# Contents of Lecture 7

- What can PRE achieve?
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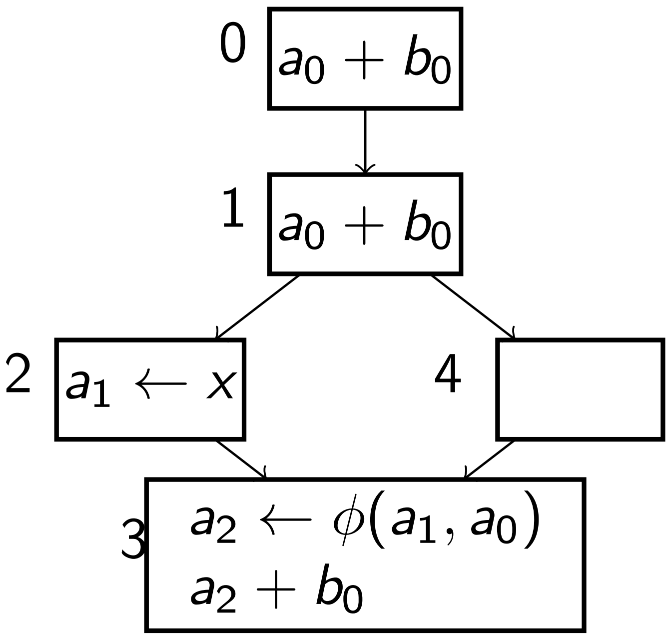
# Purpose of Partial Redundancy Elimination

- Recall that Partial Redundancy Elimination, or **PRE**, can eliminate both **full** and **partial** redundancies.
- Full redundancies: when the expression is available from all predecessor basic blocks.
- Partial redundancies: when the expression is only available from some but not all predecessor basic blocks.
- Partial redundancies also covers loops, i.e. PRE can move code out from loops.

# Partial Redundancy Elimination History

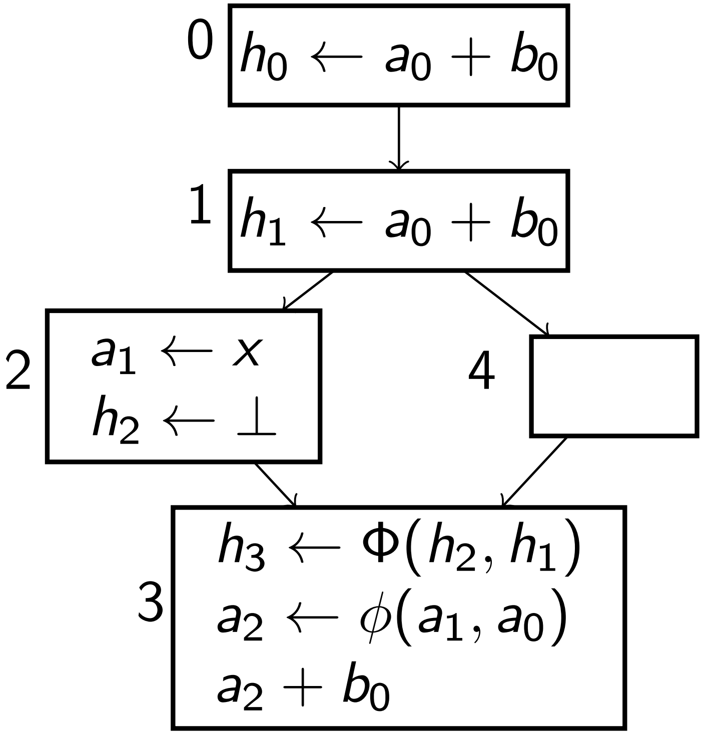
- PRE was invented by Morel and Renvoise in 1979.
- Then Fred Chow in his PhD thesis at Stanford from 1983 (with John Hennessy as supervisor) improved it.
- In 1992 Knoop et al. published a version of PRE which is optimal in the sense of minimizing register pressure. They called their algorithm **Lazy Code Motion**.
- In 1999 Kennedy and Chow and others at SGI published the SSA formulation of Lazy Code Motion and called it **SSAPRE**.
- We will first study a simpler version of it and then note that there exists an efficient variant of SSAPRE which is much faster.

# Limitations of Value Numbering




- Both hash-based and global value numbering can optimize the full redundancy in vertex 1.
- None of them can optimize the partial redundancy in vertex 3.

