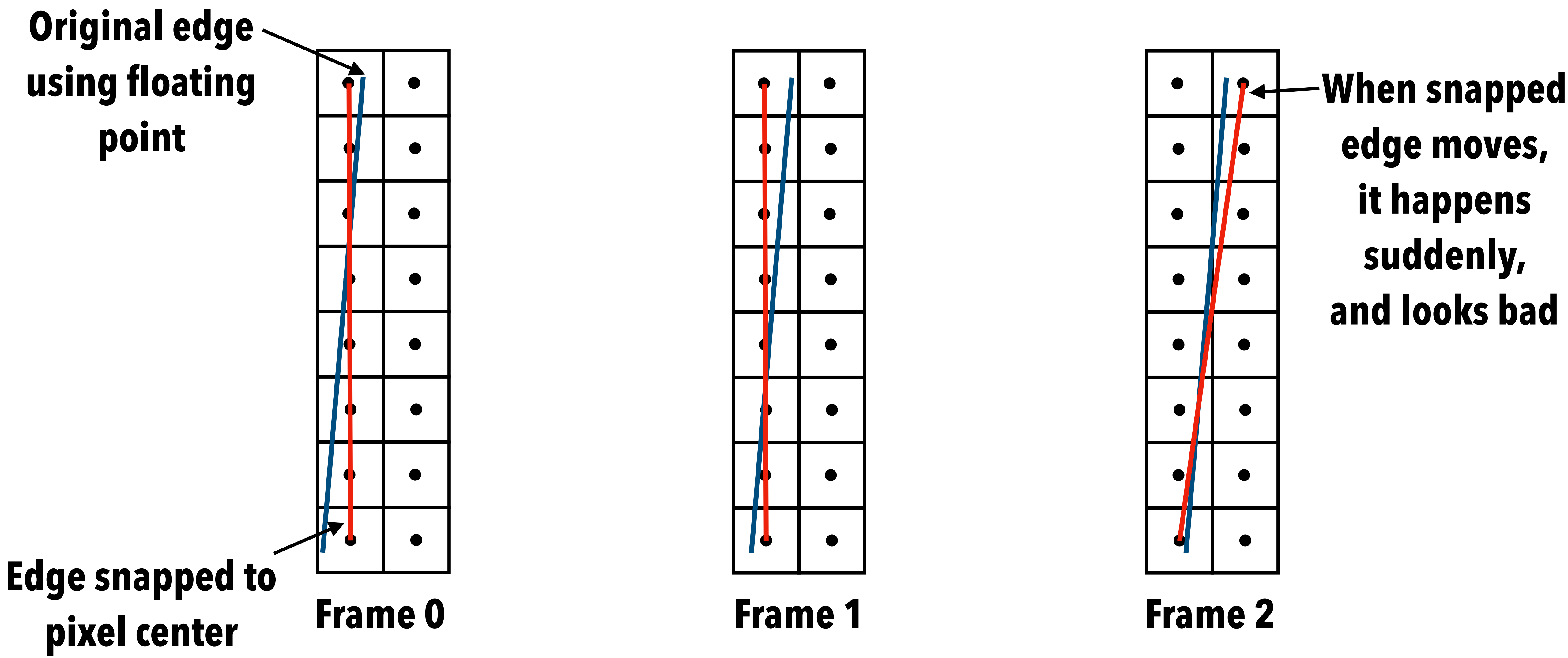
How do we compute pixel center?

- In screen space coordinates
- (v_x, v_y) are in $[0,w] \times [0,h]$



What happens if we use pixel grid for vertex positions?

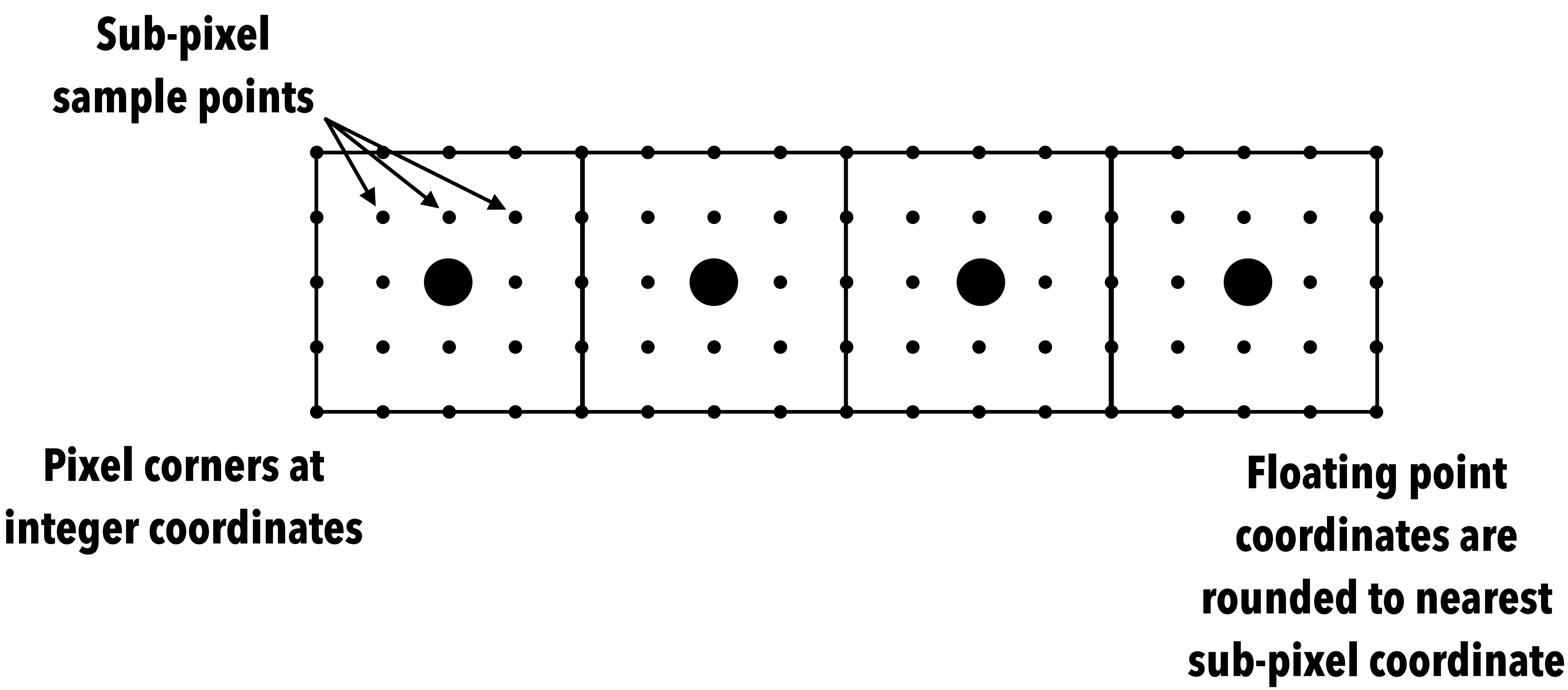
- Vertex positions are in floating point, pixels are integer positions



How to solve this without using floating point?

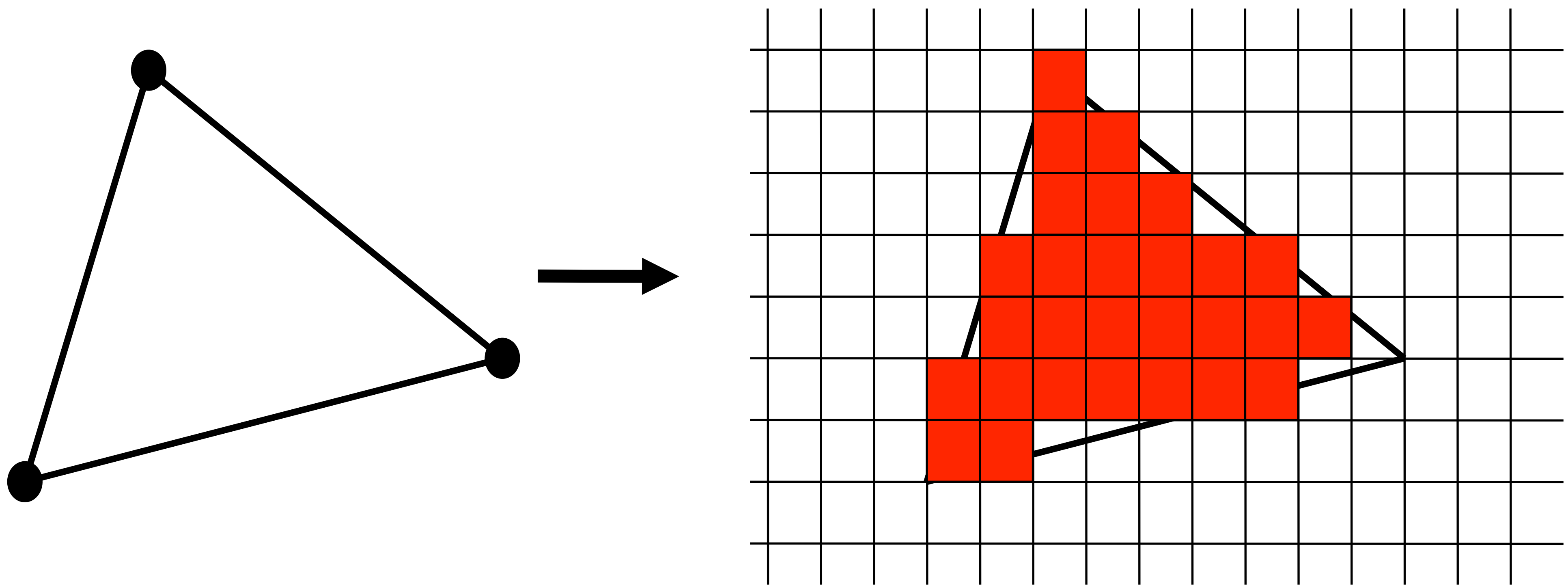
Use sub-pixel coordinates based on integers

- We can use fixed point math (integer)
  - Lowers complexity of silicon hardware design needed
- Use 2 sub-pixel fraction bits per x and y coordinate



Edge functions

Vertex positioning



Traversal

Interpolation

How do we determine if sample is inside a triangle?

