Contents of Lecture 8

- Purpose of Operator Strength Reduction, OSR
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Purpose of OSR

```
double  a[N];

for (i = 0; i < N; ++i)
        x += a[i];
```

```
double*     p = a;
double*     end = &a[N];

while (p < end)
        x += *p++;
```

- The most important purpose is to rewrite the code to the left into the code to the right.
- C/C++ compilers are required to make it possible to use the address of the array element **after** the last declared element.
- Typically, in total one extra byte might be wasted in memory due to this.
- It's **not** one extra byte per array but rather per memory segment.

Invalid C Code

```
double  a[N];

for (i = N-1; i >= 0; --i)
        x += a[i];
```

```
double*     p = &a[N];

while (--p >= a)
        x += *p;
```

- In the last iteration `p == a[-1]` in the comparison.
- The compiler is not required to make that address valid.
- The code to the right triggers undefined behavior.

Another Name for OSR

OSR is also known as Induction Variable Elimination

```
do {
    x = x + a[i];
    i = i + 1;
} while (i < N);
```

```
do {
    s = i * 4;
    t = load a+s;
    x = x + t;
    i = i + 1;
} while (i < N);
```

Basic och dependent IV

The primary goal is to get rid of the multiplication

```
do {
    s = i * 4;
    t = load a+s;
    x = x + t;
    i = i + 1;
} while (i < N);
```

- `i` is a *basic* induction variable
- Classes of *dependent* induction variables: $j \leftarrow b \times i + c$, i is a basic IV
- $s \leftarrow 4 \times i + 0$

Strength reduction

```
do {
    s = i * 4;
    t = load a+s;
    x = x + t;
    i = i + 1;

} while (i < N);
```

```
s = 4 * i;
do {

    t = load a+s;
    x = x + t;
    i = i + 1;
    s = s + 4;
} while (i < N);
```

- Initialize the dependent IV before the loop
- Increment the dependent IV just after the basic IV is incremented
- Maybe we can get rid of the basic IV now?

Linear function test replacement

```
s = 4 * i;
do {
    t = load a+s;
    x = x + t;
    i = i + 1;
    s = s + 4;
} while (i < N);
```

```
m = 4 * N;
s = 4 * i;
do {
    t = load a+s;
    x = x + t;
    s = s + 4;
} while (s < m);
```

- $s = i \times b + c$ (we have $b = 4$ and $c = 0$)
- $i = \frac{s-c}{b}$
- $i < N \Rightarrow \frac{s-c}{b} < N \Rightarrow s < N \times b + c$, if $b > 0$

Linear function test replacement

```
procedure operator_strength_reduce(ssa_graph)
    dfnum ← 0
    empty stack
    for each vertex v ∈ ssa_graph do
        visited(v) ← false
    for each vertex v ∈ ssa_graph do
        if (not visited(v))
            strong_connect(v)
end
```

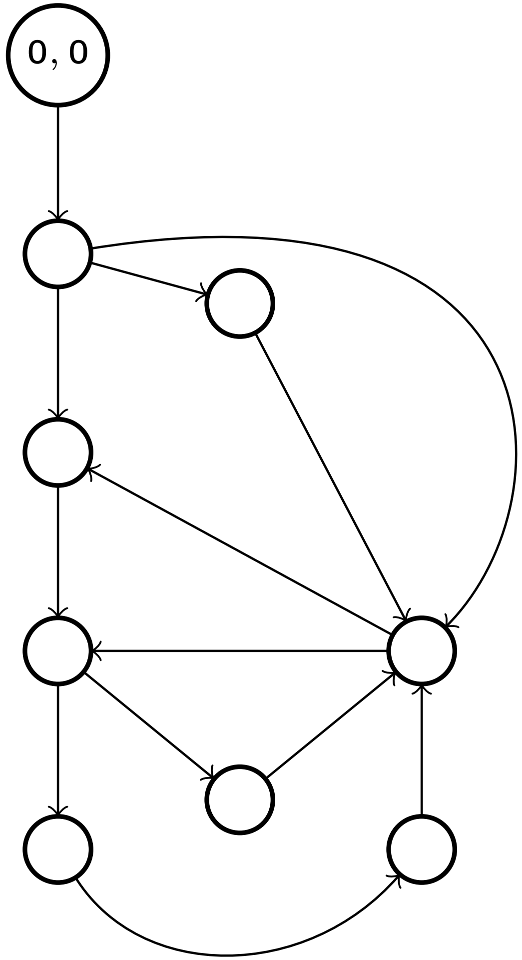
Tarjan's Algorithm: Initial Processing of 0

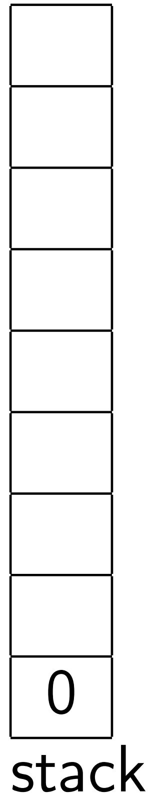
```
int     dfnum                              /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```





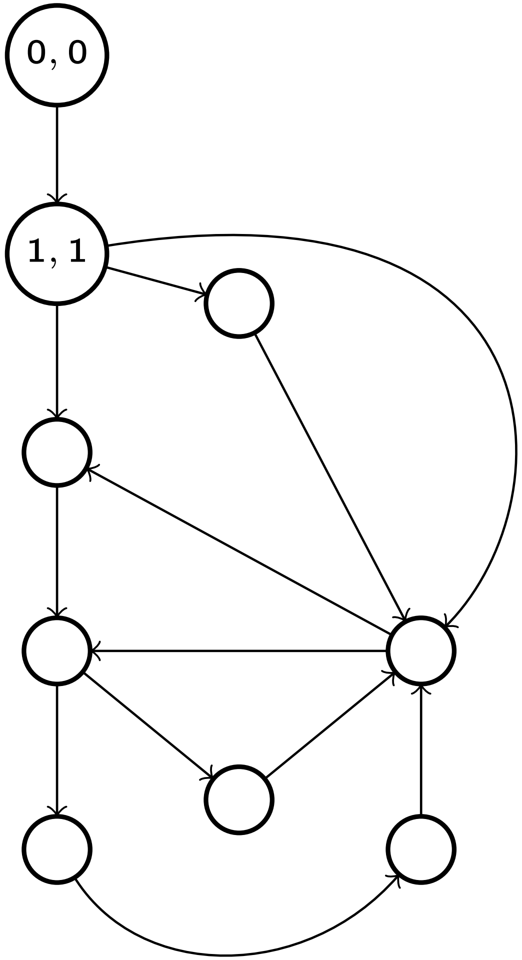
Tarjan's Algorithm: Initial Processing of 1

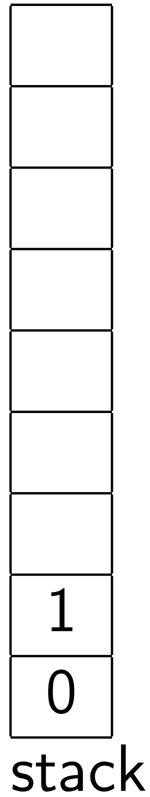
```
int     dfnum                                  /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```





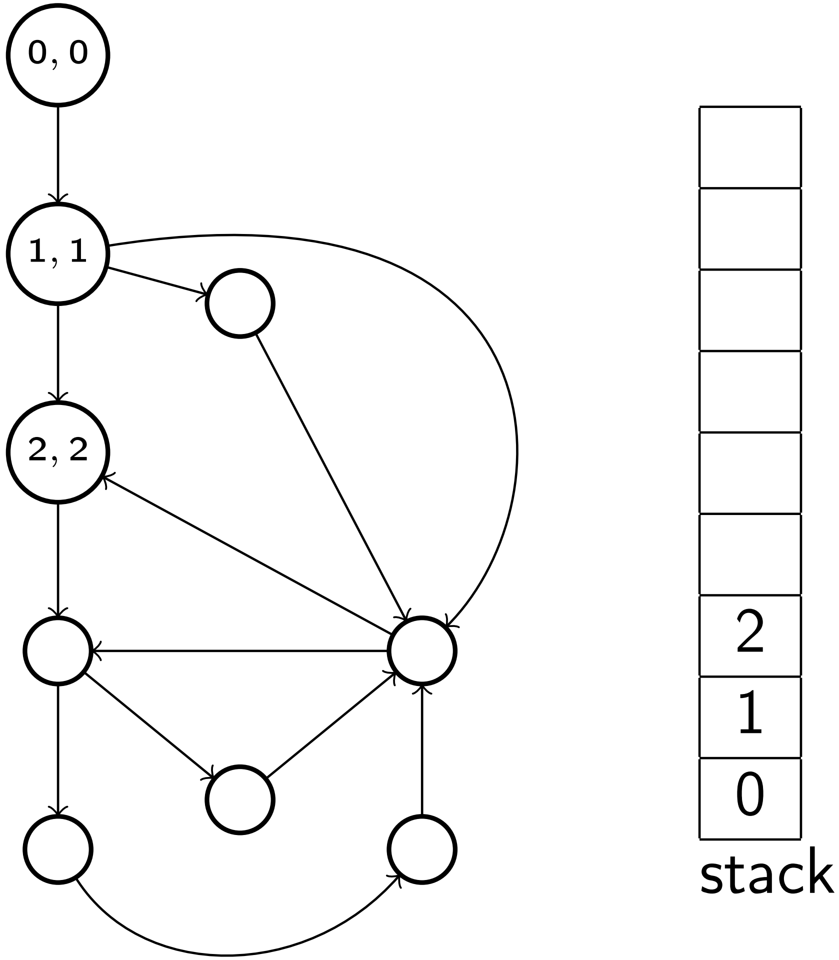
Tarjan's Algorithm: Initial Processing of 2

```
int   dfnum                          /* Depth-first search number. */

procedure strong_connect (v)
    dfn (v) ← dfnum
    lowlink (v) ← dfnum
    visited (v) ← true
    push (v)
    dfnum ← dfnum +1

    for each w ∈ succ(v) do /* operands(v) = succ(v) */
        if (not visited (w)) {
            strong_connect (w)
            lowlink (v) ← min (lowlink (v), lowlink (w))
        } else if (dfn (w) < dfn (v)  and w is on stack)
            lowlink (v) ← min (lowlink (v), dfn (w))

    if (lowlink (v) = dfn (v))
        scc ← ∅
        do
            w ← pop ()
            add w to scc
        while (w ≠ v)
        process_scc (scc)
end
```



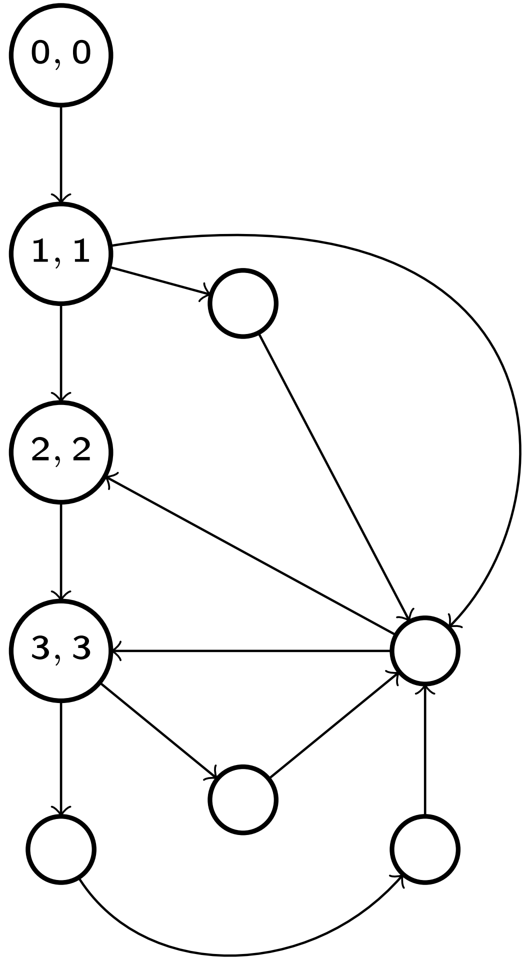
Tarjan's Algorithm: Initial Processing of 3