# The Key Idea of SSAPRE

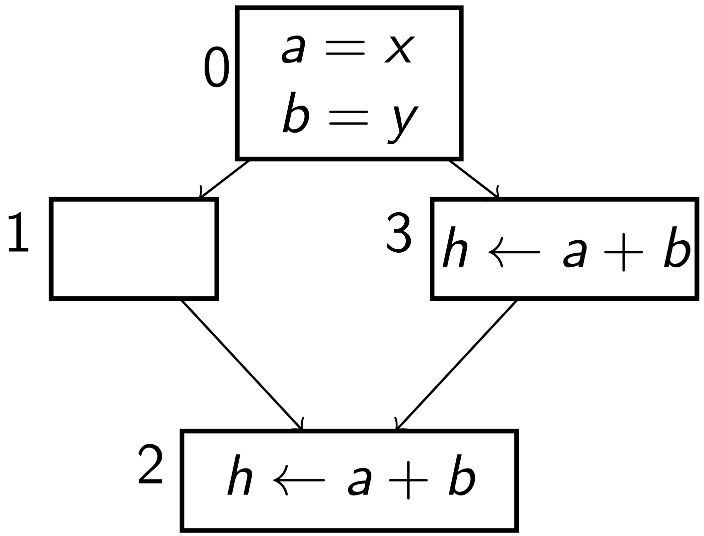



- We create $\Phi$-functions for the hypothetical variable $h$.
- After SSAPRE, $\Phi$-functions become normal $\phi$-functions and they are really the same (different notation to distinguish between them only).
- By inserting the expression $a + b$ at $\Phi$-operands with the value $\perp$ (”bottom”), the partial redundancy in vertex 3 becomes a full redundancy and can be eliminated.

# Overview of SSAPRE: $\forall$ expression $a + b$ do

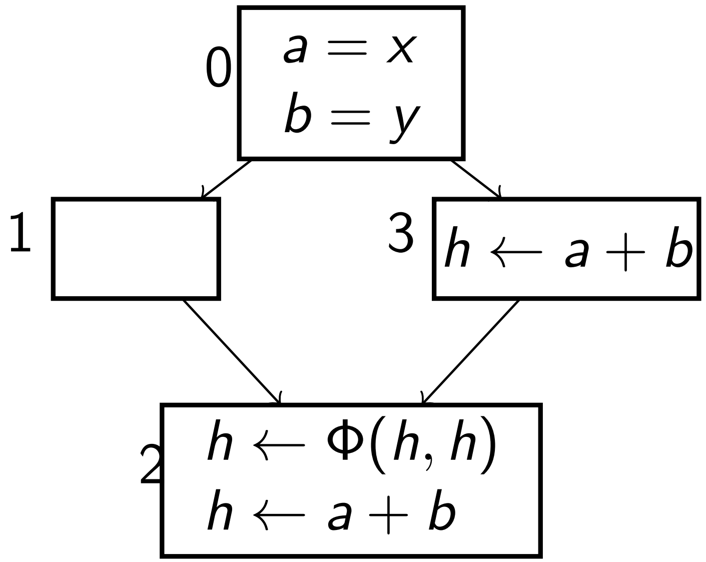
- Insert $\Phi$-functions.
- Perform SSA-renaming for the variable $h$ and **all other variables** (again).
- Compute **downsafety**, i.e. where the expression is anticipated.
- Compute **can_be_avail**, i.e. where the expression can be available, either because the expression is there or it can replace a $\perp$-operand.
- Compute **later**, i.e. if can be lazy and insert the expression further down in the control flow graph.
- Perform **finalize1**, i.e. modify the code.
- Perform **finalize2**, i.e. clean up various things.