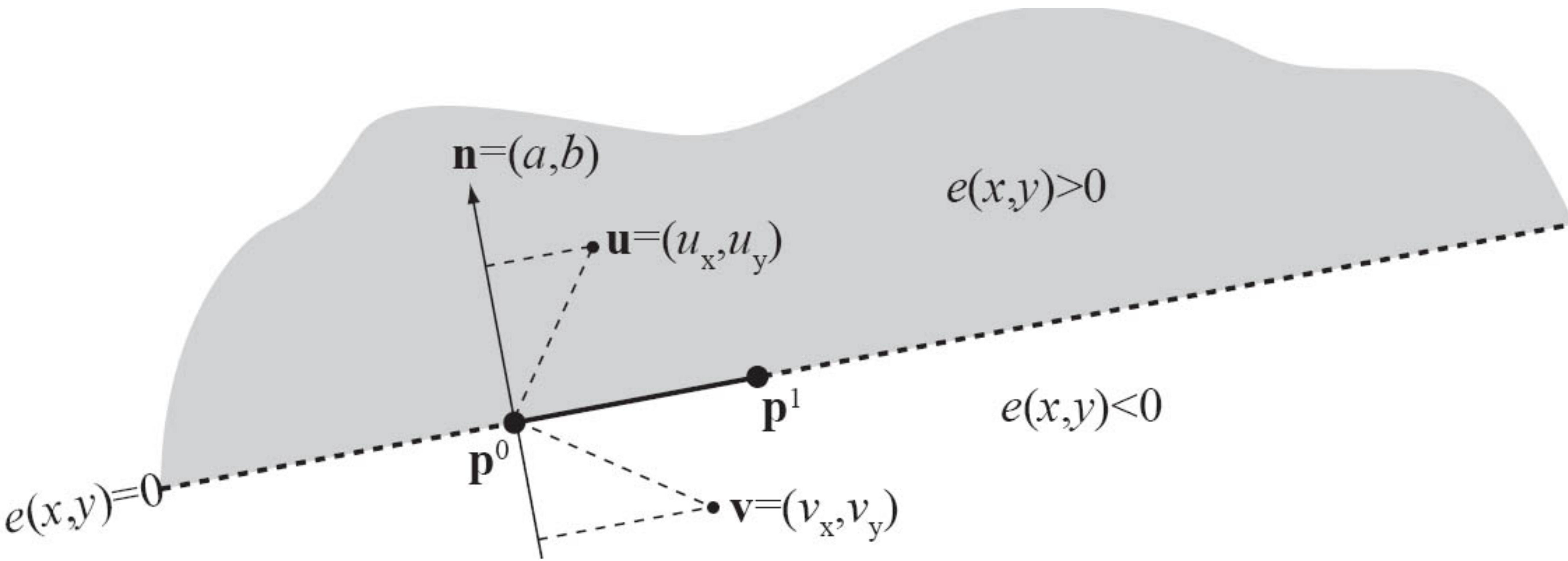
- **Convert edges into functions**
  - **Line equation** $ax + by + c$
- **Edge function for two points p0 and p1 is:**

$$e(x,y) = -(p_y^1 - p_y^0)(x - p_x^0) + (p_x^1 - p_x^0)(y - p_y^0)$$

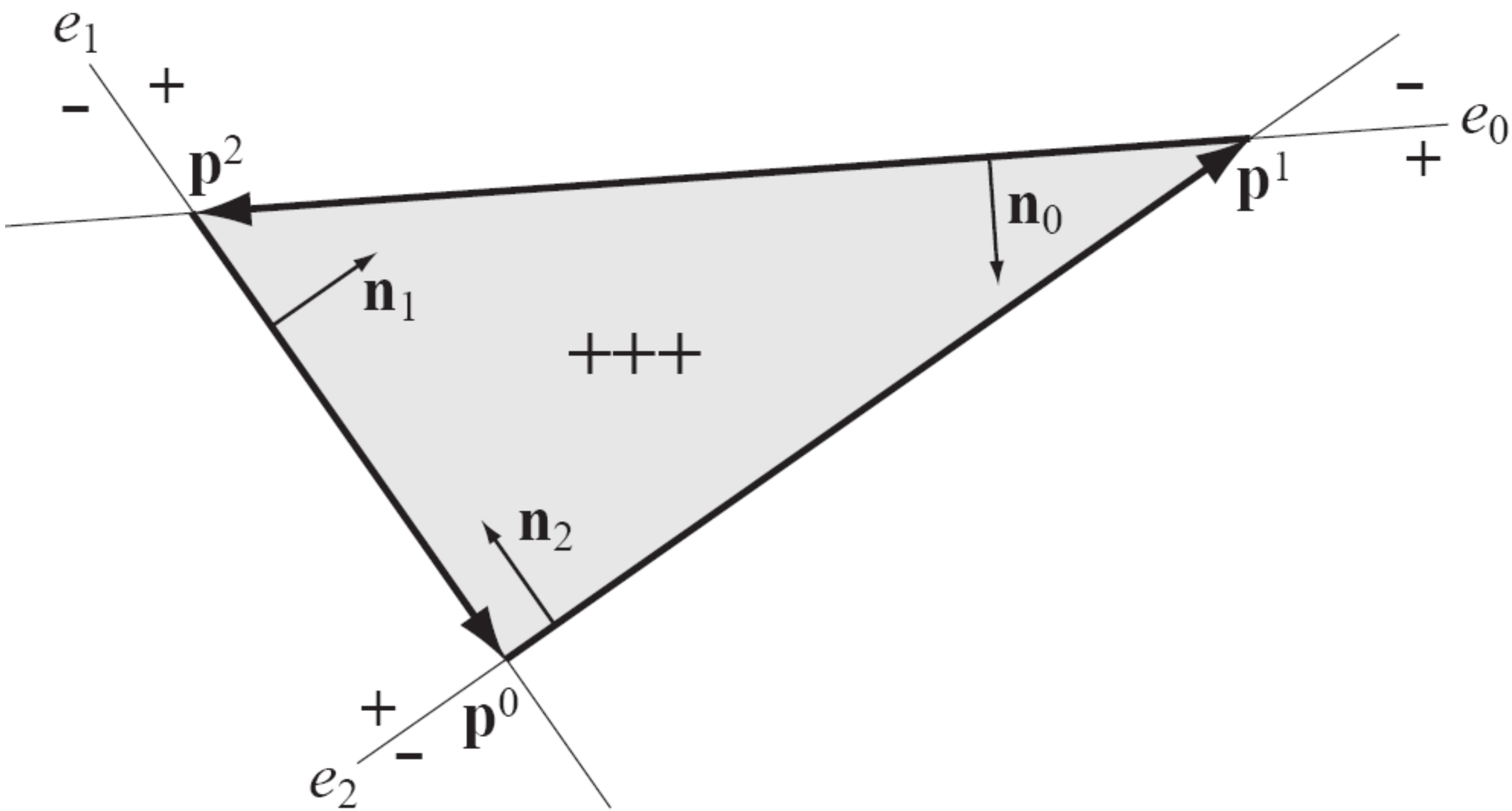
$$e(x,y) = ax + by + c = \mathbf{n}.(\mathbf{p} - \mathbf{p}_0)$$

Can be thought of as the 'normal' of the line

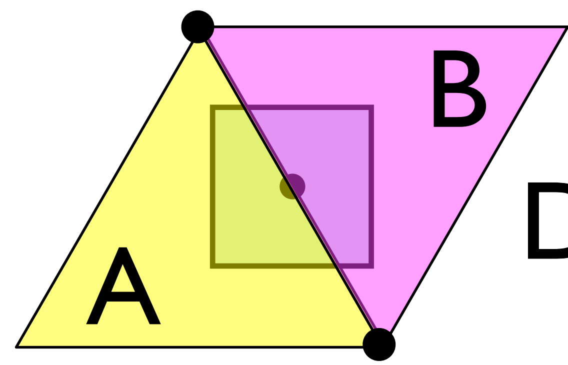
How do points relate to the edge function?



Points are inside if all edge functions are positive!



What happens to pixels exactly on an edge?



Does the pixel belong to A or B, or both ? or neither?

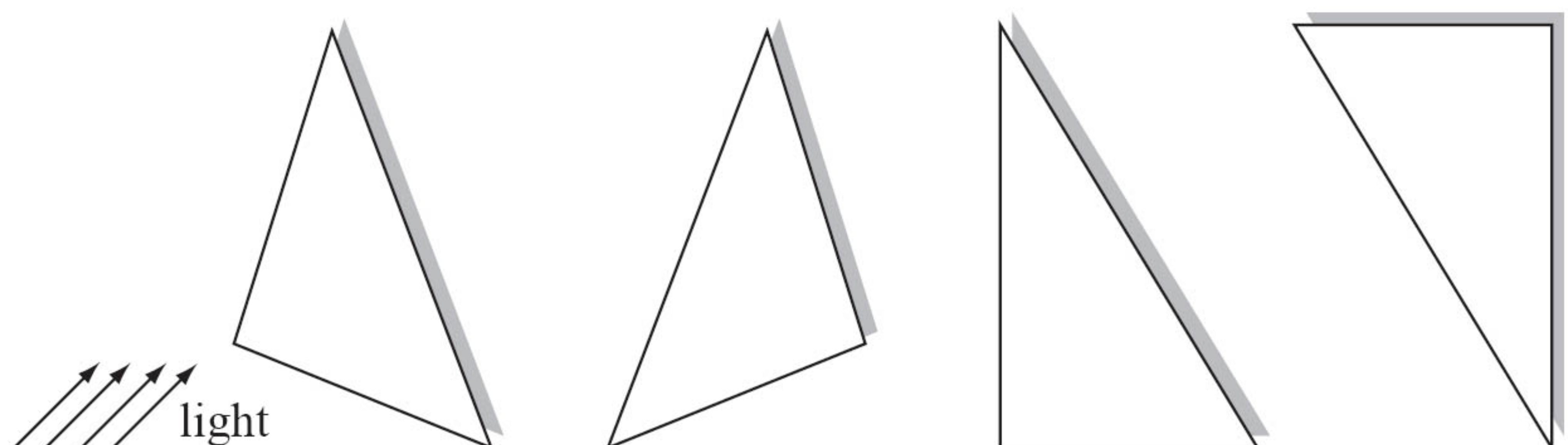
- One and only one of A or B
- Because :
  - No cracks between triangles
  - No overlapping triangles

How to decide which triangle an edge sample is in?

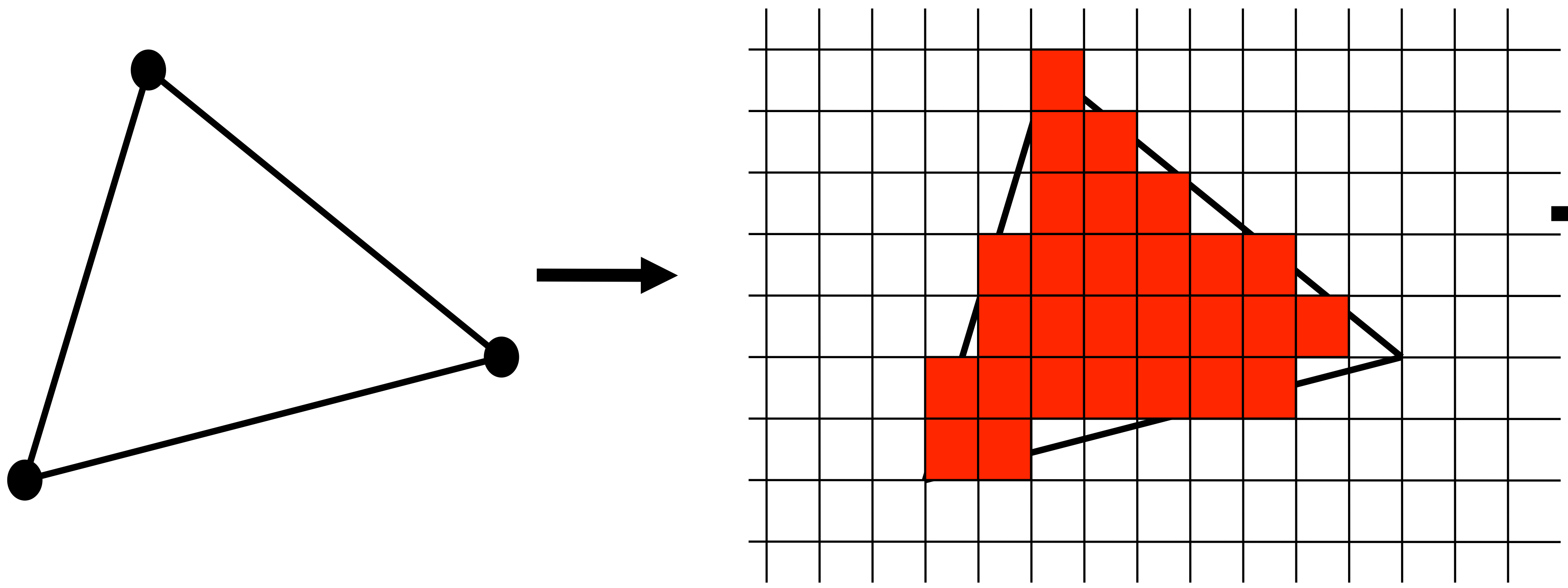
One solution (by McCool et al.)

```
bool INSIDE(e, x, y)
if e(x, y) > 0 return true;
if e(x, y) < 0 return false;
if a > 0 return true;
if a < 0 return false;
if b > 0 return true;
return false;
```