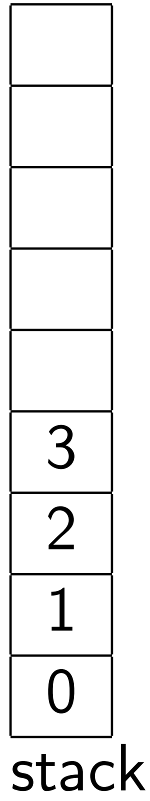
```
int     dfnum                                /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```





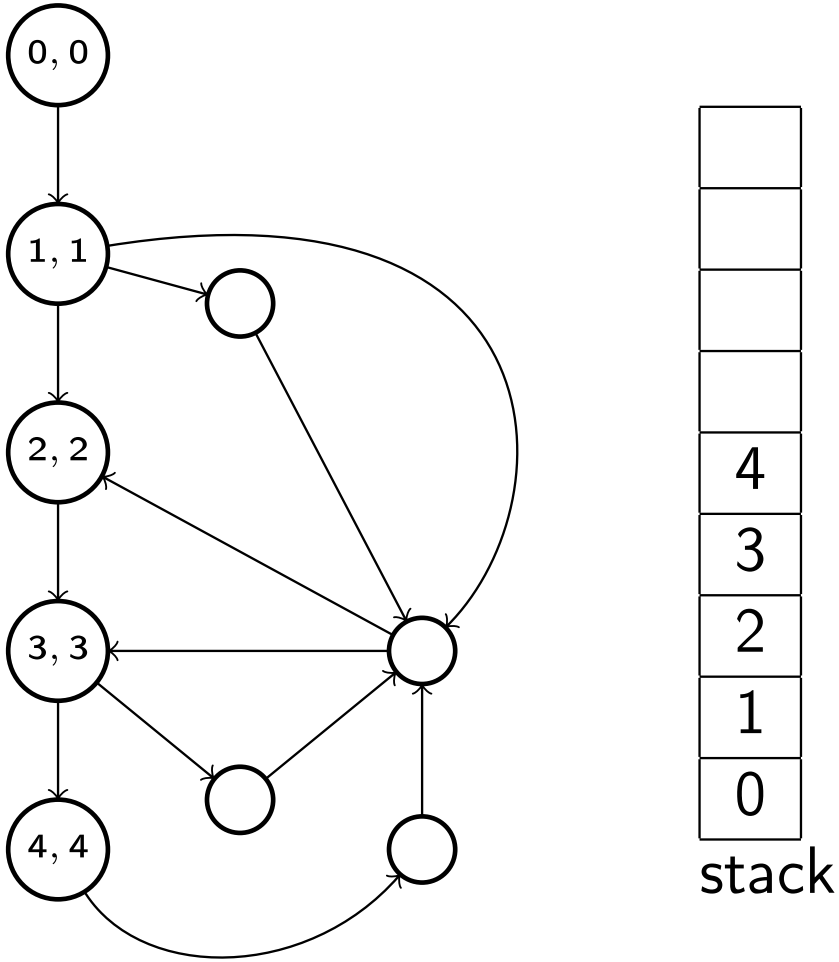
Tarjan's Algorithm: Initial Processing of 4

```
int    dfnum                              /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```



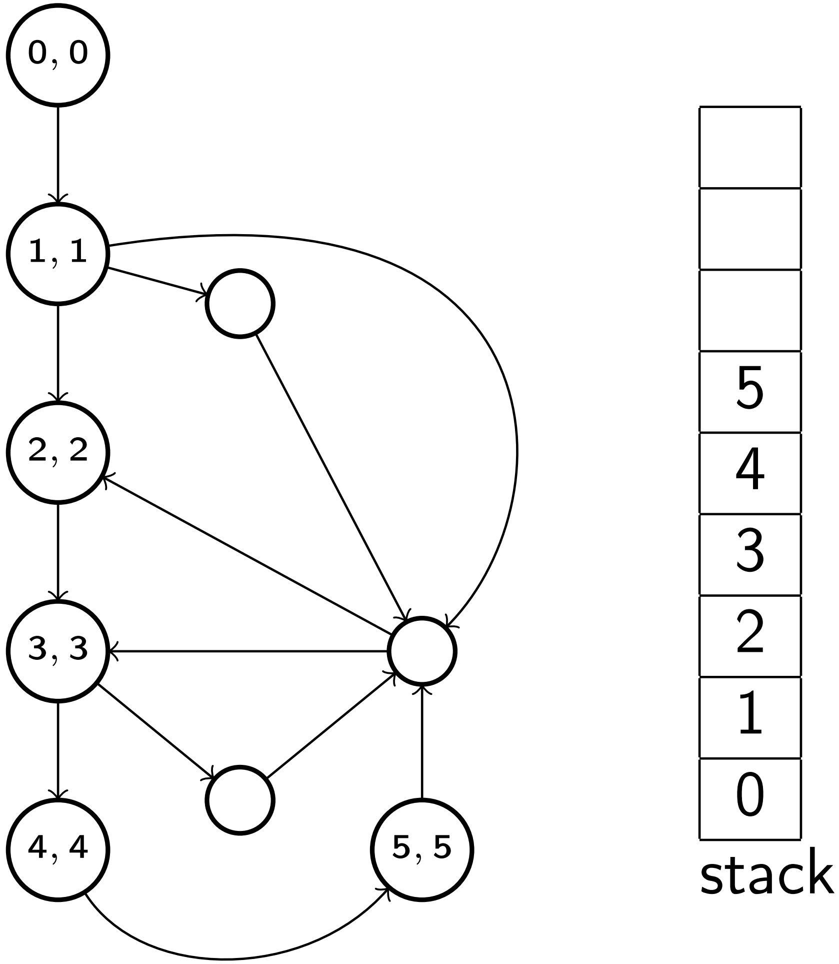
Tarjan's Algorithm: Initial Processing of 5

```
int     dfnum                          /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```



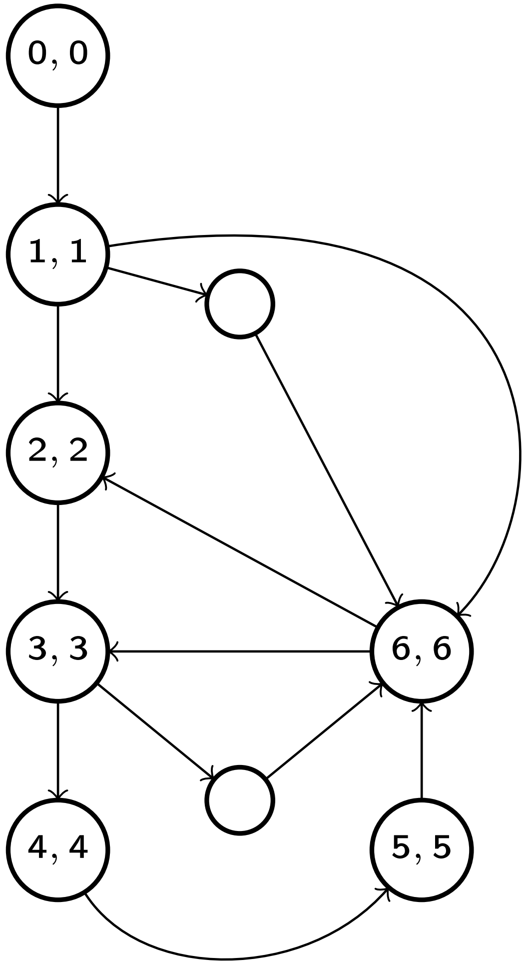
Tarjan's Algorithm: Initial Processing of 6

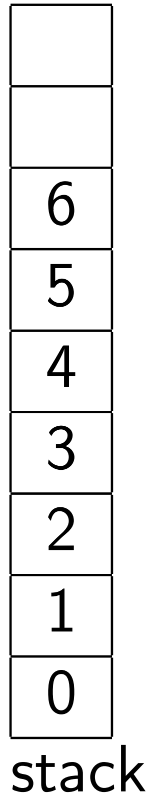
```
int     dfnum                          /* Depth-first search number. */

procedure strong_connect(v)
        dfn(v) ← dfnum
        lowlink(v) ← dfnum
        visited(v) ← true
        push(v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited(w)) {
                        strong_connect(w)
                        lowlink(v) ← min(lowlink(v), lowlink(w))
                } else if (dfn(w) < dfn(v)  and w is on stack)
                        lowlink(v) ← min(lowlink(v), dfn(w))

        if (lowlink(v) = dfn(v))
                scc ← ∅
                do
                        w ← pop()
                        add w to scc
                while (w ≠ v)
                process_scc(scc)
end
```





Tarjan's Algorithm: More Processing of 6

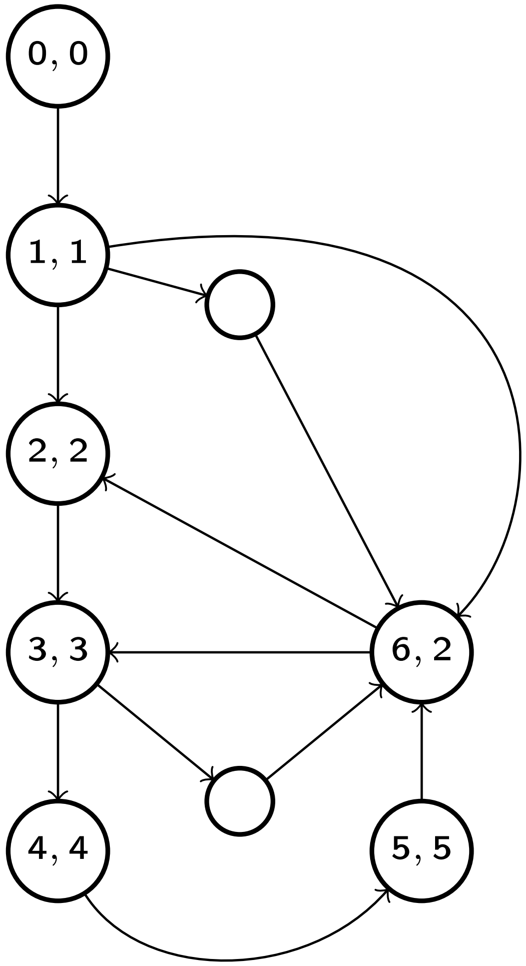
- $(6, 2) \Rightarrow$ 6 in same scc as 2.

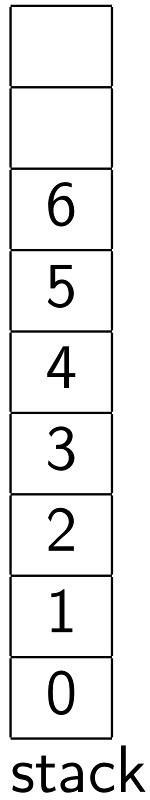
```
int     dfnum                              /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```





Tarjan's Algorithm: More Processing of 6

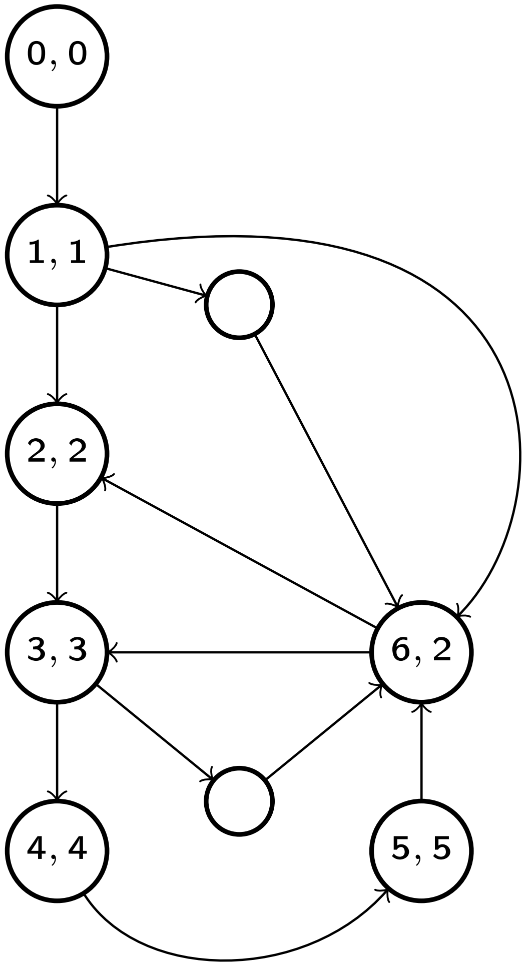
- (6, 3). no action.