# Insertion of Φ-functions



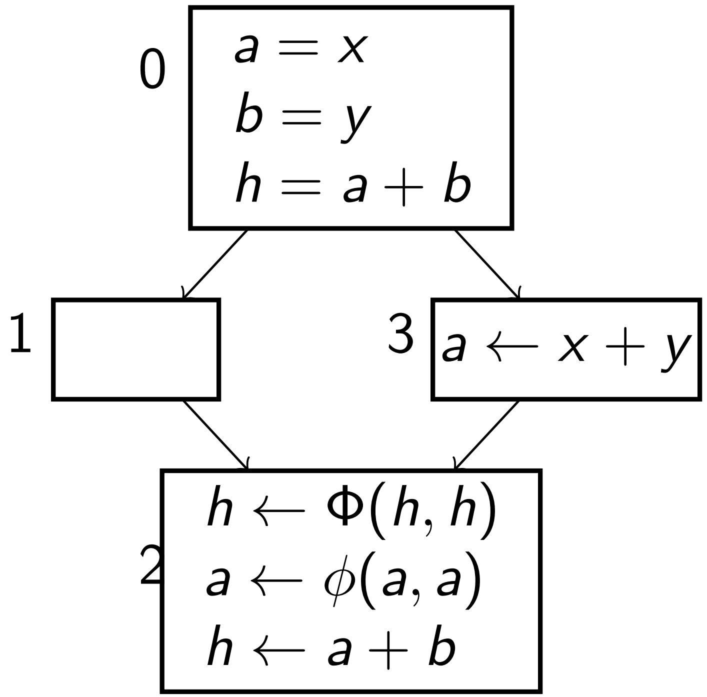

- Recall that in SSAPRE every expression assigns to a hypothetical variable $h$.
- Where should we then insert Φ-functions for $h$?
  1. In the iterated dominance frontiers of all evaluations of the expression, i.e. assignment to $h$.
  2. In the iterated dominance frontiers of all assignments to operands in the expression — since they mean $h \leftarrow \perp$

# Iterated Dominance Frontiers of Evaluations of $a + b$



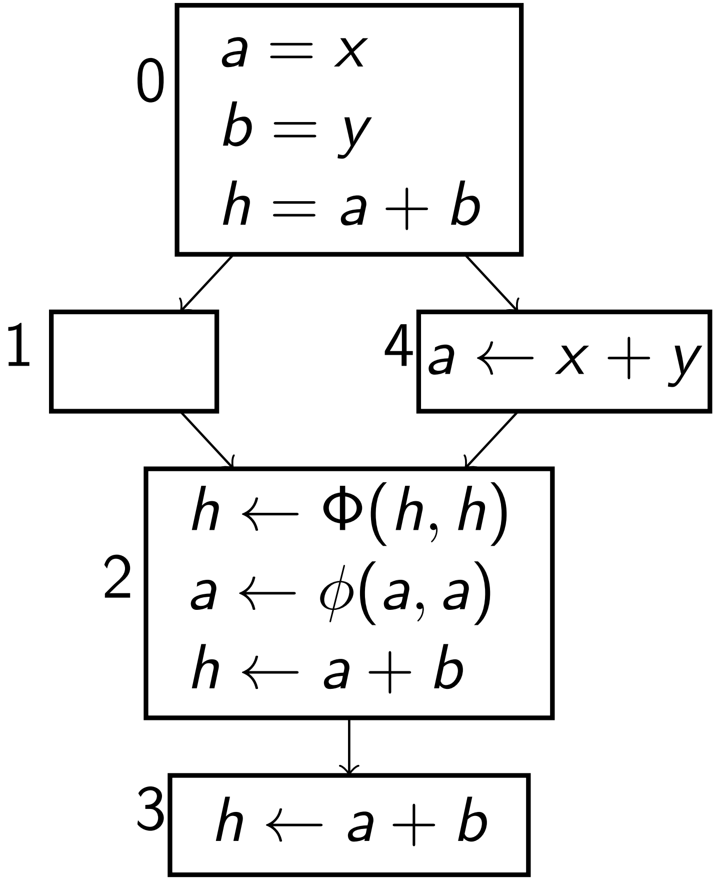

- We have already computed the dominance frontiers of each vertex.
- We thus simply have to collect the vertices which contain such an evaluation.