- Another way to think about it:
  - We exclude shadowed edges

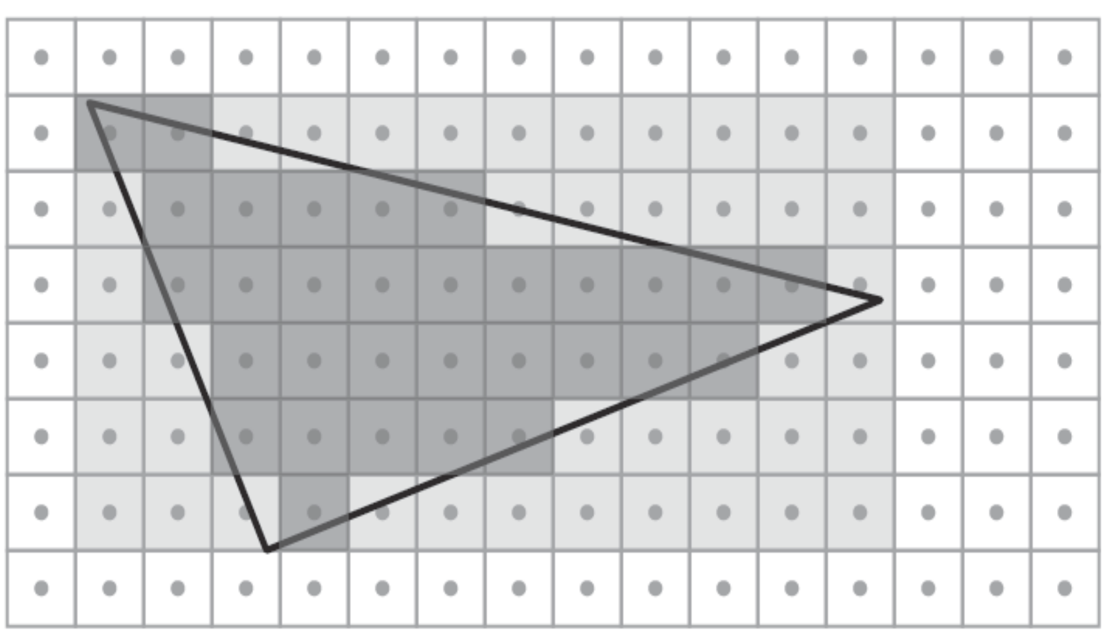


Edge functions



Triangle traversal strategies

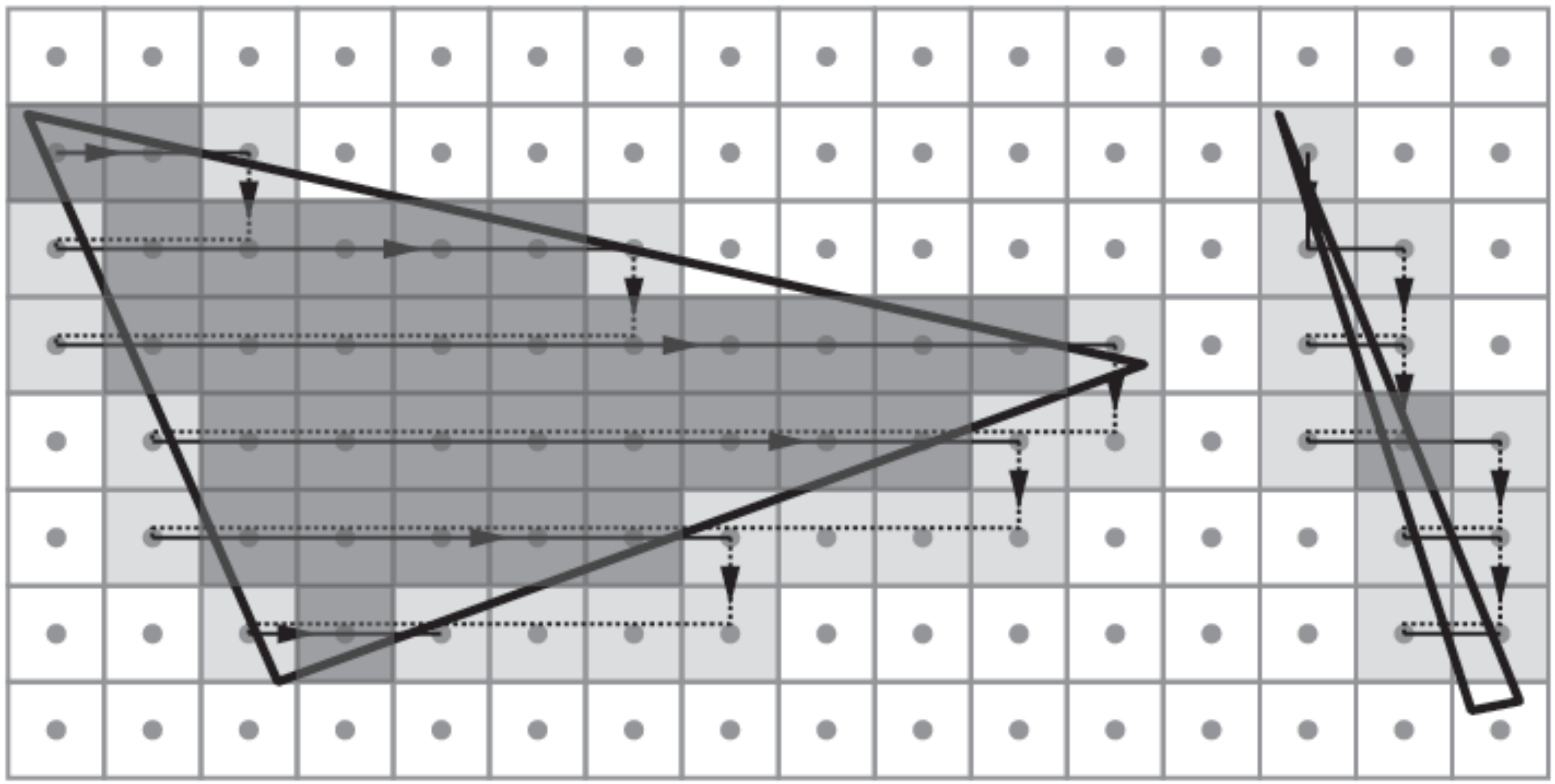
- Simple (and naive):
  - execute `Inside()` for every pixel on screen, and for every edge
- Little better: compute bounding box first
- Called ”bounding box traversal”



Visits all gray pixels

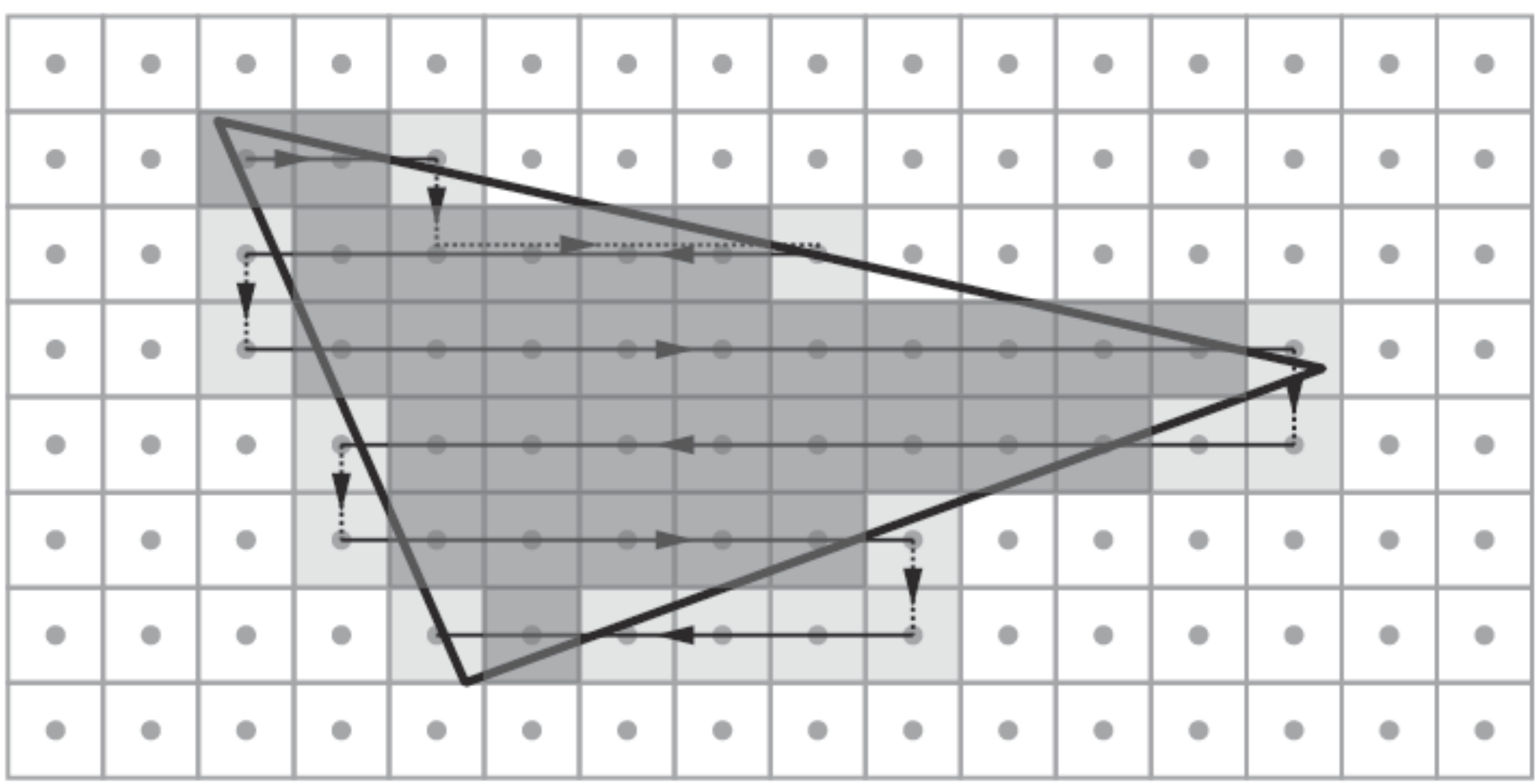
Only dark gray pixels are inside So only keep those