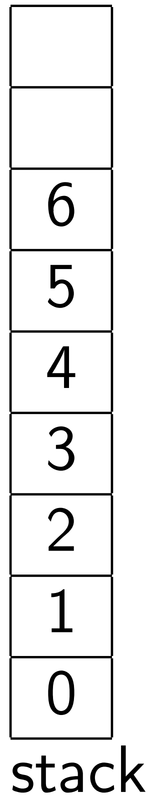
```
int       dfnum                    /* Depth-first search number. */

procedure strong_connect (v)
    dfn (v) ← dfnum
    lowlink (v) ← dfnum
    visited (v) ← true
    push (v)
    dfnum ← dfnum +1

    for each w ∈ succ(v) do /* operands(v) = succ(v) */
        if (not visited (w)) {
            strong_connect (w)
            lowlink (v) ← min (lowlink (v), lowlink (w))
        } else if (dfn (w) < dfn (v)  and w is on stack)
            lowlink (v) ← min (lowlink (v), dfn (w))

    if (lowlink (v) = dfn (v))
        scc ← ∅
        do
            w ← pop ()
            add w to scc
        while (w ≠ v)
        process_scc (scc )
end
```





Tarjan's Algorithm: More Processing of 6

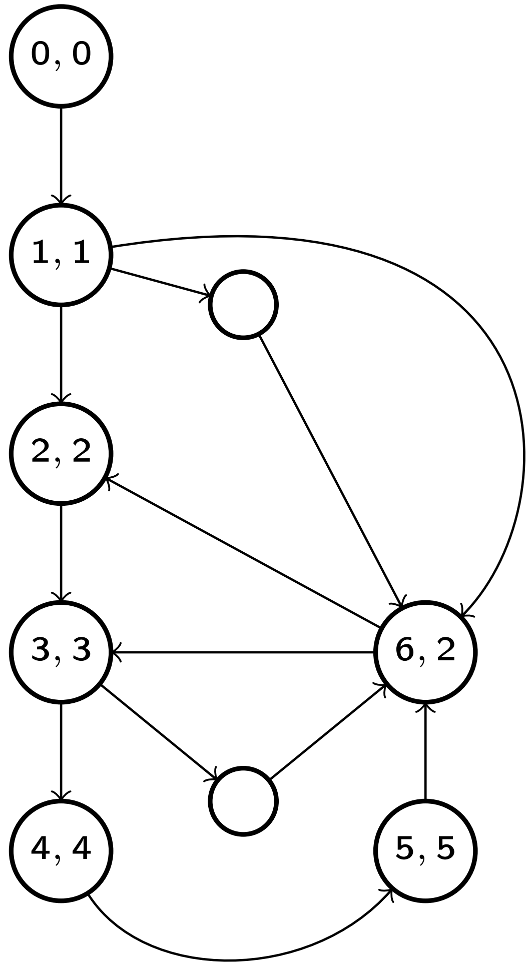
- 6 remains on the stack.

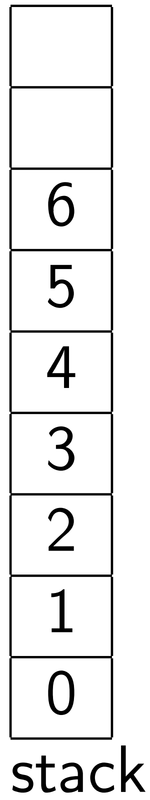
```
int     dfnum                          /* Depth-first search number. */

procedure strong_connect(v)
    dfn(v) ← dfnum
    lowlink(v) ← dfnum
    visited(v) ← true
    push(v)
    dfnum ← dfnum +1

    for each w ∈ succ(v) do /* operands(v) = succ(v) */
        if (not visited(w)) {
            strong_connect(w)
            lowlink(v) ← min(lowlink(v), lowlink(w))
        } else if (dfn(w) < dfn(v)  and w is on stack)
            lowlink(v) ← min(lowlink(v), dfn(w))

    if (lowlink(v) = dfn(v))
        scc ← ∅
        do
            w ← pop()
            add w to scc
        while (w ≠ v)
        process_scc(scc)
end
```





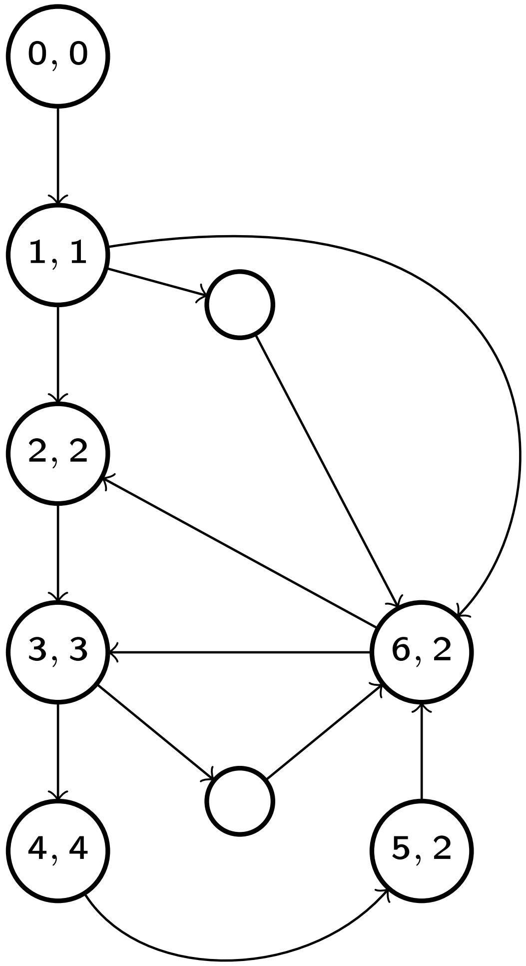
- New lowlink and remains.

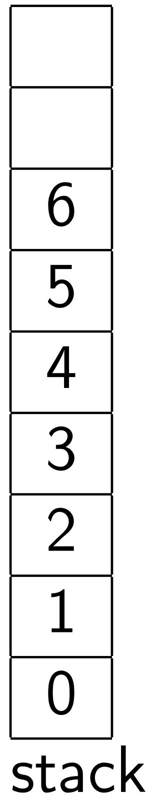
```
int     dfnum                          /* Depth-first search number. */

procedure strong_connect(v)
        dfn(v) ← dfnum
        lowlink(v) ← dfnum
        visited(v) ← true
        push(v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited(w)) {
                        strong_connect(w)
                        lowlink(v) ← min(lowlink(v), lowlink(w))
                } else if (dfn(w) < dfn(v)  and w is on stack)
                        lowlink(v) ← min(lowlink(v), dfn(w))

        if (lowlink(v) = dfn(v))
                scc ← ∅
                do
                        w ← pop()
                        add w to scc
                while (w ≠ v)
                process_scc(scc)
end
```





Tarjan's Algorithm: More Processing of 4

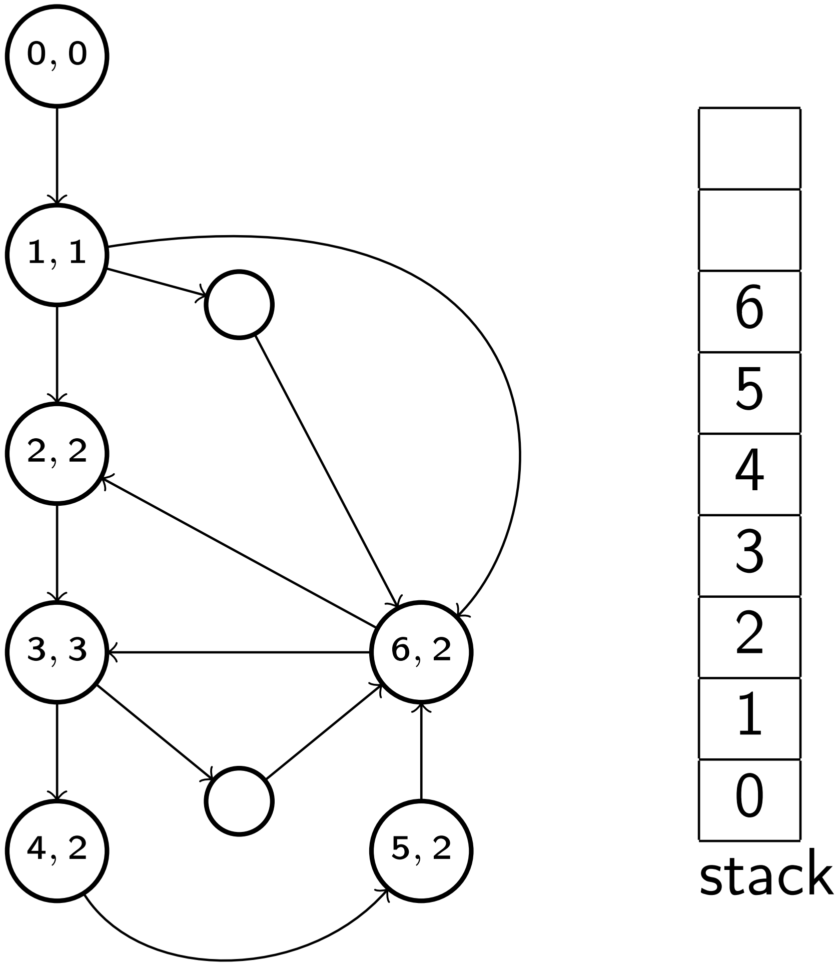
- New lowlink and remains.

```
int     dfnum                          /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc )
end
```



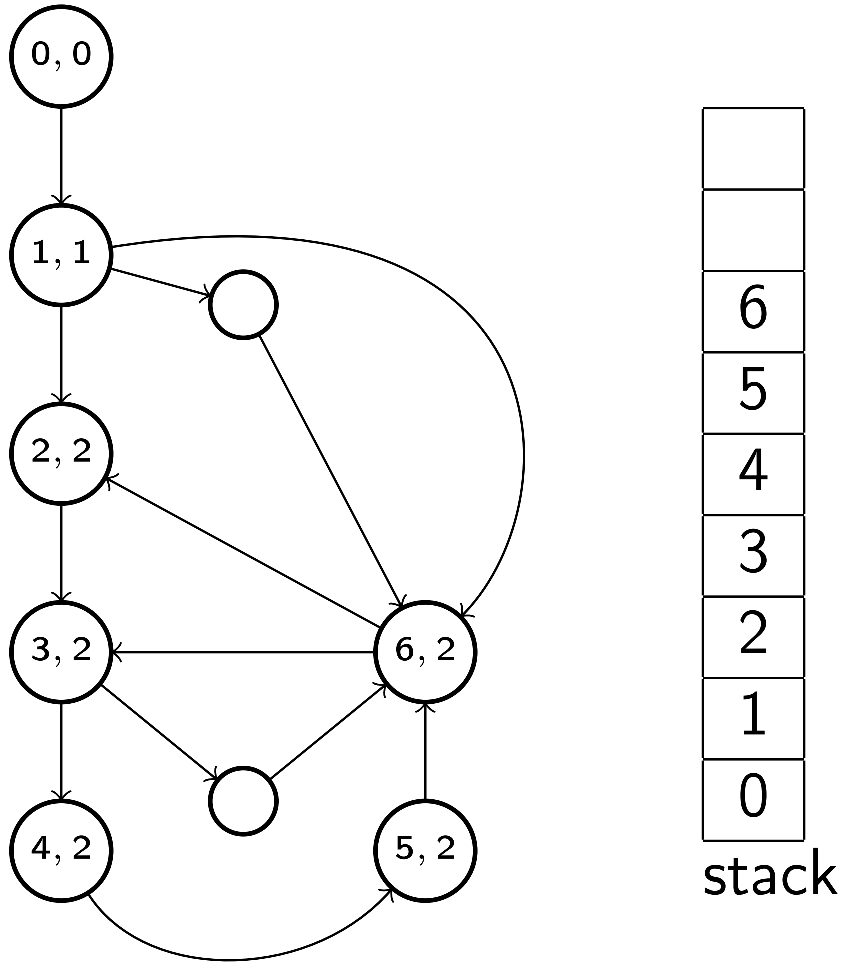
- New lowlink. Next 7.

```
int     dfnum                              /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```



Tarjan's Algorithm: Processing of 7

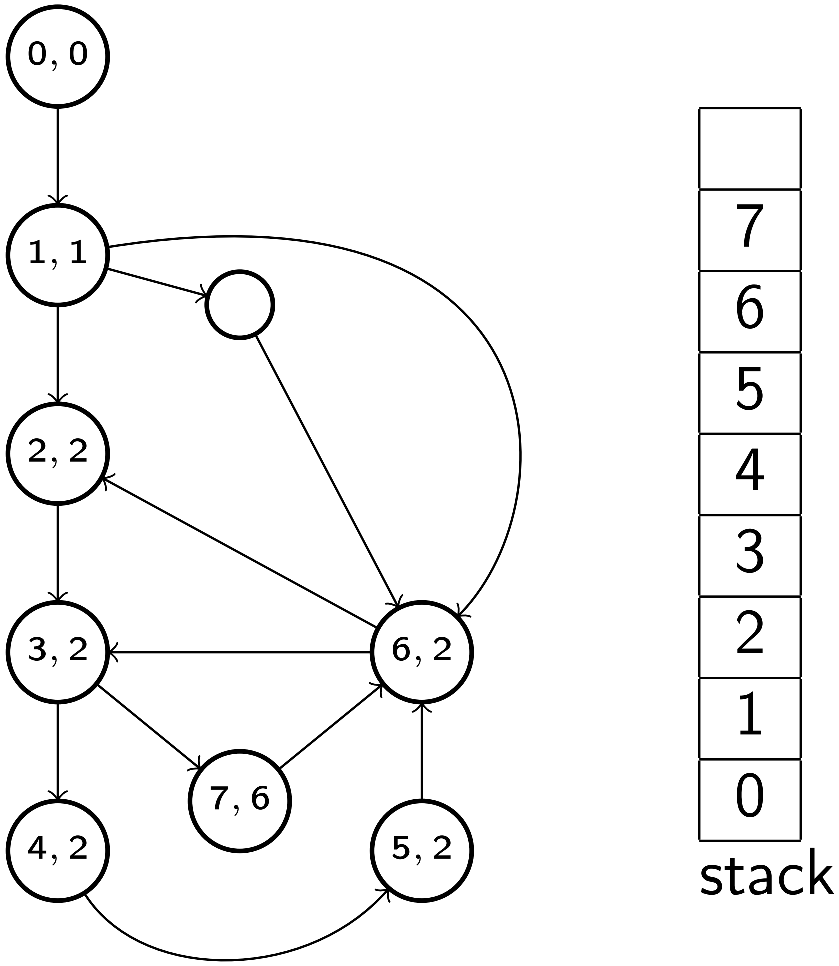
- Lowlink is set.