# Iterated Dominance Frontiers of $h \leftarrow \perp$



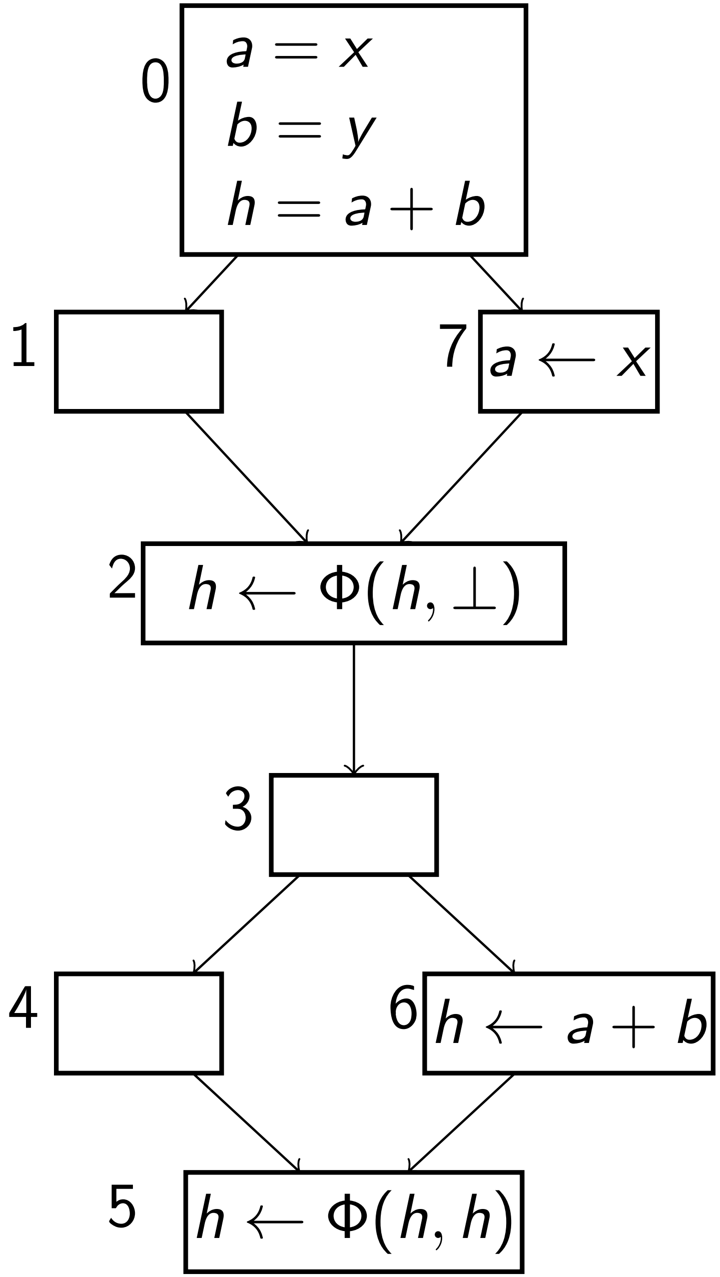

- Although we can collect all vertices with assignments to $a$ or $b$, and find the iterated dominance frontiers of these, there is a simpler way.
- Every vertex for which we will insert a $\Phi$-function due to an $h \leftarrow \perp$ must contain a $\phi$-function to any of the variables in the expression, i.e. $\phi(a)$ or $\phi(b)$.
- So we simply look for $\phi(a)$ and $\phi(b)$, and insert $\Phi(h)$ in the same vertex.
- Recall that $\phi$-functions are parallel copy statements.

# Anticipated Expressions




- An expression is **anticipated** at a point $p$ in the control flow graph if it is certain it will be evaluated with all operands having the same value on all paths from $p$.
- At the end of vertex 0, $a + b$ is not anticipated since $a$ might be assigned a new value in vertex 4.
- At the end of vertices 1 and 4 the expression is anticipated due to the evaluation in vertex 2 which certainly will be evaluated.
- The word "evaluated" here means "executed".