Backtrack traversal



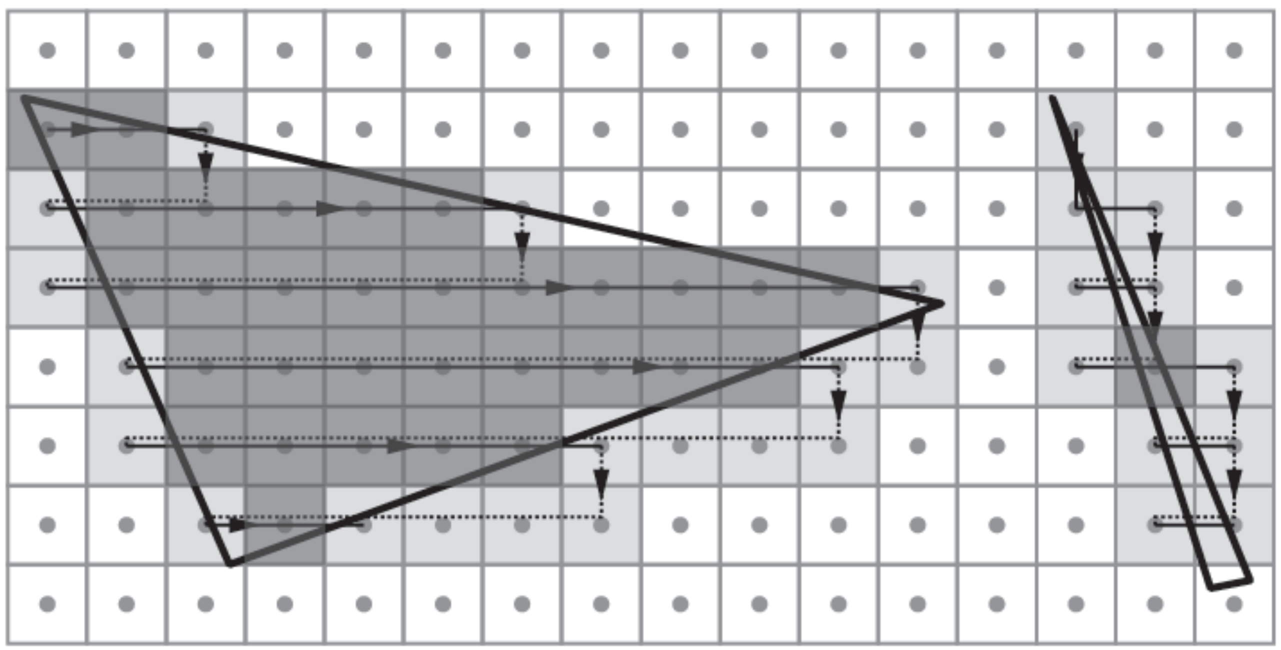
- Was used for mobile graphics chip
  - by Korean research group (KAIST)
- Advantage: only traverse from left to right
  - Could make for more efficient memory accesses
  - Could backtrack at a faster pace (because no mem acc)

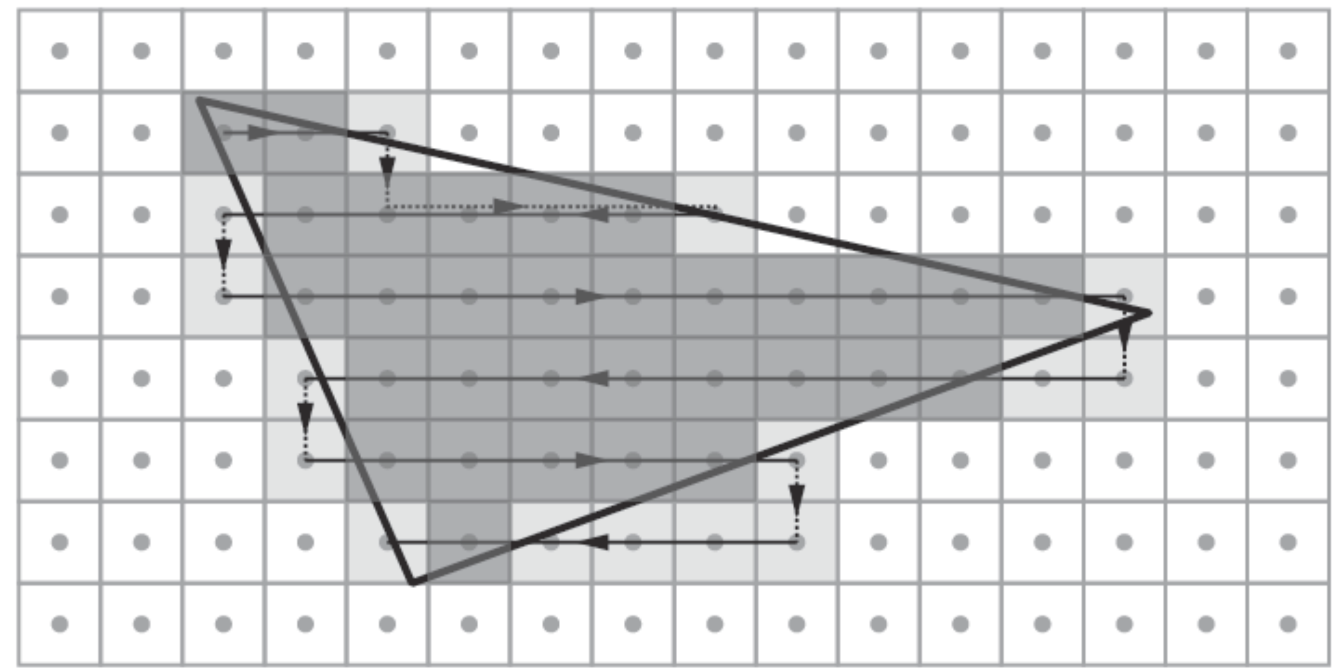
Zigzag traversal



- Simple technique that avoids backtracking
- Still visits outside pixels
  - see the last scanline

Side by side comparison Backtrack vs zigzag



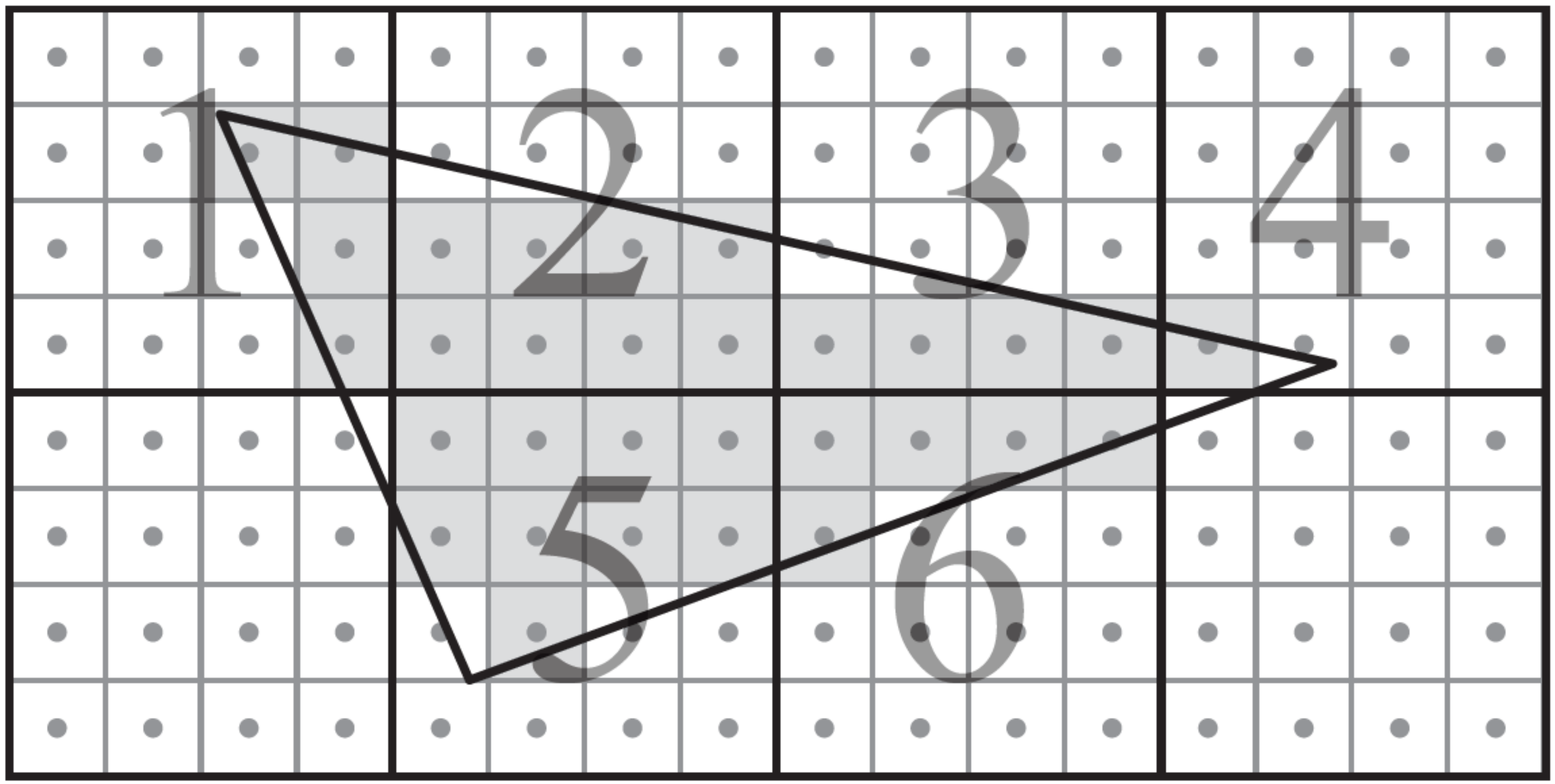


Backtrack never visits unnecessary pixels to the left

Zigzag never visits unnecessary pixels to the left on even scanlines and to the right on odd scanlines (and avoids backtracking)

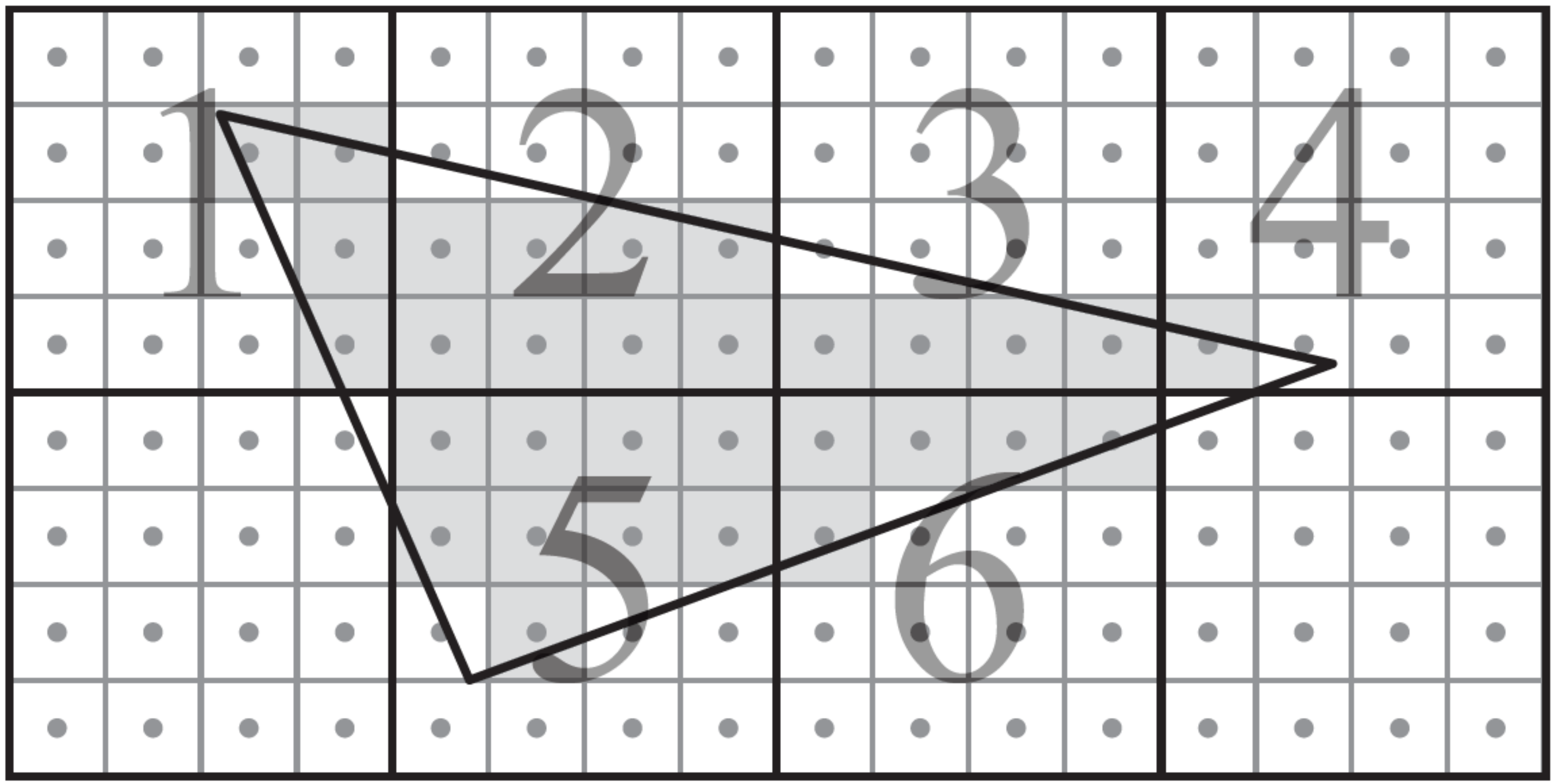
Tiled traversal

4x4 tiles



- Divide screen into tiles
  - each tile is *w* x *h* pixels
- 8x8 tile size is common in desktop GPUs