```
int    dfnum                          /* Depth-first search number. */

procedure strong_connect (v)
    dfn (v) ← dfnum
    lowlink (v) ← dfnum
    visited (v) ← true
    push (v)
    dfnum ← dfnum +1

    for each w ∈ succ(v) do /* operands(v) = succ(v) */
        if (not visited (w)) {
            strong_connect (w)
            lowlink (v) ← min (lowlink (v), lowlink (w))
        } else if (dfn (w) < dfn (v)  and w is on stack)
            lowlink (v) ← min (lowlink (v), dfn (w))

    if (lowlink (v) = dfn (v))
        scc ← ∅
        do
            w ← pop ()
            add w to scc
        while (w ≠ v)
        process_scc (scc)
end
```



Tarjan's Algorithm: More Processing of 2

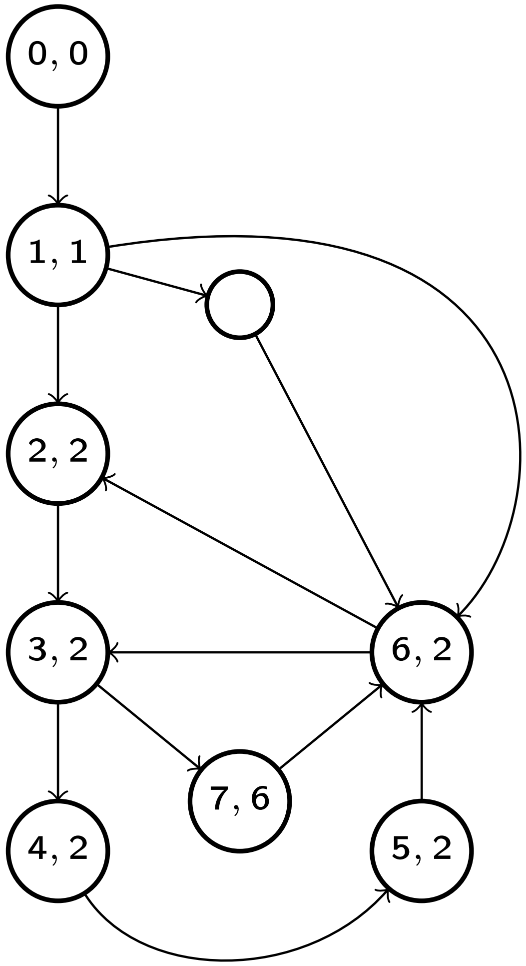
- Remove SCC from stack

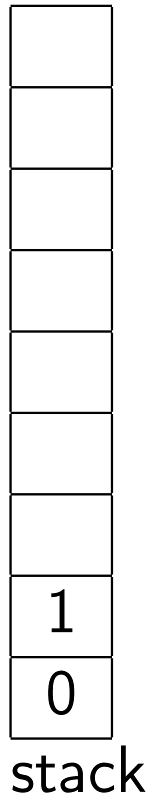
```
int     dfnum                          /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc )
end
```





Tarjan's Algorithm: Initial Processing of 8

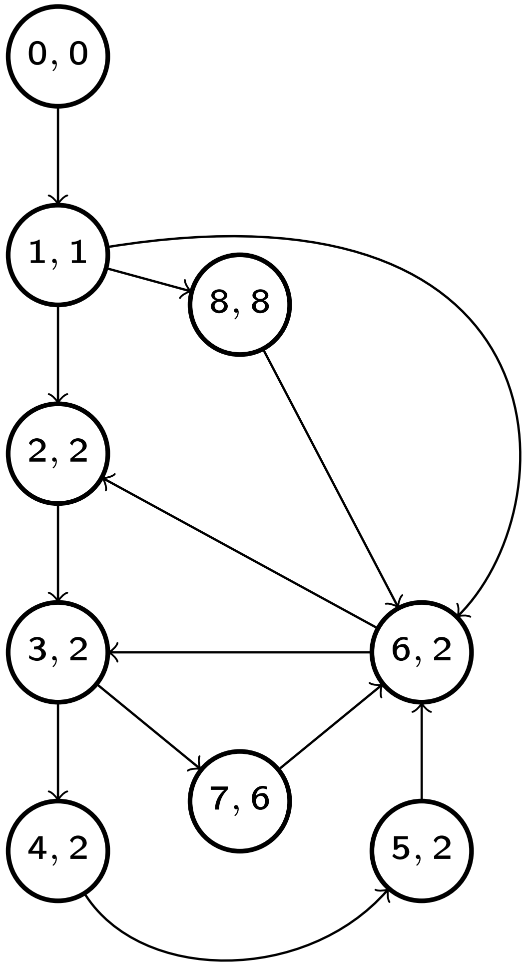
- No path from 2 to 8.

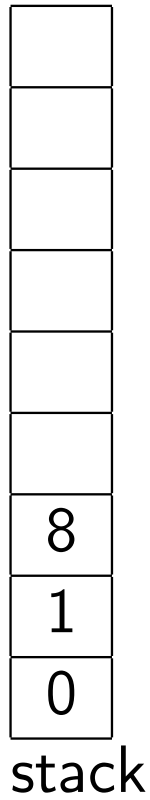
```
int   dfnum                            /* Depth-first search number. */

procedure strong_connect (v)
    dfn (v) ← dfnum
    lowlink (v) ← dfnum
    visited (v) ← true
    push (v)
    dfnum ← dfnum +1

    for each w ∈ succ(v) do /* operands(v) = succ(v) */
        if (not visited (w)) {
            strong_connect (w)
            lowlink (v) ← min (lowlink (v), lowlink (w))
        } else if (dfn (w) < dfn (v)  and w is on stack)
            lowlink (v) ← min (lowlink (v), dfn (w))

    if (lowlink (v) = dfn (v))
        scc ← ∅
        do
            w ← pop ()
            add w to scc
        while (w ≠ v)
        process_scc (scc )
end
```





Tarjan's Algorithm: More Processing of 8

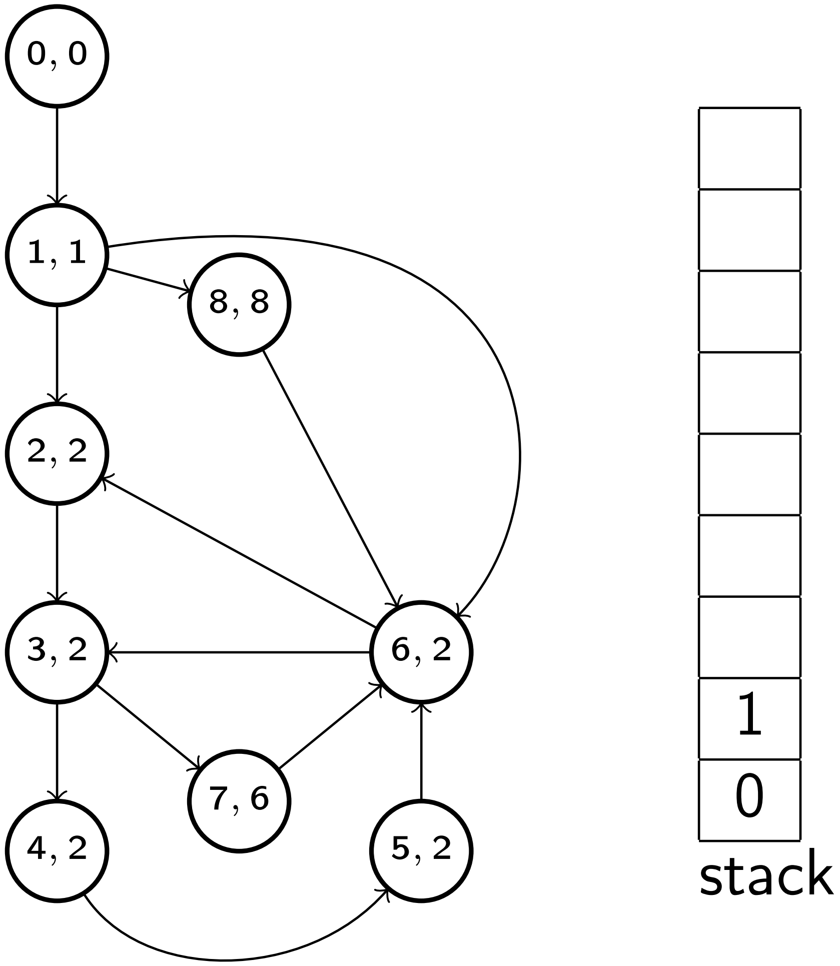
- 8 is its own SCC.

```
int     dfnum                            /* Depth-first search number. */

procedure strong_connect(v)
        dfn(v) ← dfnum
        lowlink(v) ← dfnum
        visited(v) ← true
        push(v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited(w)) {
                        strong_connect(w)
                        lowlink(v) ← min(lowlink(v), lowlink(w))
                } else if (dfn(w) < dfn(v) and w is on stack)
                        lowlink(v) ← min(lowlink(v), dfn(w))

        if (lowlink(v) = dfn(v))
                scc ← ∅
                do
                        w ← pop()
                        add w to scc
                while (w ≠ v)
                process_scc(scc)
end
```



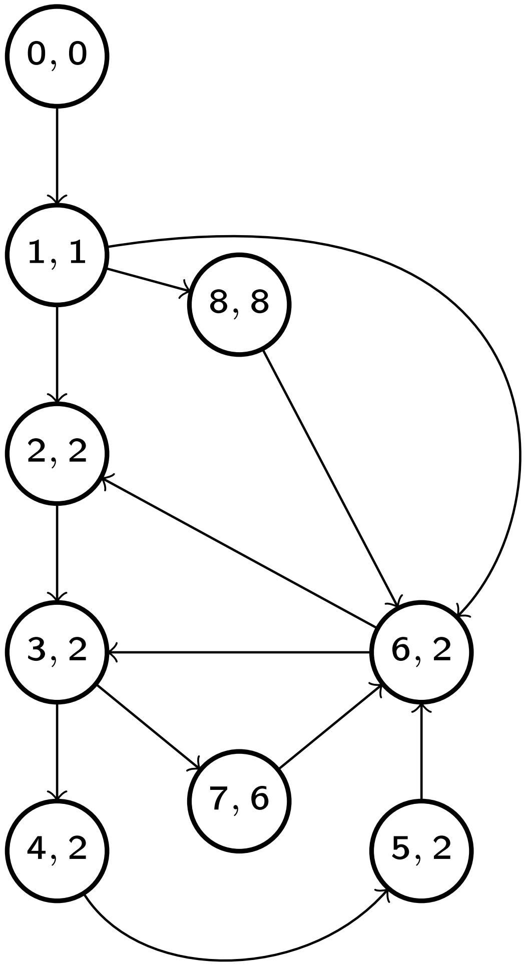
- 1 is its own SCC.

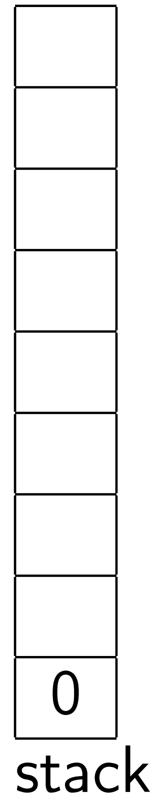
```
int     dfnum                       /* Depth-first search number. */

procedure strong_connect (v)
        dfn (v) ← dfnum
        lowlink (v) ← dfnum
        visited (v) ← true
        push (v)
        dfnum ← dfnum +1

        for each w ∈ succ(v) do /* operands(v) = succ(v) */
                if (not visited (w)) {
                        strong_connect (w)
                        lowlink (v) ← min (lowlink (v), lowlink (w))
                } else if (dfn (w) < dfn (v)  and w is on stack)
                        lowlink (v) ← min (lowlink (v), dfn (w))