# The Main Rule of the Game of PRE

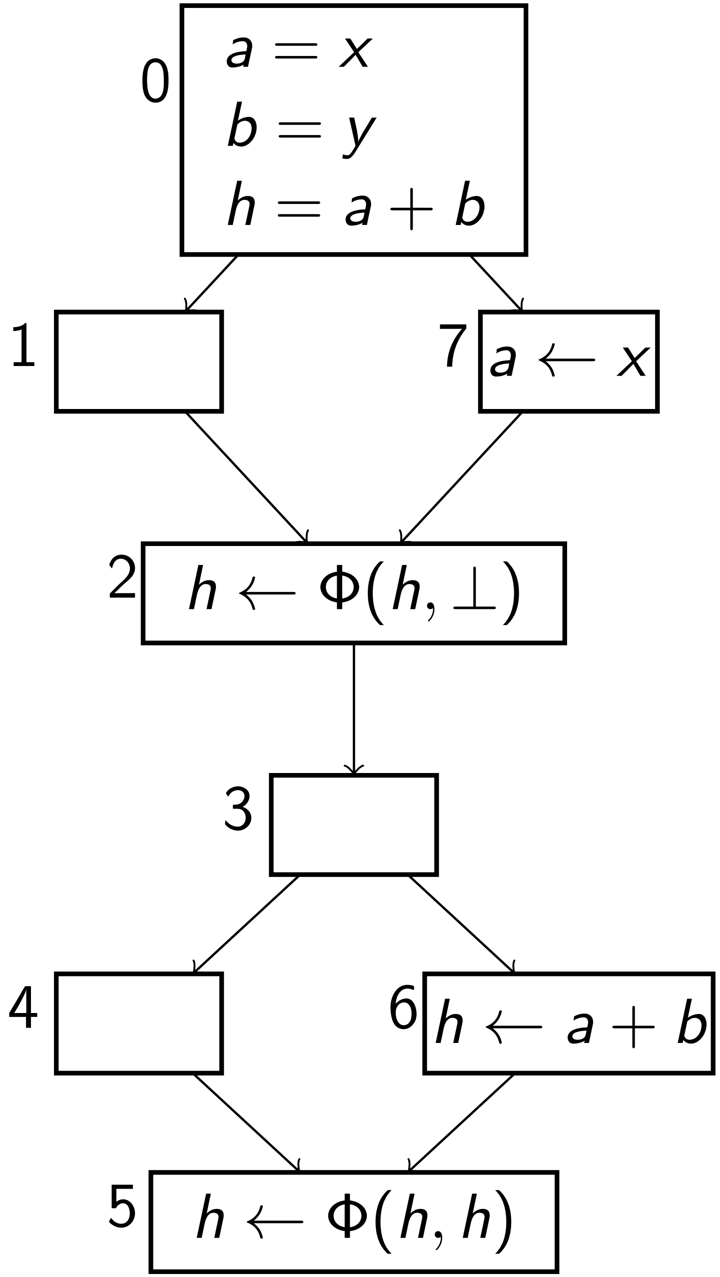



- No matter what, PRE may never transform a function so it will execute additional instructions due to PRE.
- Should the $\perp$ in vertex 2 be replaced with $h \leftarrow a + b$?
- No, it's not safe to insert the expression since the expression is **not anticipated** by the $\Phi$-function.
- The path $(0, 7, 2, 3, 4, 5)$ would execute $a + b$ at the end of vertex 7 (for the $\Phi$-operand) without any purpose.
- Actually, a $\Phi$-operand is regarded as belonging to the predecessor vertex.

# Occurrences

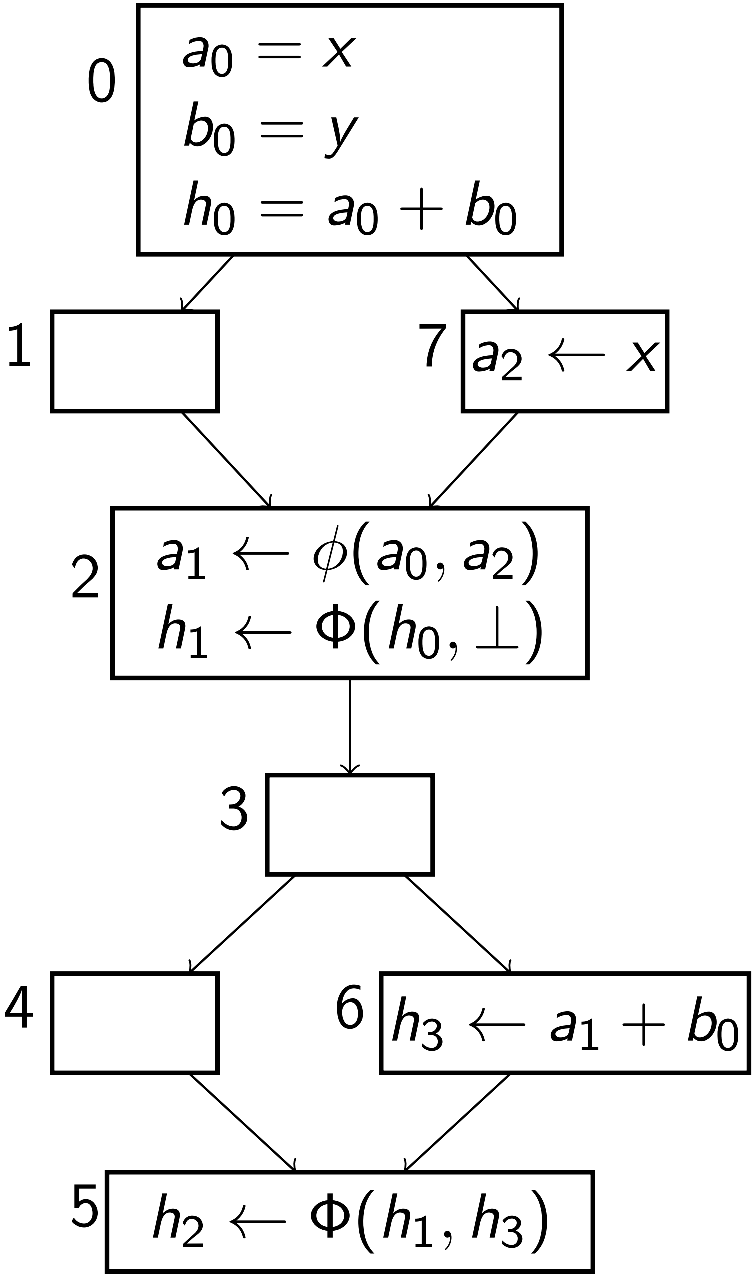
- There are three main types of so called **occurrences** of an expression:
  1. A **real occurrence**, i.e. the expression $a + b$,
  2. A $\Phi$-**function occurrence**, and
  3. A $\Phi$-**operand occurrence**.
- Note that $\Phi$-operands are placed in the predecessor basic block.