Tiled traversal



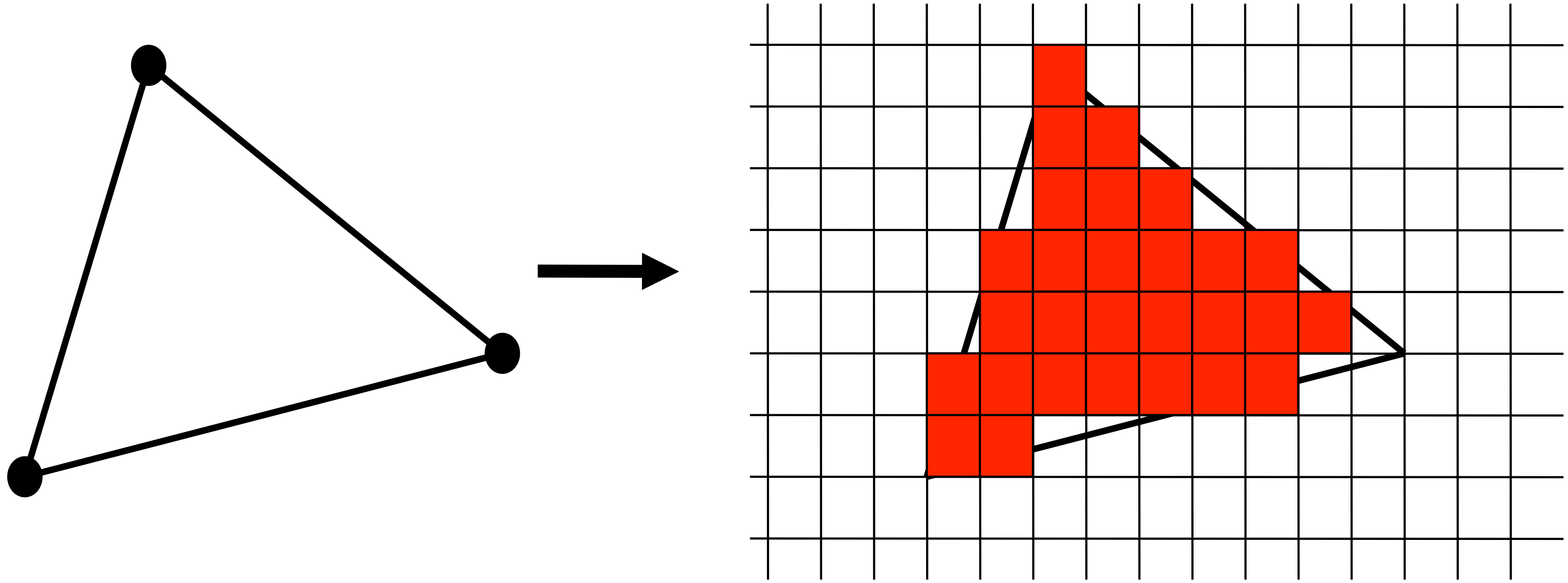
- Gives better texture cache performance
- Enables simple culling (Zmin & Zmax)
- Real-time buffer compression (color and depth)

Is tiled traversal that different?

- We need:
  - 1 : Traverse to tiles overlapping triangle
  - 2 : Test if tile overlaps with triangle
  - 3 : Traverse pixels inside tile
- We only need new algorithm for part 2
- Can use Haines and Wallace's box line intersection test (EGSR94)

Edge functions

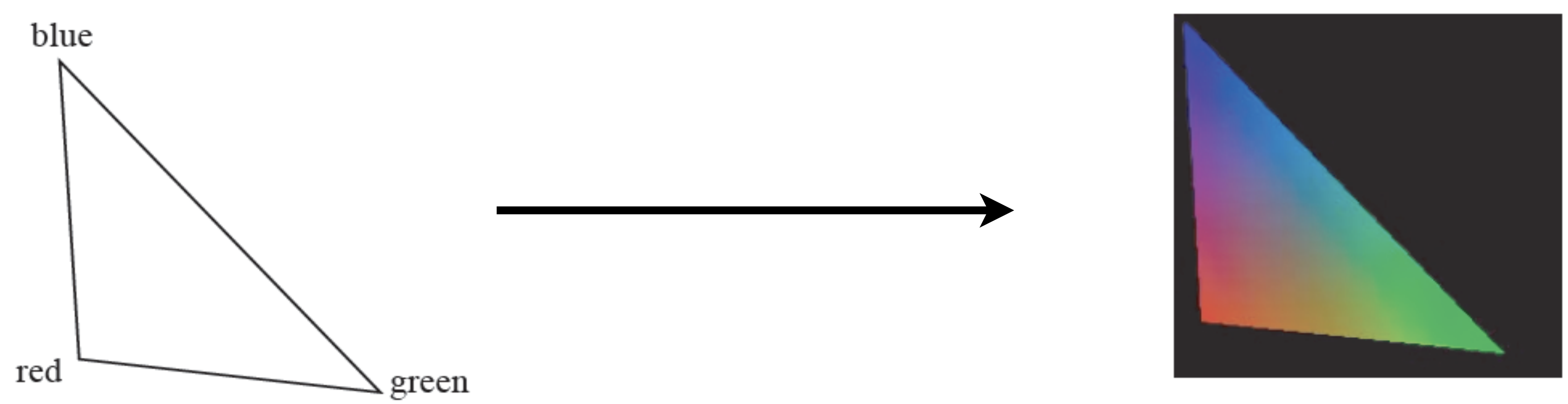
Vertex positioning

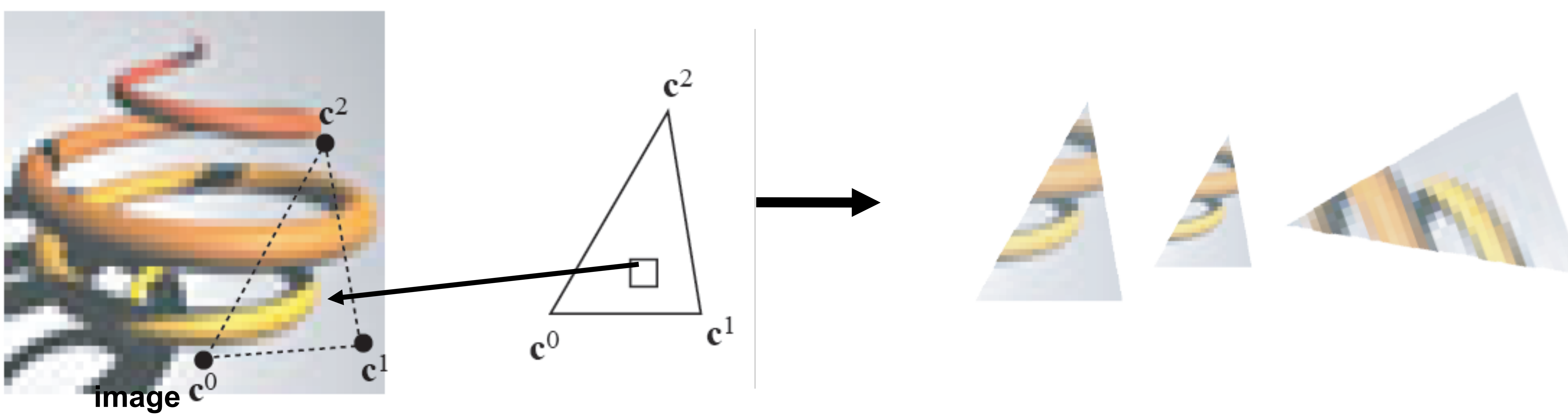


Traversal

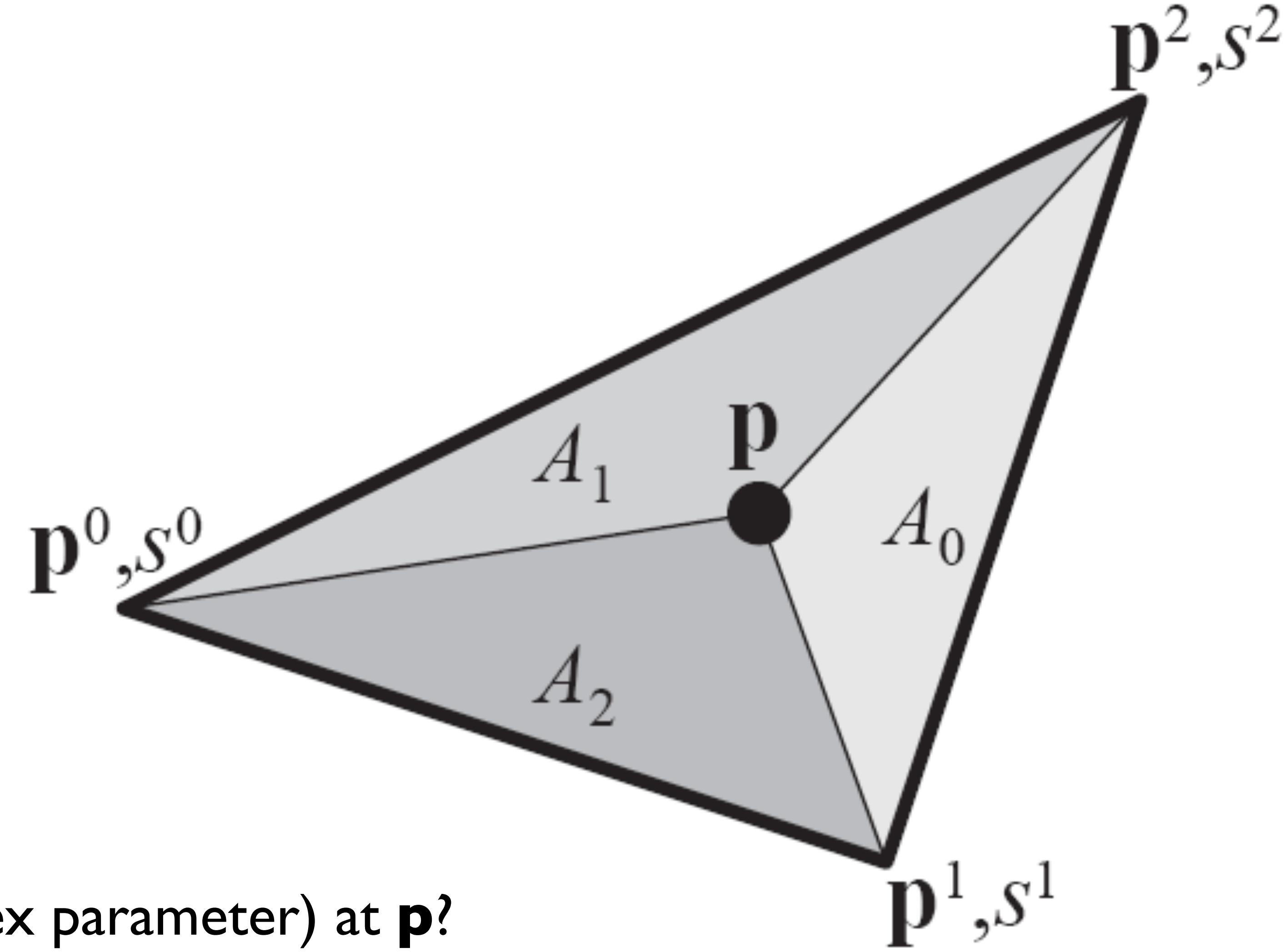
Interpolation

How can we interpolate parameters across triangles?