        if (lowlink (v) = dfn (v))
                scc ← ∅
                do
                        w ← pop ()
                        add w to scc
                while (w ≠ v)
                process_scc (scc)
end
```





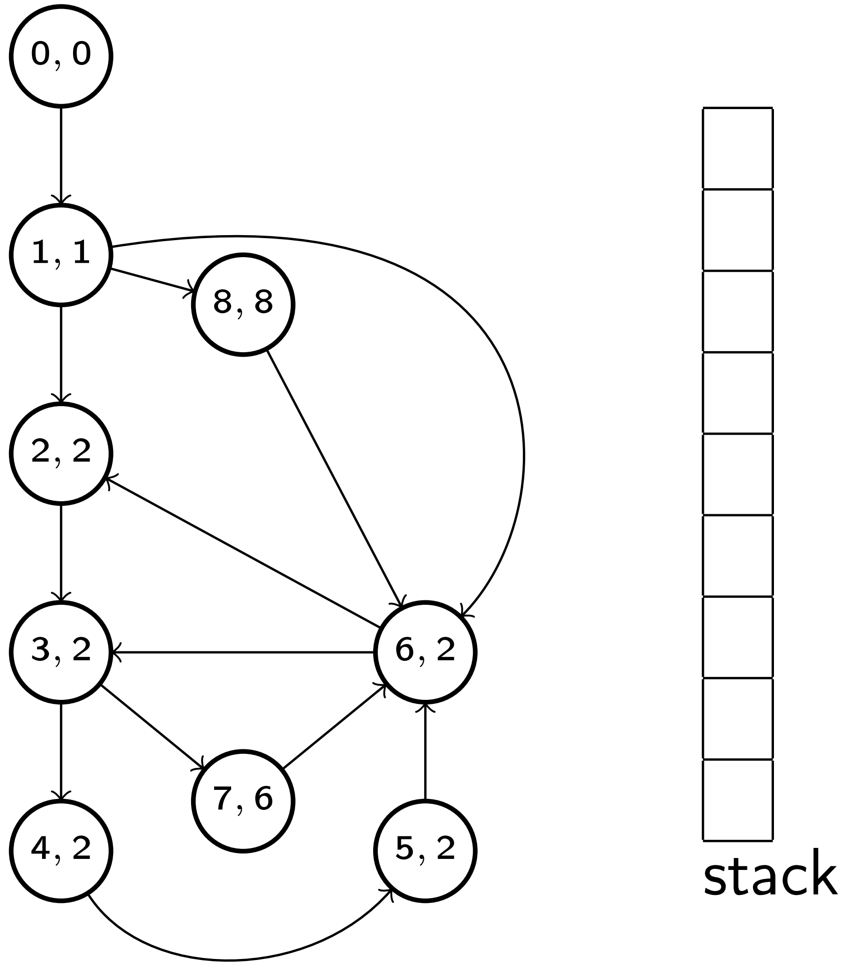
- 0 is its own SCC.

```
int     dfnum                          /* Depth-first search number. */

procedure strong_connect (v)
    dfn (v) ← dfnum
    lowlink (v) ← dfnum
    visited (v) ← true
    push (v)
    dfnum ← dfnum +1

    for each w ∈ succ(v) do /* operands(v) = succ(v) */
        if (not visited (w)) {
            strong_connect (w)
            lowlink (v) ← min (lowlink (v), lowlink (w))
        } else if (dfn (w) < dfn (v)  and w is on stack)
            lowlink (v) ← min (lowlink (v), dfn (w))

    if (lowlink (v) = dfn (v))
        scc ← ∅
        do
            w ← pop ()
            add w to scc
        while (w ≠ v)
        process_scc (scc )
end
```

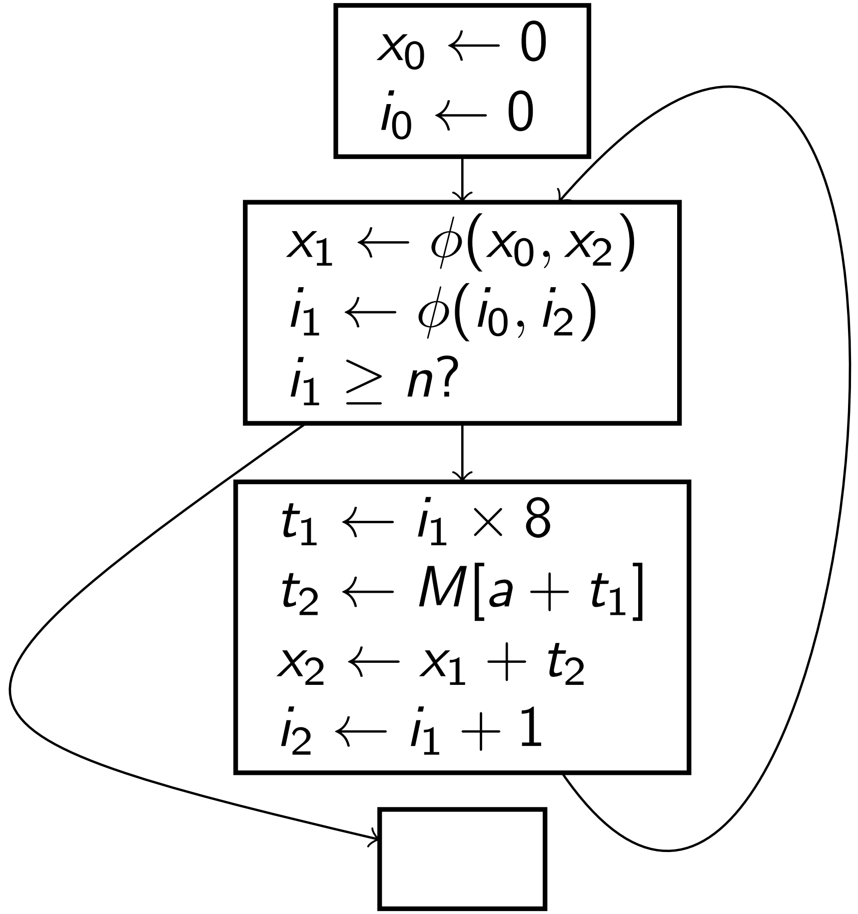


Tarjan's Algorithm: Remarks

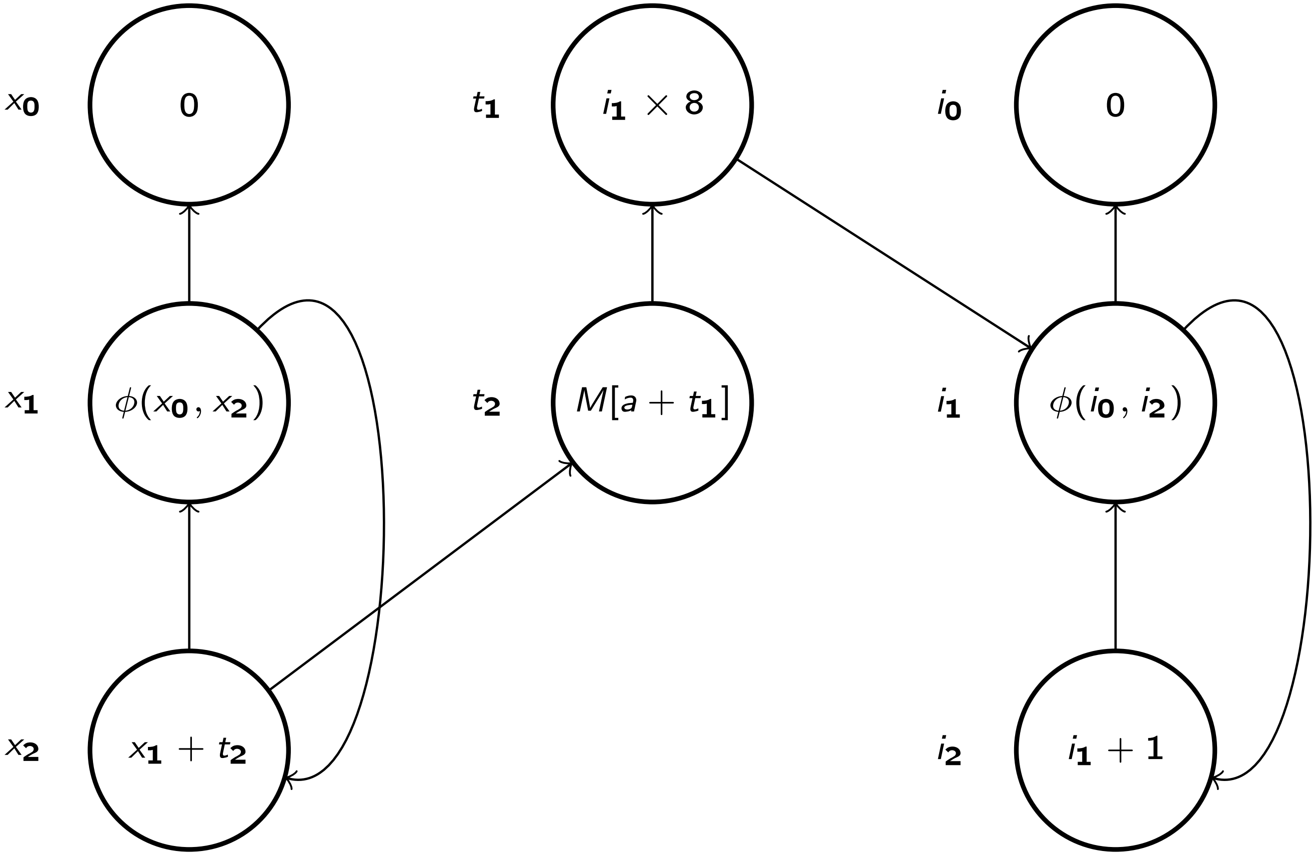
- Consider the edge (v, w).
- When w is not yet visited we must visit it by calling $strong\_connect(w)$.
- If w has been visited, we have two main cases:
  1. w is not on the stack, because it has already found its SCC.
  2. w is on the stack, because it's waiting for being popped.
     - If $dfn(w) < dfn(v)$ then v must set it's lowlink so it does not think it is its own SCC.
     - If $dfn(w) \geq dfn(v)$ then no more information for v is available. There is another path from v to w due to which they will belong to the same SCC.

```
double  a[N];