# Attributes of Φ-functions



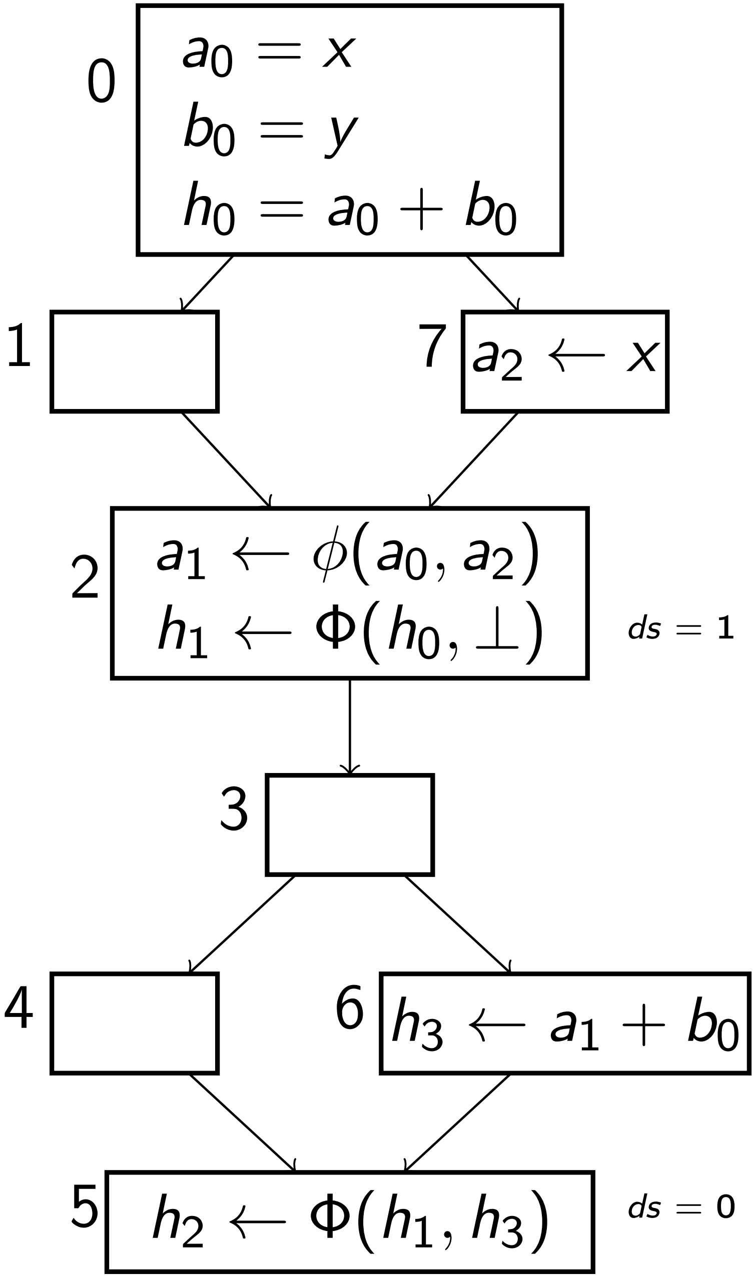

- Each Φ-function has a number of boolean attributes:
  - **downsafe** or **ds**
  - **can_be_available** or **cba**
  - **later**
  - **will_be_available** or **wba**
- If a Φ-function is downsafe, it's OK to replace a $\perp$ operand with the expression.
- We will soon see how downsafe is computed.
- A Φ-operand has the boolean attribute **has_real_use** which is true if the value comes from a real occurrence.