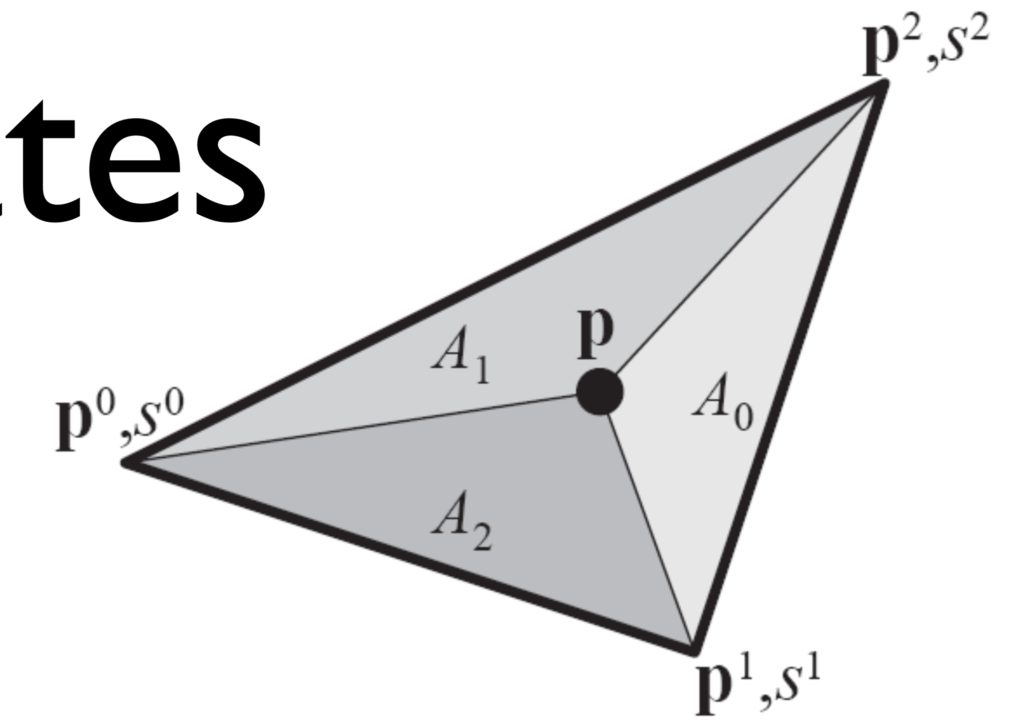


How can we interpolate parameters across triangles?



- What is *s* (vertex parameter) at **p**?
- S should vary smoothly across triangle
- Use **barycentric interpolation**
  - First we compute barycentric coordinates, (*u,v,w*)

Barycentric Coordinates



Proportional to the signed areas of the subtriangles formed by p and the vertices

Area computed using cross product, e.g.:

$$A_1 = \frac{1}{2}((p_x - p_x^0)(p_y^2 - p_y^0) - (p_y - p_y^0)(p_x^2 - p_x^0))$$

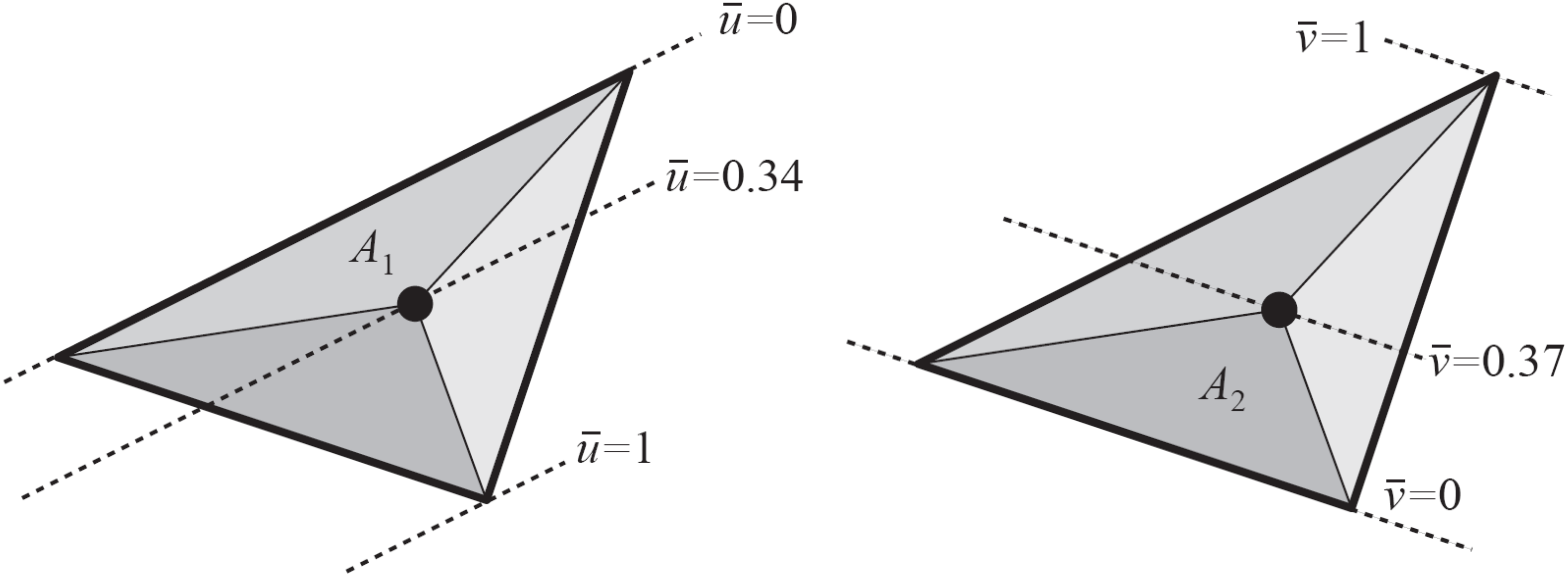
In graphics, we use barycentric coordinates normalized with respect to triangle area:

$$(\bar{u}, \bar{v}, \bar{w}) = \frac{(A_1, A_2, A_0)}{A_\Delta} \qquad A_\Delta = A_0 + A_1 + A_2$$

Not perspective correct

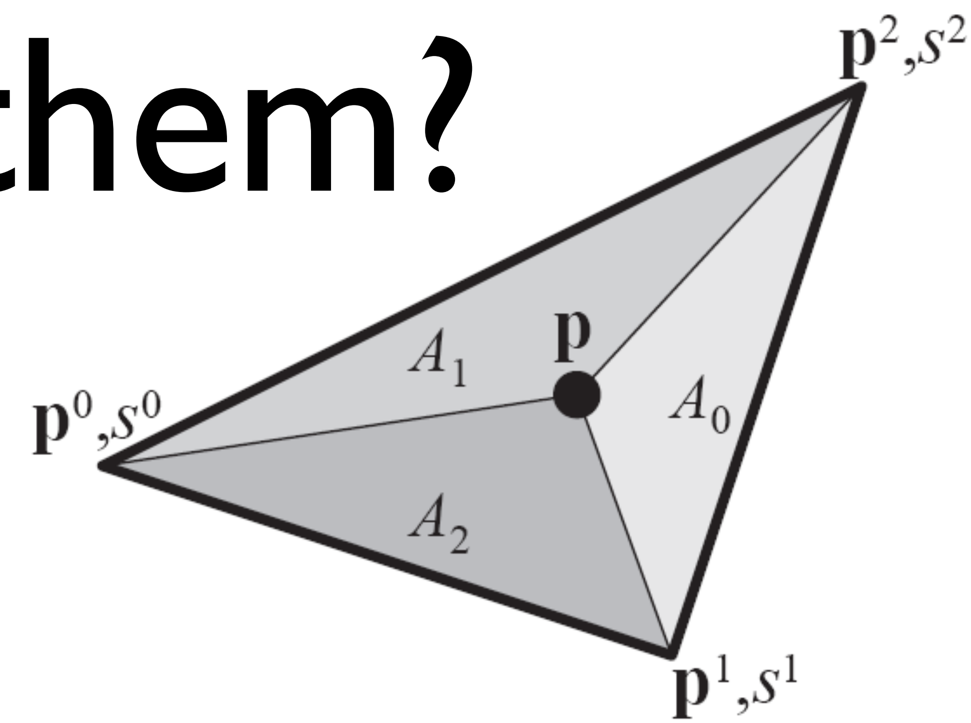
$$\bar{u} + \bar{v} + \bar{w} = 1 \qquad \bar{w} = 1 - \bar{u} - \bar{v}$$

What do barycentric coordinates look like?



- Constant on lines parallel to an edge
  - because the height of the subtriangle is constant

How to use them?