for (i = 0; i < N; ++i)
        x += a[i];
```

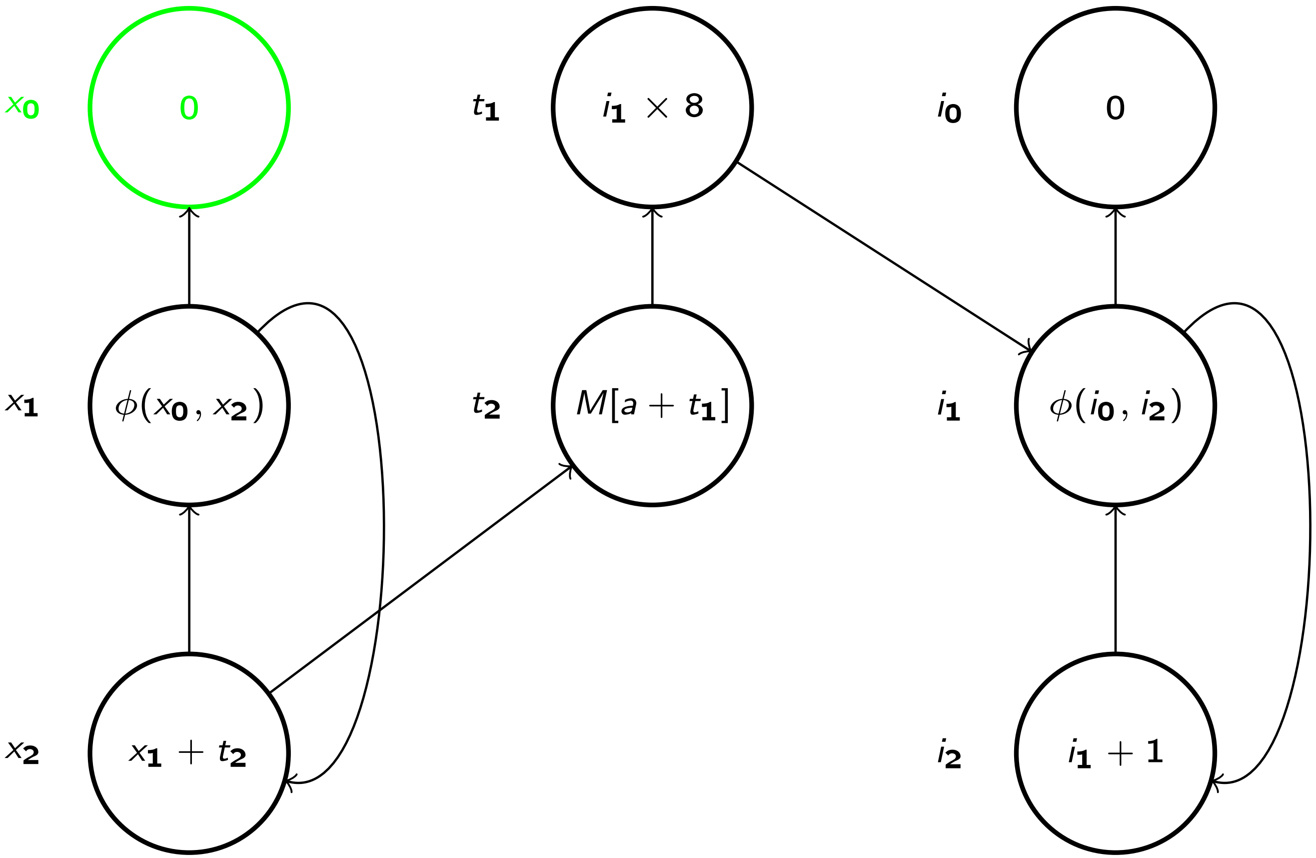


The SSA Graph of Loop



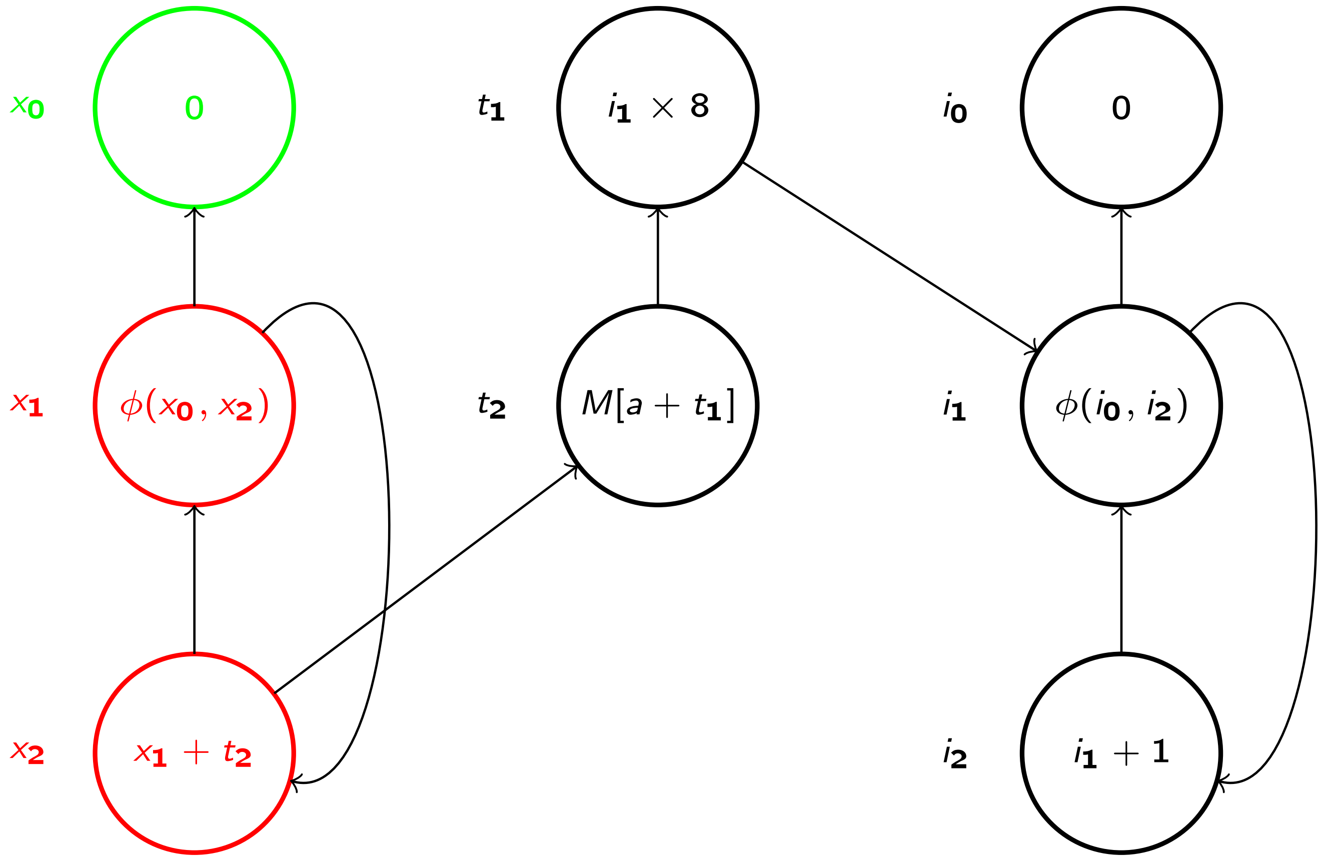
- We first find all strongly connected components of the SSA graph.
- We want to copy the SCC of i and modify the copy for t_1.
- Therefore we want to have processed i before processing t_1.
- Let us start with x.

Processing of x_0



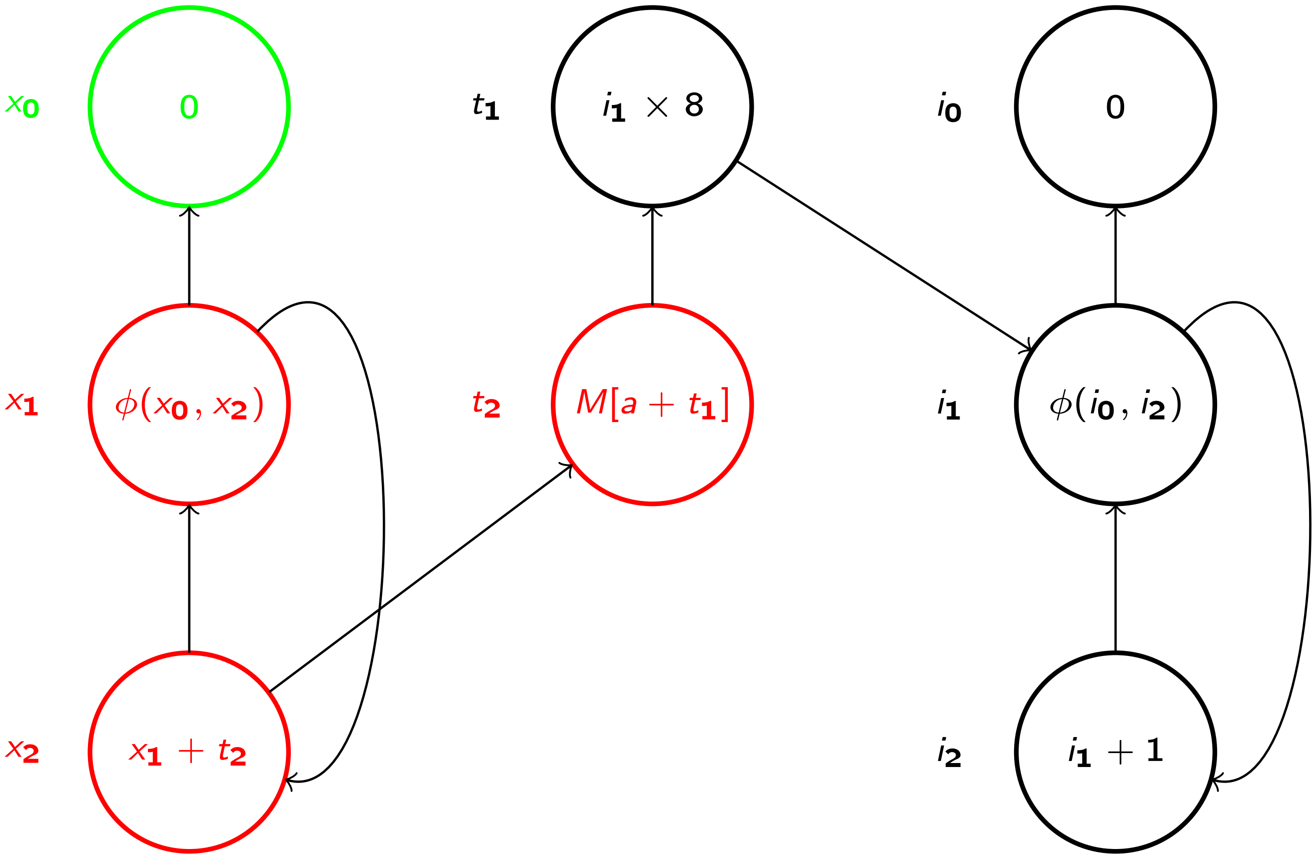
- $SCC_0 = \{x_0\}$. Empty stack.
- Nodes processed in a SCC are green.
- Next processing x_1.

Processing of x_1 and x_2



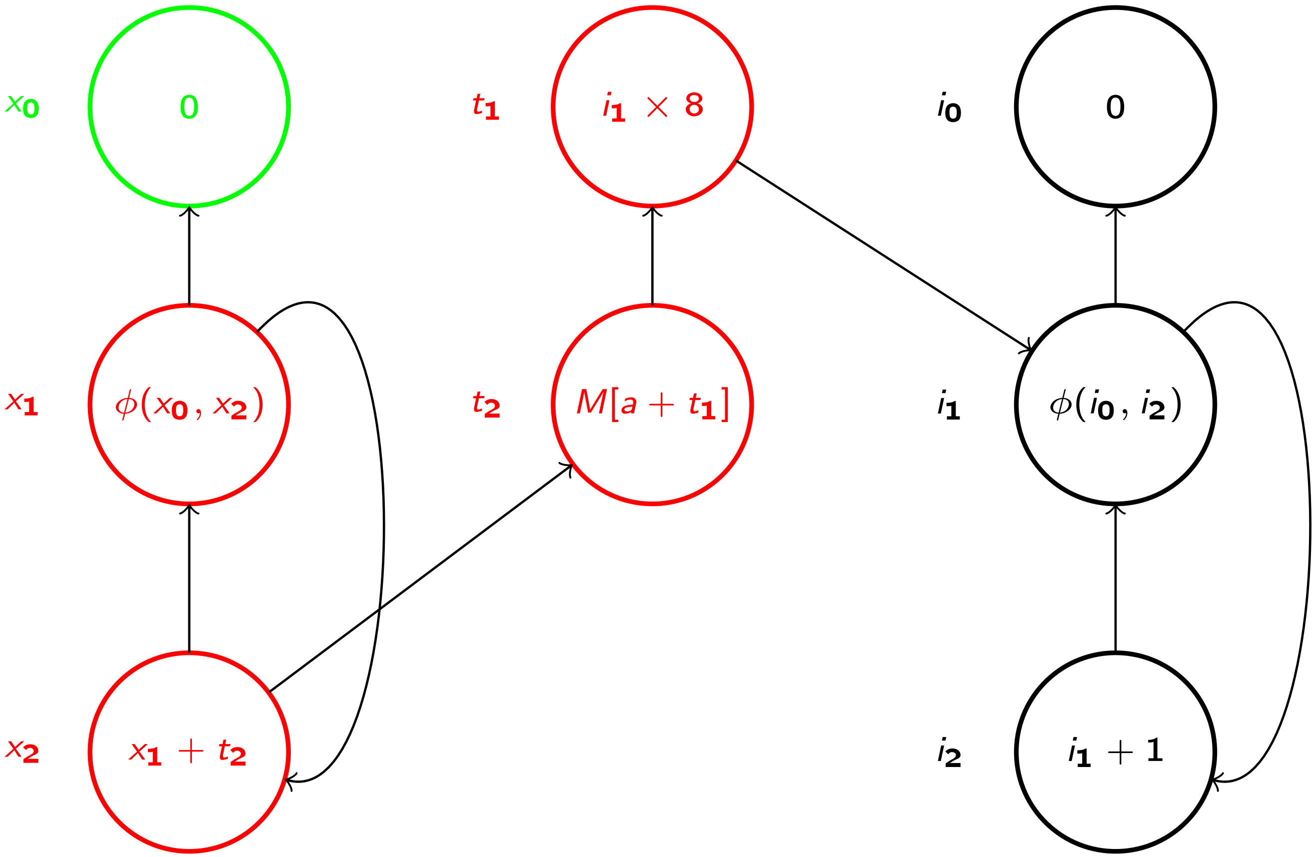
- x_1 and x_2 are pushed and then the search continues with t_2.
- Nodes on the stack are red.
- Next processing t_2.

Processing of t_2



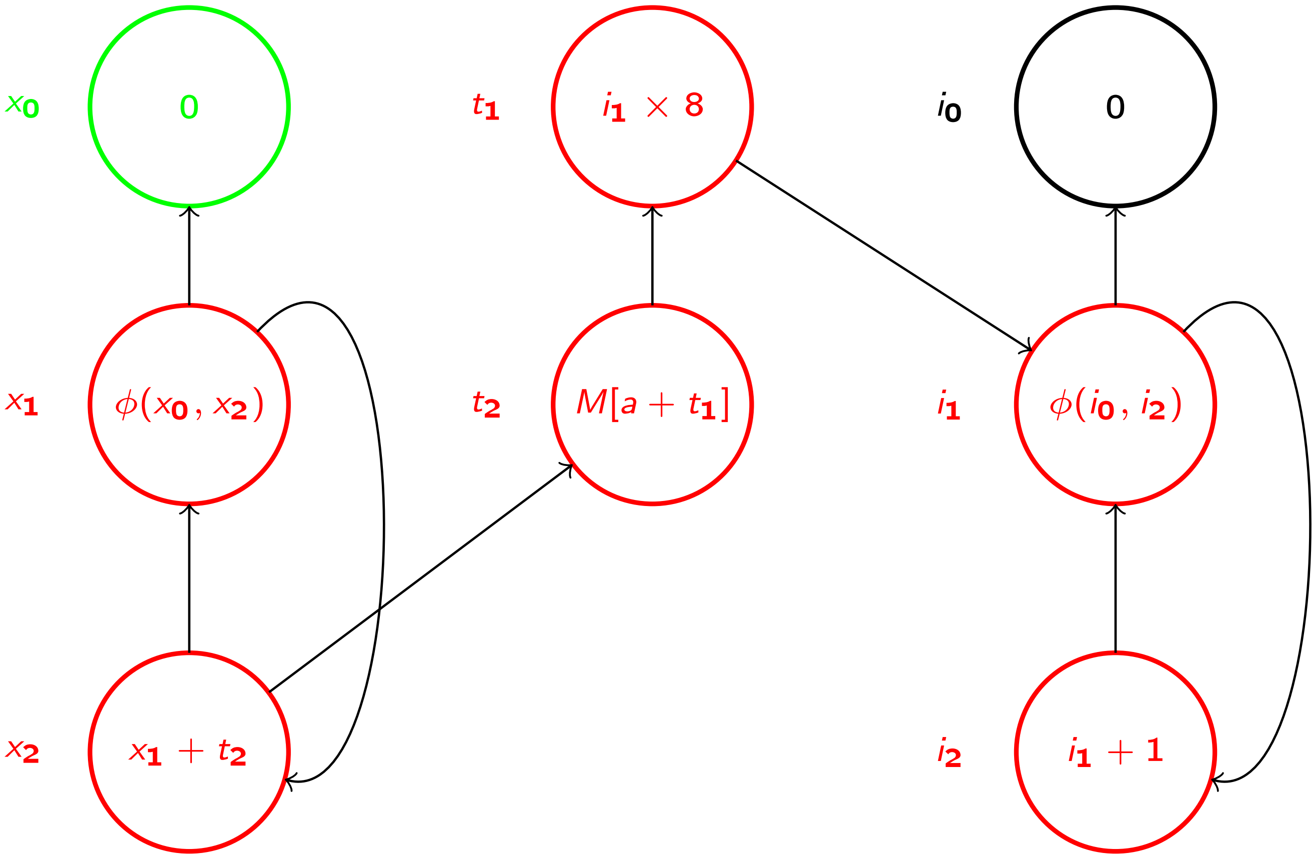
- Next processing t_1.

Processing of t_1



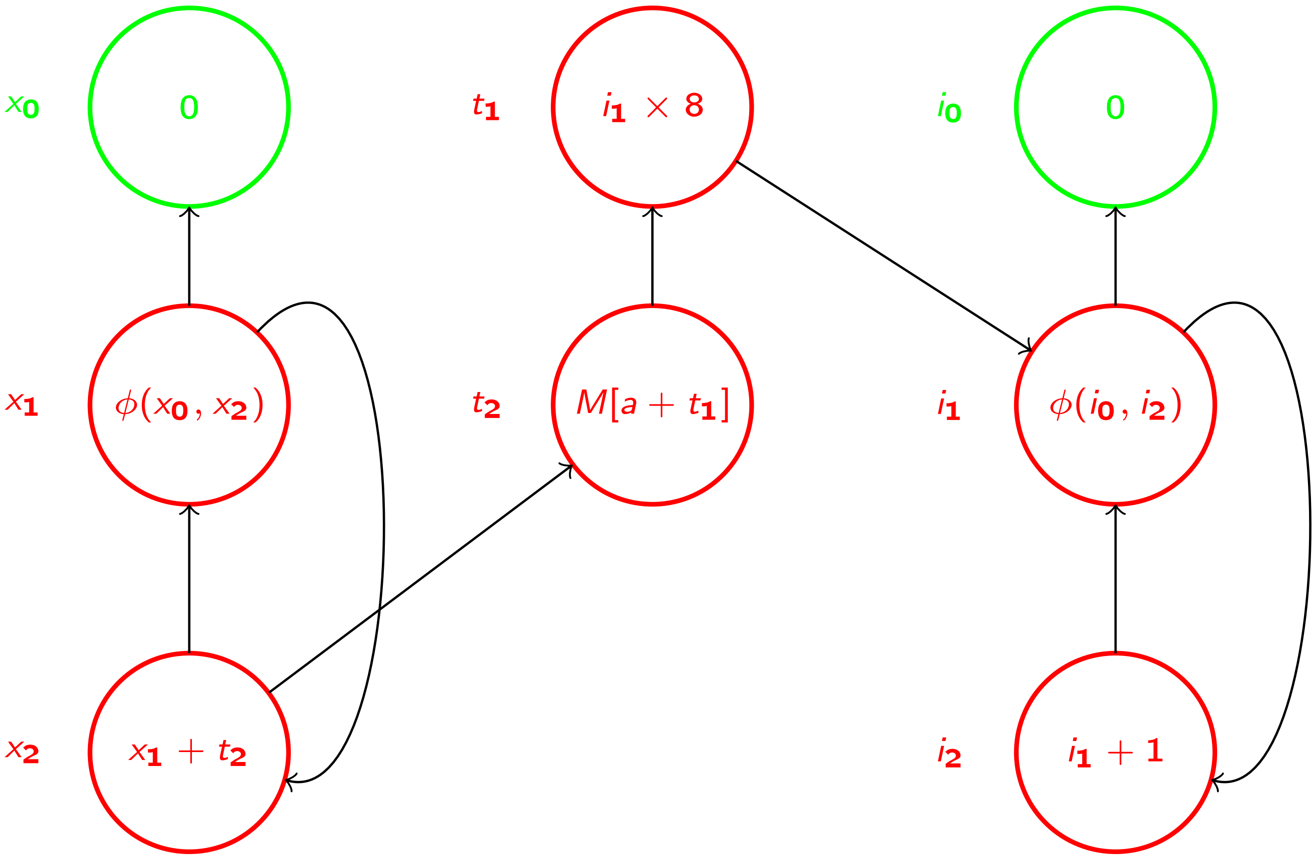
- Next processing i_2.

Processing of i_2 and i_1

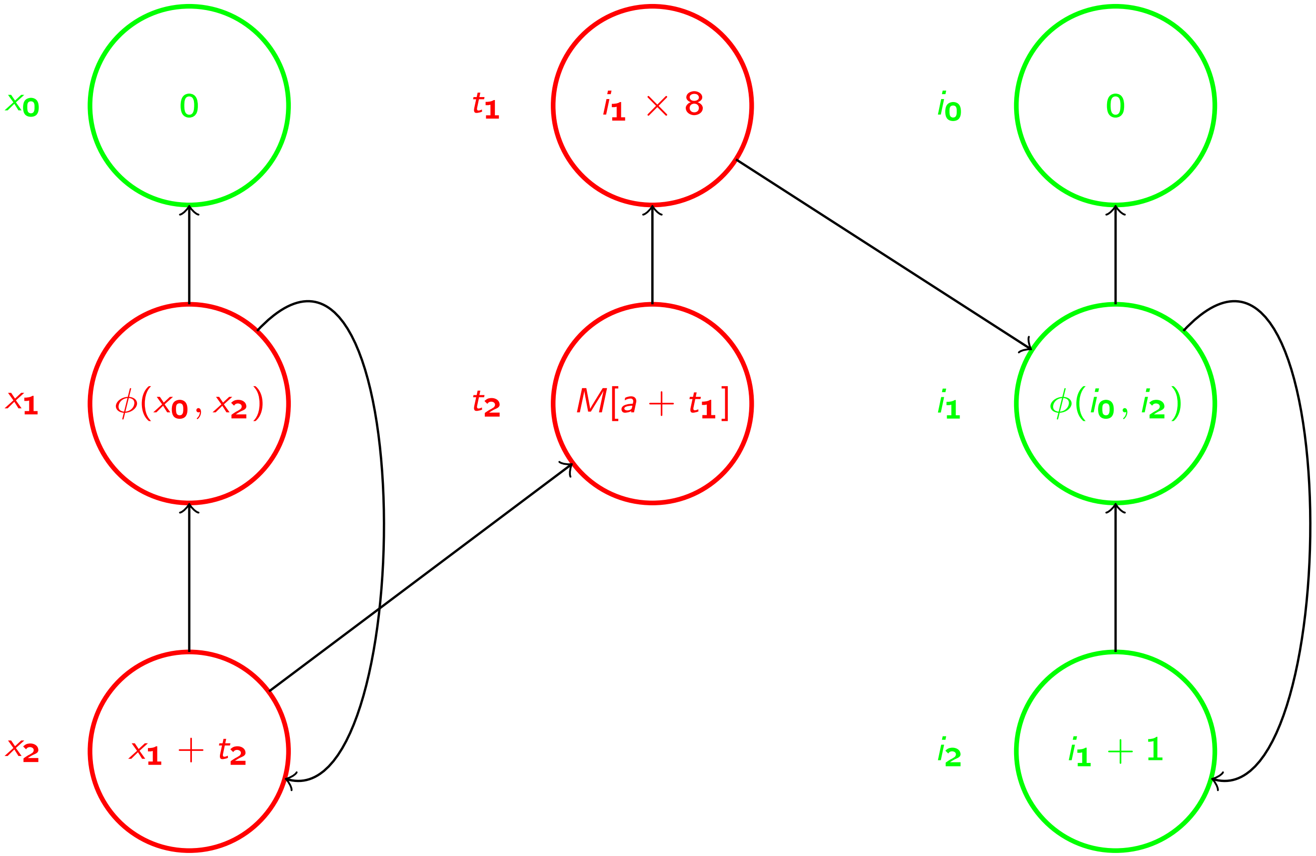


- Next processing i_0.

Processing of i_0

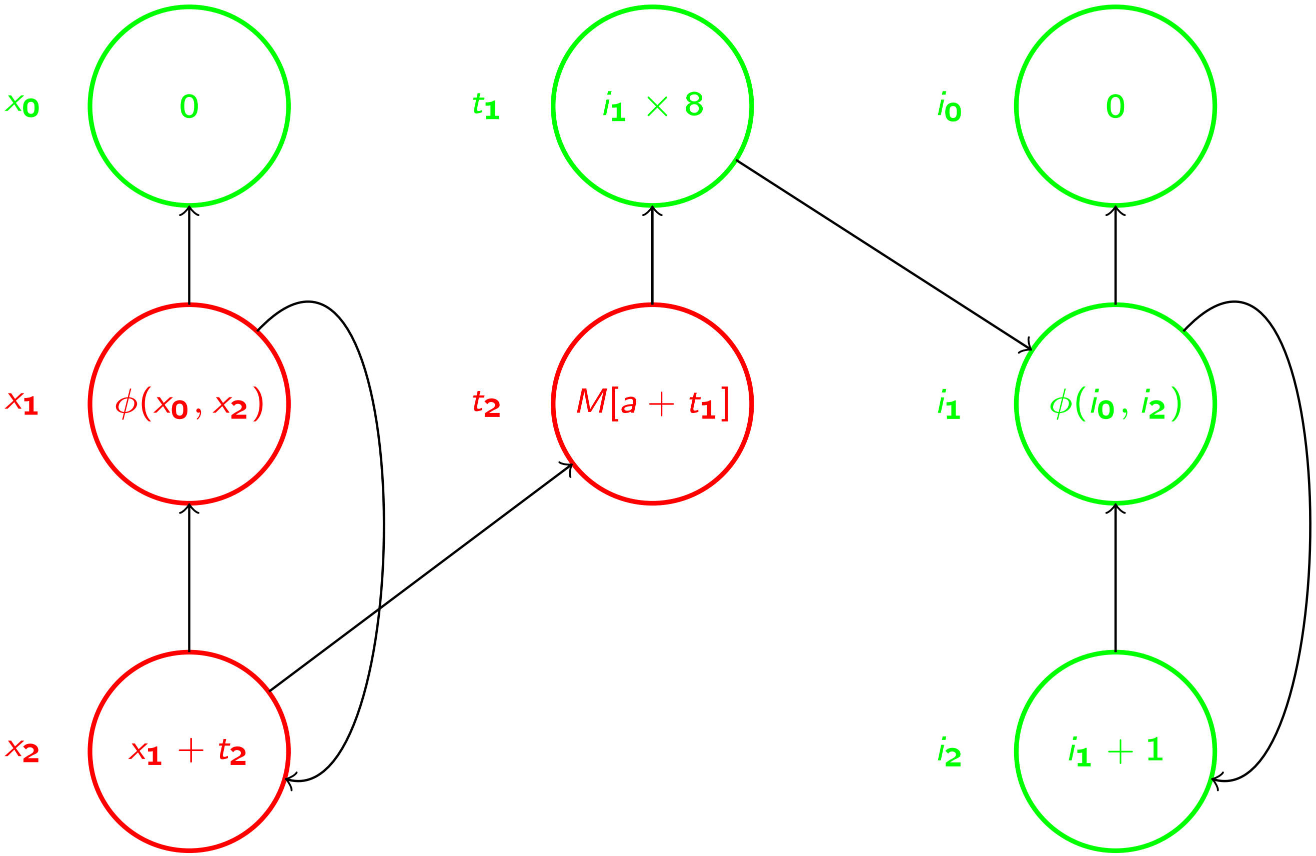


- $SCC_1 = \{i_0\}$
- Next more processing in i_2.



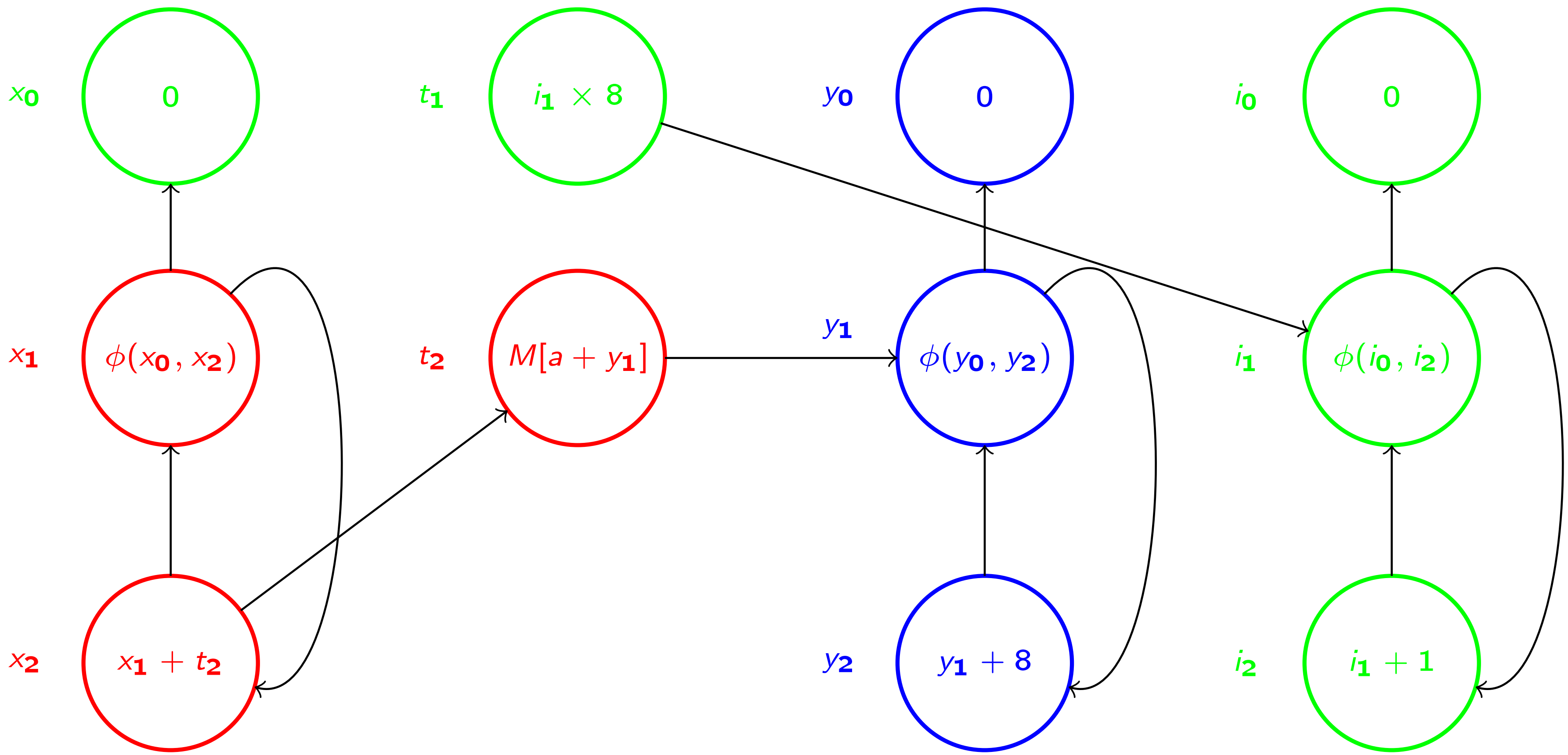
- $SCC_2 = \{i_1, i_2\}$