# Renaming



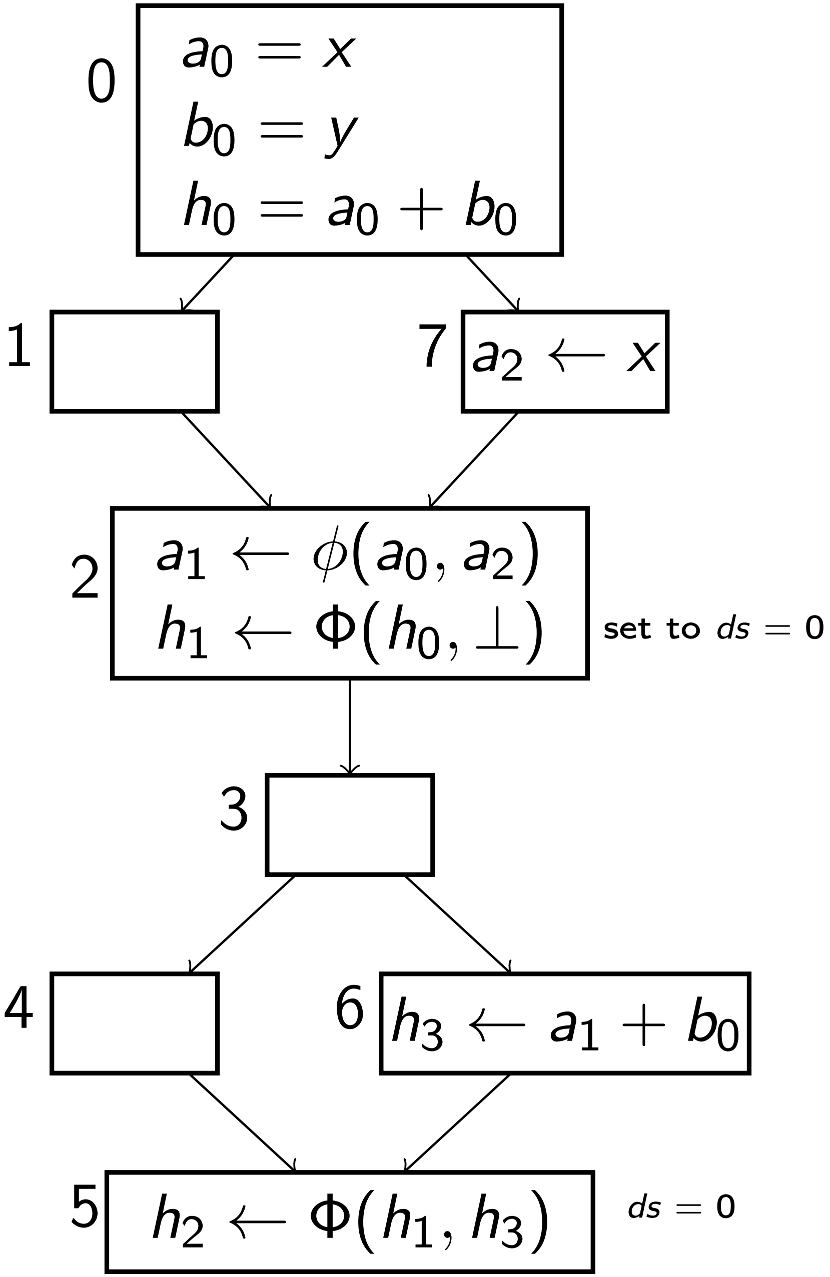

- Renaming traverses the dominator tree and links uses with definitions of $h$ variables.
- At a $\Phi$-function occurrence, a new version of $h$ is always created.
- At a $\Phi$-operand occurrence it is noted if the value comes from a real occurrence, in which case **has_real_use** is set to true.
- At a real occurrence, a new version of $h$ is created if the top of stacks of $a$, $b$, and $h$ don't have the same versions.
- Both real and $\Phi$-function occurrences are pushed on the rename stack of $h$.