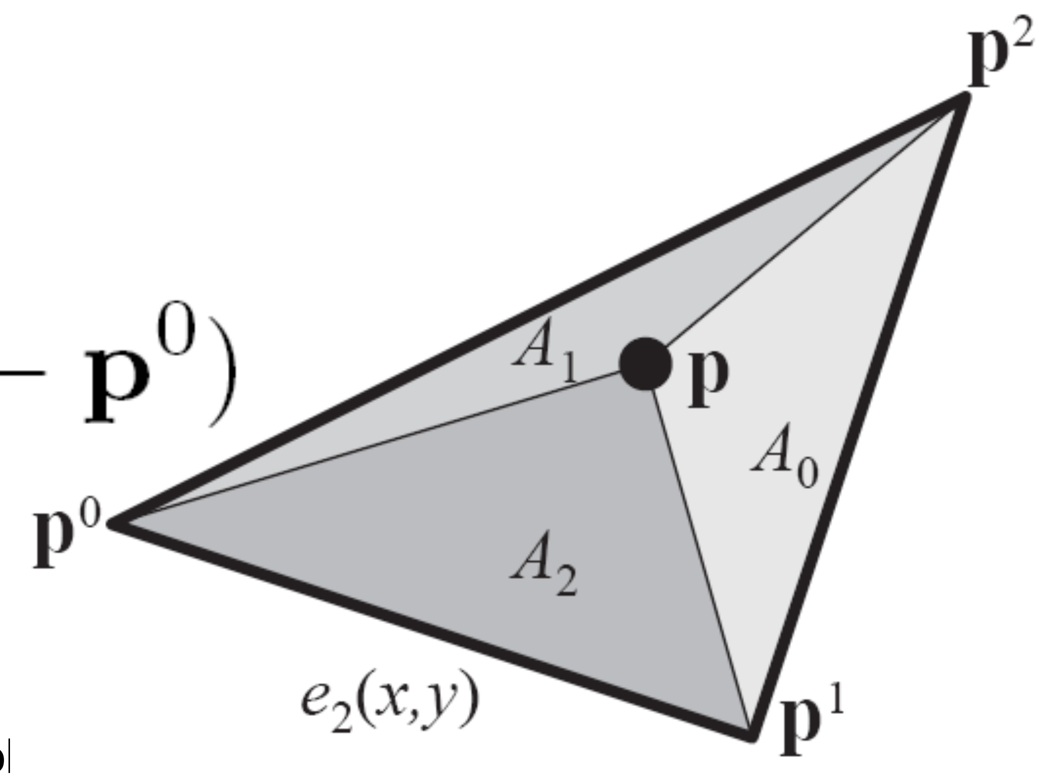
Interpolate vertex parameters s0, s1, s2

$$
\begin{aligned}
s &= \bar{w}s_0 + \bar{u}s_1 + \bar{v}s_2 = (1 - \bar{u} - \bar{v})s_0 + \bar{u}s_1 + \bar{v}s_2 \\
&= s_0 + \bar{u}(s_1 - s_0) + \bar{v}(s_2 - s_0).
\end{aligned}
$$

Barycentric coordinates from edge functions (1)

- The a and b parameters of an edge function must be proportional to the normal
  - We can use the edge functions directly to compute barycentric coordinates as well!
- Focus on edge, e_2:

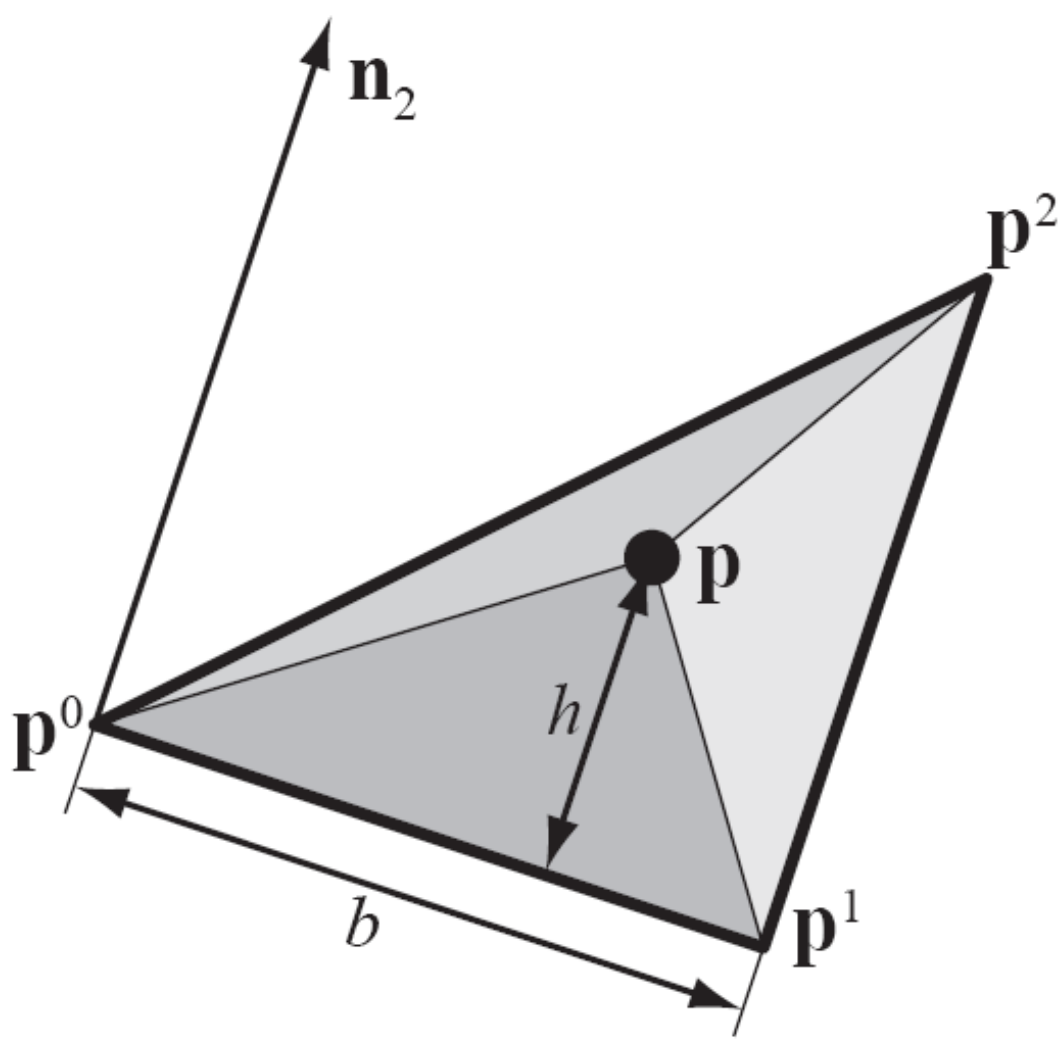
$$e_2(x, y) = e_2(\mathbf{p}) = \mathbf{n}_2 \cdot (\mathbf{p} - \mathbf{p}^0)$$



Barycentric coordinates from edge functions (2)

- From definition of dot product:

$$e_2(x, y) = e_2(\mathbf{p}) = \mathbf{n}_2 \cdot (\mathbf{p} - \mathbf{p}^0) \quad \Leftrightarrow$$
$$e_2(\mathbf{p}) = ||\mathbf{n}_2||\,||\mathbf{p} - \mathbf{p}^0||\cos\alpha$$



- We can show that $||\mathbf{n}_2|| = b$ (base of triangle)
- $||\mathbf{p}\text{-}\mathbf{p}^0||\cos\alpha$ is the length of projection of $\mathbf{p}\text{-}\mathbf{p}^0$ onto $\mathbf{n}_2$ i.e., h (height of triangle)

Barycentric coordinates from edge functions (3)

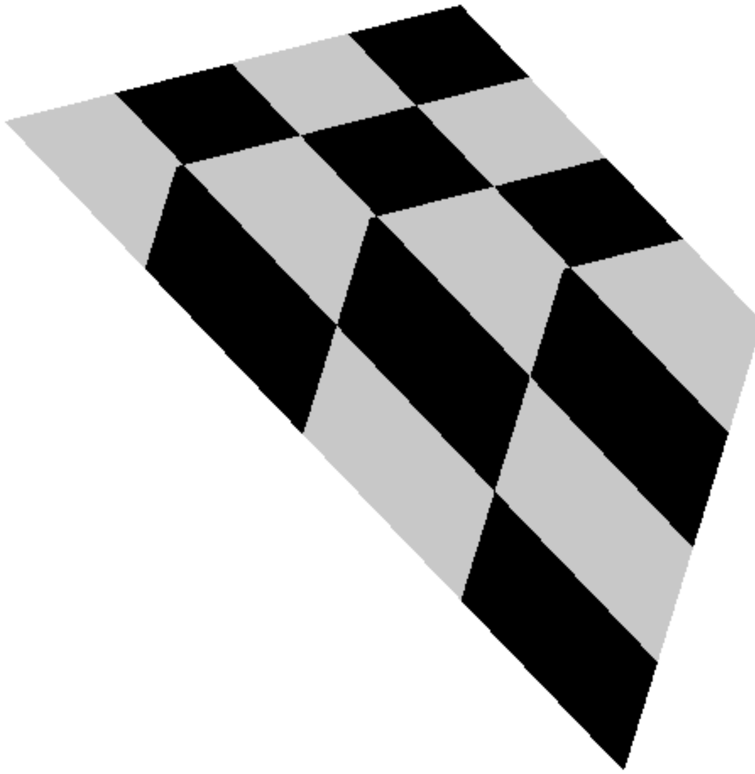
- This means:

$$\bar{u} = \frac{e_1(x,y)}{2A_\Delta}$$

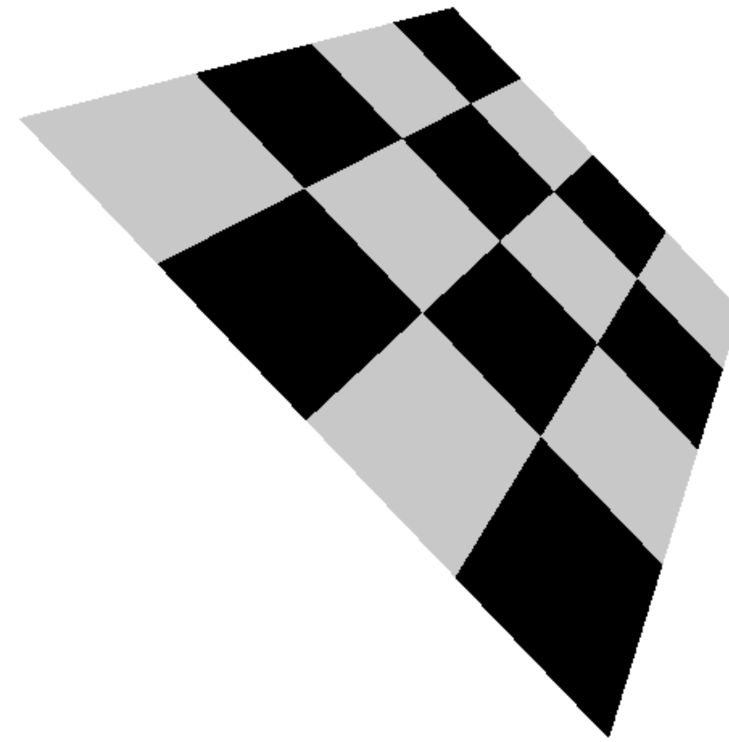
$$\bar{v} = \frac{e_2(x,y)}{2A_\Delta}$$

- And $1/(2A_\Delta)$ can be computed in the triangle setup (once per triangle)

Resulting interpolation



With barycentric coordinates, i.e., without perspective correction



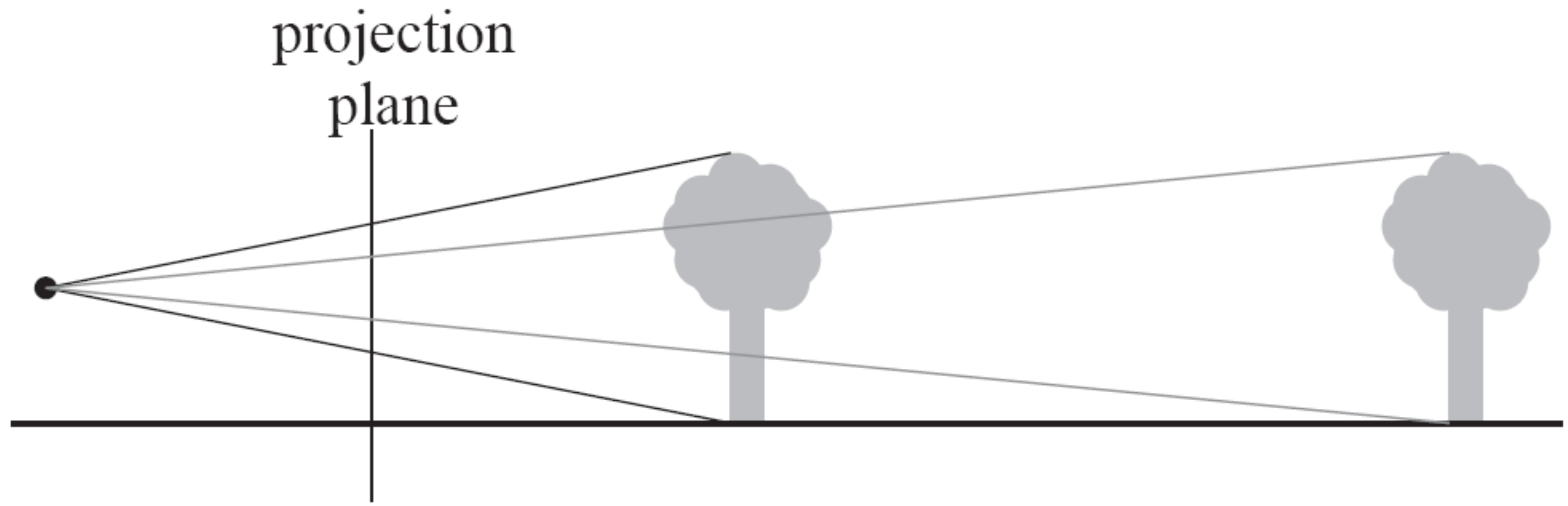
With perspective correction

- Looks even worse when animated...
- Clearly, perspective correction is needed!