- SCC_2 is an **induction variable** due it consists of a ϕ-function and an add with a **region constant**.
- A region constant is not modified in a loop, i.e. it's a number or its definition strictly dominates the loop header.

Replacing $i_1 \times 8$



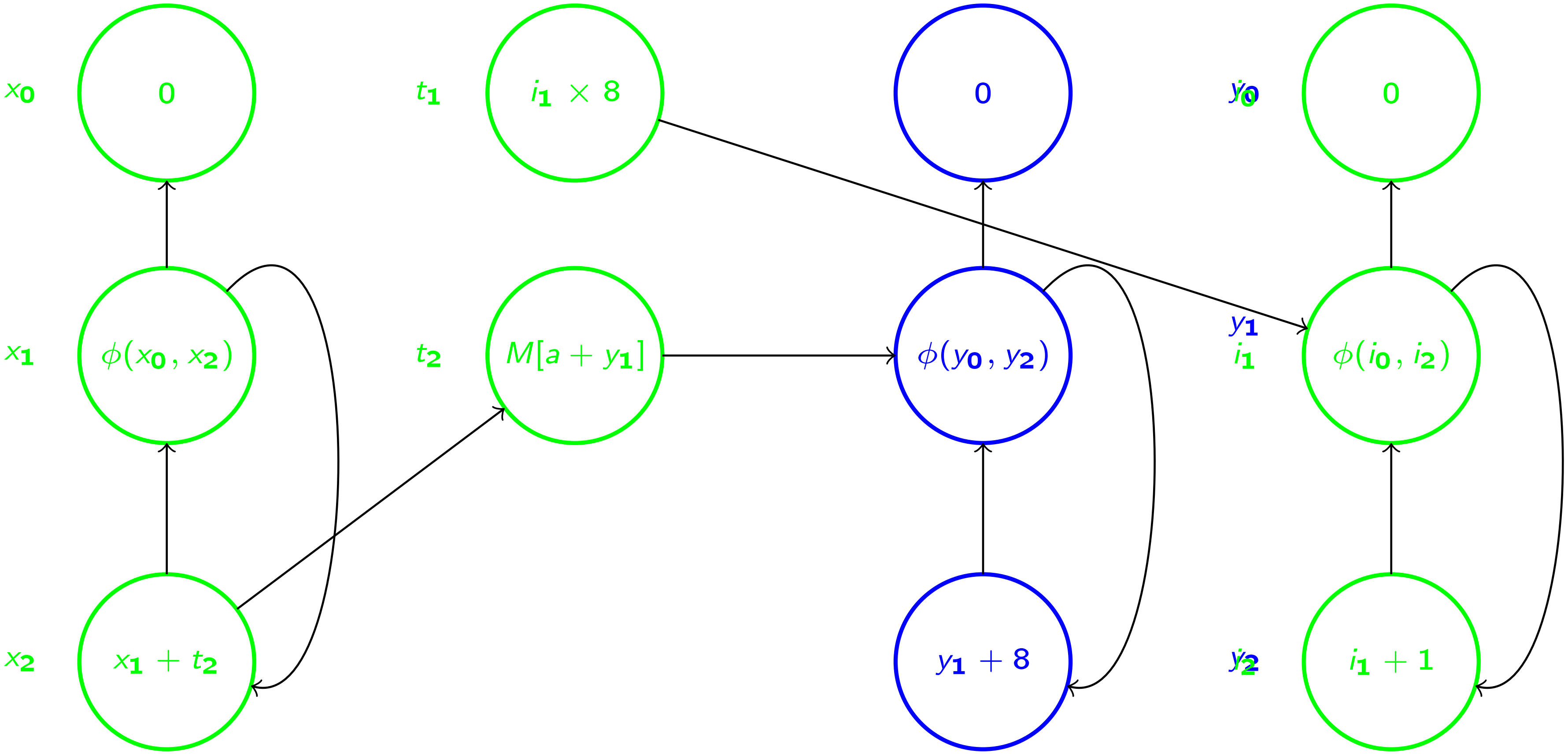
- $SCC_3 = \{t_1\}$
- SCC_3 is a multiplication of an induction variable and a region constant.
- Therefore SCC_3 is replaced by a modified copy of SCC_2 with $\phi(i)$.

Modifying a Copy of SCC_2 to Compute t_1



- $SCC_4 = \{y_1, y_2\}$
- Due to the replacement, the assignment to t_1 becomes dead code.
- There is a very beautiful algorithm to remove t_1 and other dead code that we will look at during the next lecture.

Also $a + t_1$ can be Replaced



- Due to Tarjan's algorithm we can start in any node and be sure we have already processed the operand nodes, when a variable's definition is going to be replaced.
- Not only multiplications but also some additions can be replaced, but we don't show this in the example.

Processing of a new SCC

- When nodes have been popped from the stack and collected in a SCC, the following is performed.
- A SCC has the attribute **header** which is the header of a loop in the control flow graph.