# Initialization of Downsafe




- Recall that a $\Phi$-function is downsafe if all paths from it evaluate $a + b$ (with the same variable versions).
- Thus, if there is a path from a $\Phi$-function to the exit vertex that $\Phi$-function is not downsafe unless the expression was evaluated.
- When renaming comes to the exit vertex, it checks the top of the stack of $h$.
- If the top is a $\Phi$-function, it is marked with $\mathbf{ds = 0}$.

# Computing Downsafety




- After the initialization of downsafety during rename, the downsafety is computed for all Φ-functions.
- What should be done?
- A Φ-function with $ds = 0$ should tell other Φ-functions that also they are not downsafe!
- A Φ-function with $ds = 0$ and with a Φ-operand that is defined by a Φ-function and for which **has_real_use = 0**, should reset its downsafety and continue the recursion.
- In this example both Φ-functions have $ds = 0$.

# Computing Downsafety

**procedure** *reset_downsafe*$(x)$
    **if** (*has_real_use*$(x)$ **or** *def*$(x)$ is not a $\Phi$)
        **return**
    $f \leftarrow$ *def*$(x)$
    **if** (**not** *down_safe*$(f)$)
        **return**
    *down_safe*$(f) \leftarrow$ false
    **for** each operand $\omega$ of $f$ **do**
        *reset_downsafe*$(\omega)$

**procedure** *downsafety*
    **for** each $f \in \mathcal{F}$ **do**
        **if** (**not** *down_safe*$(f)$)
            **for** each operand $\omega$ of $f$ **do**
                *reset_downsafe*$(\omega)$

# Compute Can Be Available

**procedure** *compute_can_be_avail*
    **for** each $f \in \mathcal{F}$ in the program **do**
        $can\_be\_avail(f) \leftarrow$ true
    **for** each $f \in \mathcal{F}$ in the program **do**
        if (**not** $down\_safe(f)$
            **and** $can\_be\_avail(f)$
            **and** $\exists$ an operand of $f$ that is $\perp$)
            $reset\_can\_be\_avail(f)$
**end**

# Reset Can Be Available

**procedure** *reset_can_be_avail*$(g)$
    *can_be_avail*$(g) \leftarrow$ false
    **for** each $f \in \mathcal{F}$ with operand $\omega$ with $g = def(\omega)$ **do**
        **if** (**not** *has_real_use*$(\omega)$
            **and not** *downsafe*$(f)$
            **and** *can_be_avail*$(f)$)
            *reset_can_be_avail*$(f)$
**end**

# Computing Later

**procedure** *reset_later*$(g)$
    *later*$(g) \leftarrow$ false
    **for** each $f \in \mathcal{F}$ with operand $\omega$ with $g =$*def*$(\omega)$ **do**
        **if** (*later*$(f)$)
            *reset_later*$(f)$
**end**

**procedure** *compute_later*
    **for** each $f \in \mathcal{F}$ **do**
        *later*$(f) \leftarrow$ *can_be_avail*$(f)$
    **for** each $f \in \mathcal{F}$ **do**
        **if** (*later*$(f)$ **and**
            $\exists$ an operand $\omega$ of $f$ such that *def*$(\omega) \neq \bot$ **and** *has_real_use*$(\omega)$)
                *reset_later*$(f)$
**end**

**procedure** *will_be_avail*
    *compute_can_be_avail*
    *compute_later*
**end**

# Finalize1

```
procedure finalize1(g)
    let E ← the current expression
    for each redundancy class x of E do
        avail_def[x] = ⊥
    for each occurrence ψ of E in preorder DT traversal order do
        x ← class(ψ)
        if (ψ is a Φ occurrence) {
            if (will_be_avail(ψ))
                avail_def[x] = ψ
        } else if (ψ is a real occurrence) {
            if (avail_def[x] is ⊥  or avail_def[x] does not dominate ψ)
                reload(ψ) ←false
                avail_def[x] = ψ
            } else {
                reload(ψ) ←true
                def(ψ) ←avail_def[x]
            }
        } else {
            /* ψ is a Φ operand occurrence. */
            let f be the Φ in the successor vertex of this operand
            if (will_be_avail(f)) {
                if (ψ satisfies insert) {
                    insert E at the end of the vertex containing ψ
                    def(ψ) ← inserted occurrence
                } else
                    def(ψ) ←avail_def[x]
            }
        }
end
```