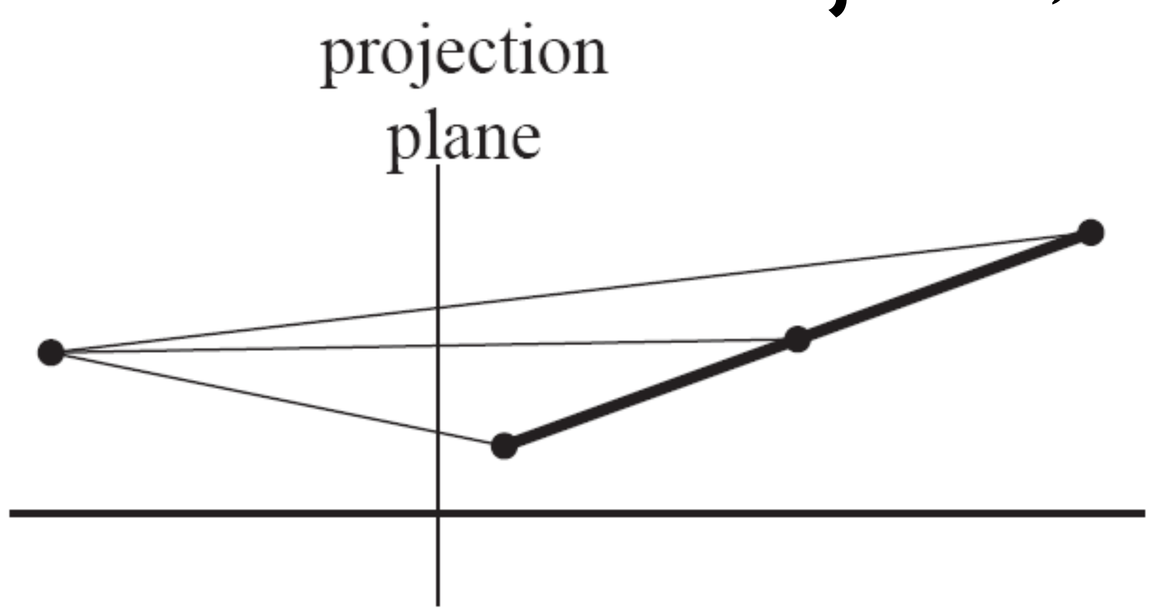
Which is which?

Perspective-correct interpolation

- Why?
  - Things farther away appear smaller!



- And even inside objects, of course:



Remember homogeneous coordinates

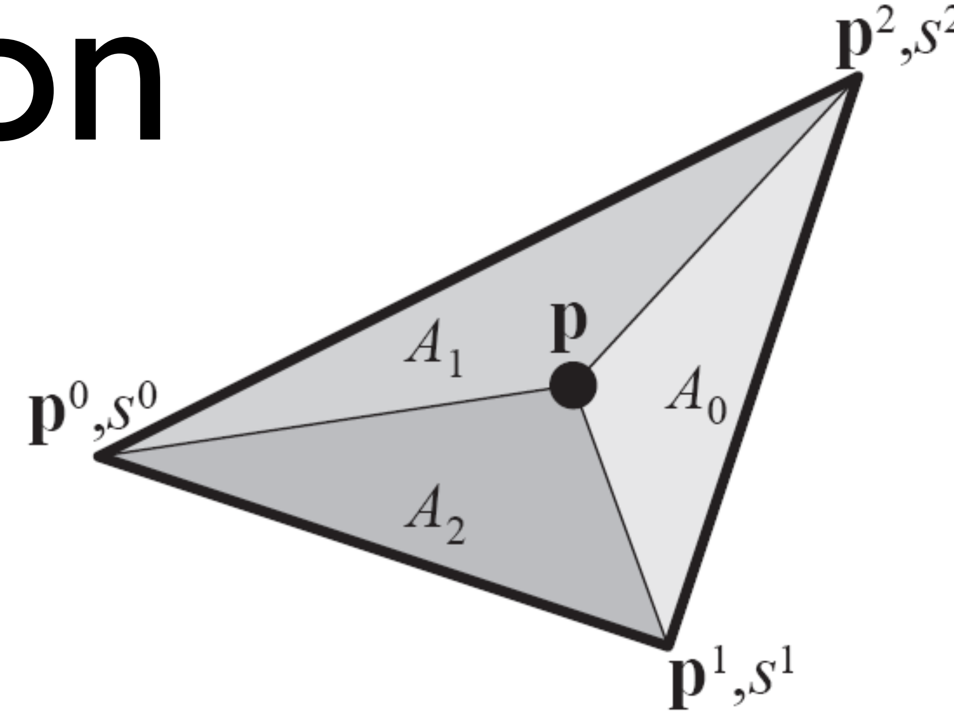
$$\mathbf{Mv} = \mathbf{h} = \begin{pmatrix} h_x \\ h_y \\ h_z \\ h_w \end{pmatrix} \Longrightarrow \begin{pmatrix} h_x/h_w \\ h_y/h_w \\ h_z/h_w \\ h_w/h_w \end{pmatrix} = \begin{pmatrix} h_x/h_w \\ h_y/h_w \\ h_z/h_w \\ 1 \end{pmatrix} = \mathbf{p}$$

$\mathbf{M}$ is a projection matrix

$\boldsymbol{p} = (p_x, p_y, p_z, 1)$ in screen space

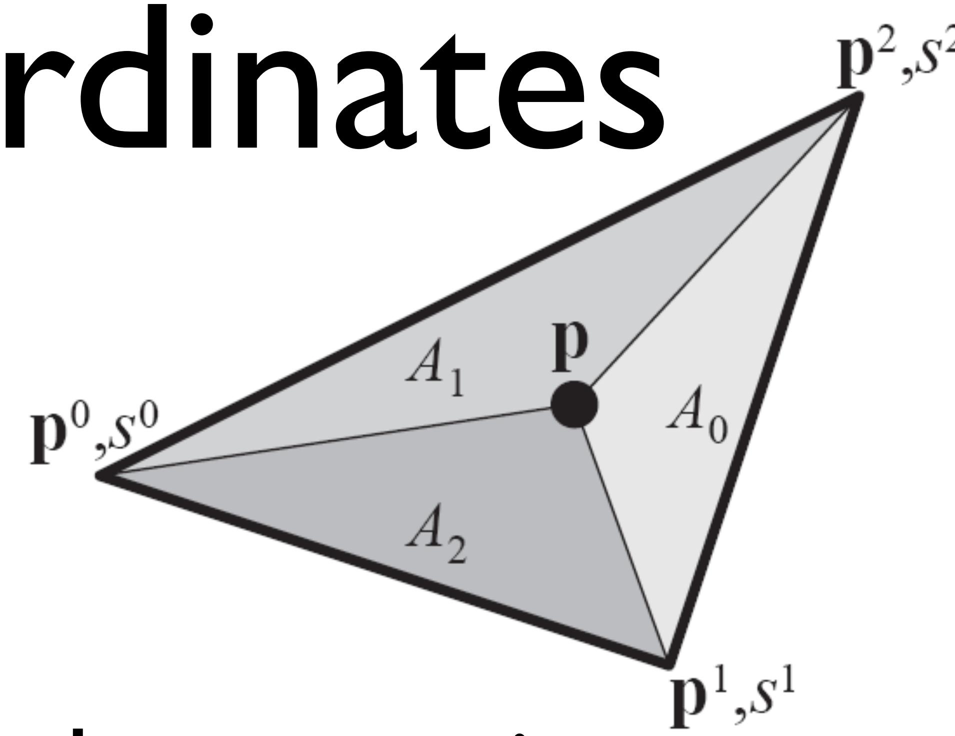
Perspective correct interpolation

$$\frac{s/w}{1/w} = \frac{sw}{w} = s$$



- An overly simplified way to think of it
- Linearly interpolate
  - s/w in screen space
  - 1/w in screen space
- Then divide

Perspective correct interpolation coordinates



- Compute perspective correct barycentric coordinates *(u,v,w)* first
- Then interpolate vertex parameters

$$s(p_x, p_y) = (1-u-v)s^0 + us^1 + vs^2 = s^0 + u(s^1 - s^0) + v(s^2 - s^0)$$

Perspectively correct barycentric coordinates

Recall perspective correction

$$u(p_x, p_y) = \frac{\hat{s}(p_x, p_y)}{\hat{o}(p_x, p_y)}$$

$$\begin{aligned} \hat{s}(p_x, p_y) &= (1 - \bar{u} - \bar{v})\frac{0}{h_w^0} + \bar{u}\frac{1}{h_w^1} + \bar{v}\frac{0}{h_w^2} \\ \hat{o}(p_x, p_y) &= (1 - \bar{u} - \bar{v})\frac{1}{h_w^0} + \bar{u}\frac{1}{h_w^1} + \bar{v}\frac{1}{h_w^2} \end{aligned}$$

Simplify: $$u(p_x, p_y) = \frac{\frac{e_1}{h_w^1}}{\frac{e_0}{h_w^0} + \frac{e_1}{h_w^1} + \frac{e_2}{h_w^2}}$$

$$\begin{aligned} u &= \frac{f_1}{f_0 + f_1 + f_2} \\ v &= \frac{f_2}{f_0 + f_1 + f_2} \end{aligned}$$

$$f_0 = \frac{e_0(x, y)}{h_w^0}, \qquad f_1 = \frac{e_1(x, y)}{h_w^1}, \qquad f_2 = \frac{e_2(x, y)}{h_w^2}$$

Once per triangle vs Once per pixel

Triangle setup

	Notation	Description
1	$a_i, b_i, c_i,\ i \in [0,1,2]$	Edge functions
2	$\frac{1}{2A_\Delta}$	Half reciprocal of triangle area
3	$\frac{1}{h_w^i}$	Reciprocal of w-coordinates

Per pixel (simple)

	Notation	Description
1	$e_i(x, y)$	Evaluate edge functions at (x, y)
2	$(\bar{u}, \bar{v})$	Barycentric coordinates (Equation 3.13)
3	$d(x, y)$	Per-pixel depth (Equation 3.14)
4	$f_i(x, y)$	Evaluation of per-pixel f-values (Equation 3.21)
5	(u, v)	Perspectively-correct interpolation coordinates (Equation 3.22)
6	$s(x, y)$	Interpolation of all desired parameters, s^i (Equation 3.15)

What's next

- Read chapter 2 & 3 in Graphics Hardware notes
  - Rasterization and interpolation
- Monday - Ray Tracing lab seminar