```
procedure process_scc(scc)
    if (scc has a single member n)
        if (valid_form(n))
            replace(n, iv, rc)
        else
            header(n) ← ⊥
    else
        classify(scc)
end
```

Valid Forms of Definition for Replacement

- *iv* is induction variable
- *rc* is region constant

function *valid_form*(n)
    if (n is of form $x \leftarrow iv \times rc$
        or n is of form $x \leftarrow rc \times iv$
        or n is of form $x \leftarrow iv \pm rc$
        or n is of form $x \leftarrow rc + iv$)
        return true
    else
        return false
end

Definition of Region Constant

function *region_const*(*x*, *header*)
  return *x* is constant **or** *vertex*(*x*) strictly dominates *header*
end

```
scanf("%d %d", &a, &b);
while (i < n) {
        x += u[a * i + b];
        i += 1;
}
```

- The variables `a` and `b` are region constants in the loop.

Reverse Post Order

```
int         i

procedure dfs(v)
    visited(v) ← true
    for each w ∈ succ(v) do
        if (not visited(w))
            dfs(w)
    i ← i − 1
    rpo(v) ← i
end


procedure compute_rpo(CFG)
    i ← |V|
    for each vertex v do
        visited(v) ← false
    dfs(s)
end
```

Classification of SCC as Induction Variable

```
procedure classify (scc)
    for each n ∈ scc do
        if (rpo (vertex (n)) < rpo (header))
            header ← vertex (n)
    for each n ∈ scc do
        if (operator (n) ∉ {φ, +, −, move})
            scc is not an induction variable
        else
            for each operand ω ∈ operands(n) do
                if (ω ∉ scc  and not region_const (ω, header))
                    scc is not an induction variable
    if (scc is an induction variable)
        for each n ∈ scc do
            header (n) ← header
    else
            for each n ∈ scc do
                if (valid_form (n))
                    replace (n, iv, rc)
                else
                    header (n) ← ⊥
end
```

Replace

procedure *replace*(*operation*, *iv*, *rc*)

result ← *reduce*(*opcode*(*operation*), *iv*, *rc*)

replace *operation* with mov using *result* as source

header(*operation*) ← *header*(*iv*)

end

Reduce

```
function reduce(operation, iv, rc)
    result ← lookup(opcode, iv, rc)
    if (result is not found)
        result ← new_temp()
        install(opcode, iv, rc, result)
        new_def ← copy_def(iv, result)
        for each operand ω in new_def do
            if (ω is an induction variable)
                replace ω with reduce(opcode, ω, rc)
            else if (opcode = × or new_def is a φ)
                replace ω with apply(opcode, ω, rc)
    return result
end
```

Apply

```
function apply(opcode, op1, op2)
    result ← lookup(opcode, op1, op2)
    if (result is not found)
        if (op1 is an induction variable and op2 is a region constant)
            result ← reduce(opcode, op1, op2)
        else if (op2 is an induction variable and op1 is a region constant)
            result ← reduce(opcode, op2, op1)
        else
            result ← new_temp()
            install(opcode, op1, op2, result)
            choose the location where the operation will be inserted
            perform constant folding if possible
            create a new operation at the chosen location
    return result
end
```