Contents of Lecture 3

- Translation to SSA Form
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Translation to SSA Form

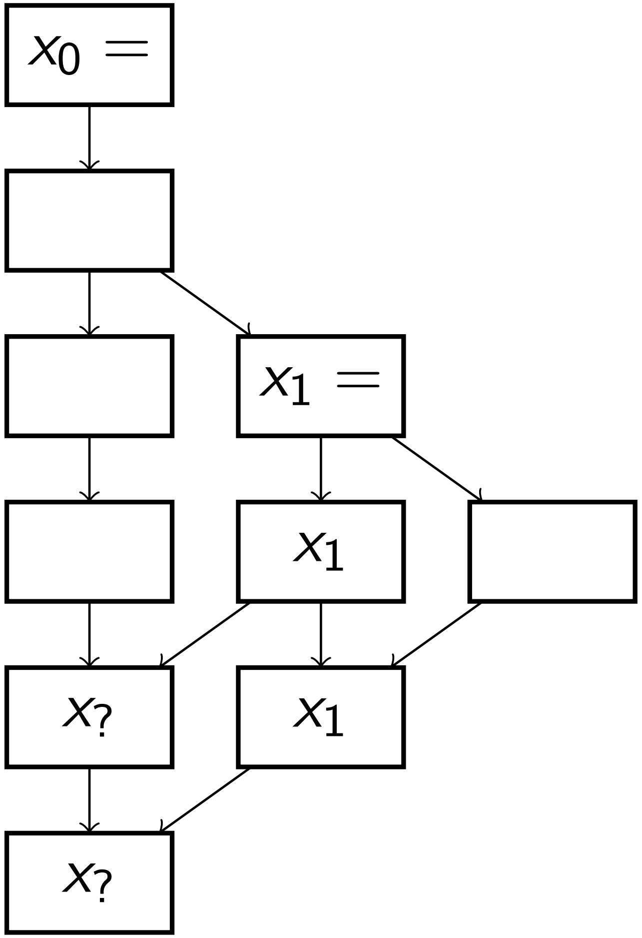
A function is translated to SSA Form in the following steps

1. Compute the dominator tree DT of the function.
2. Compute the dominance frontier of each vertex in the CFG.
3. Insert ϕ-functions.
4. Rename variables while traversing the dominator tree.

A Trick

- We want to insert a ϕ-function where two paths from assignments meet.
- This formulation of the problem was difficult to use to find an efficient algorithm.
- The following which makes it easier to answer the question of where to insert ϕ-functions:
- **Trick:** Every variable is given a assignment in the start vertex.
- That is, a variable x is given an assignment x_0 in the start vertex.
- No assembler code is produced for the assignment though.

Why would x_0 help???

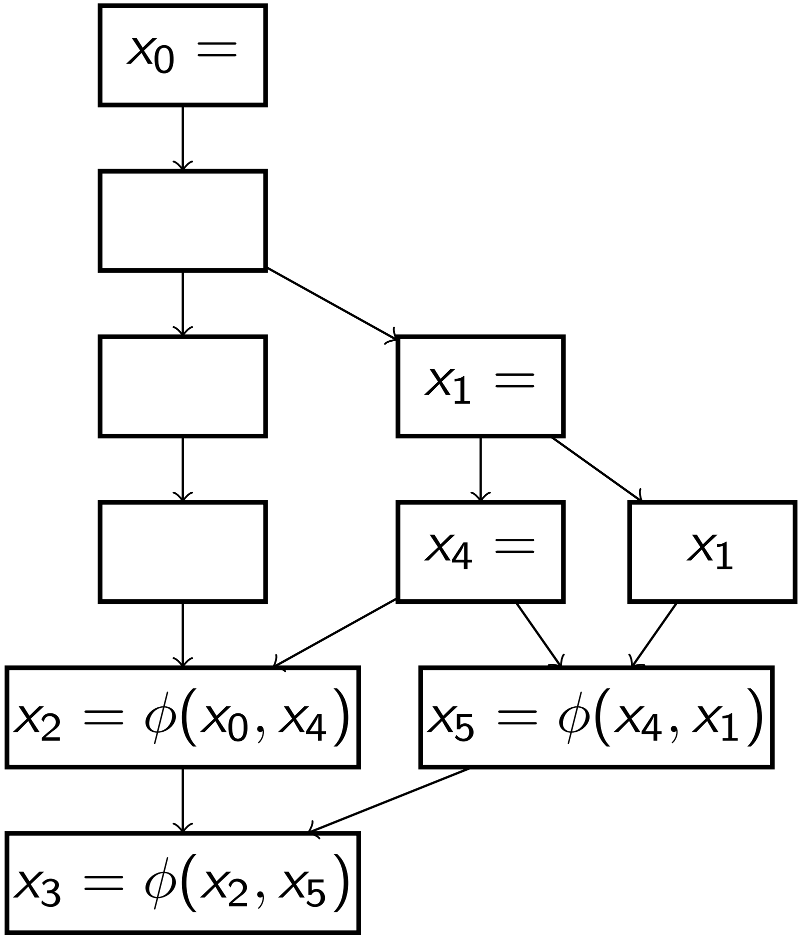


- With the assignment to x_0 we can see that two paths from assignments join in the vertices with $x_?$.
- Therefore each of them needs a ϕ-function.
- Another way to see this is that these vertices are just outside what is dominated by the vertex with $x_1 =$.

Dominance frontier

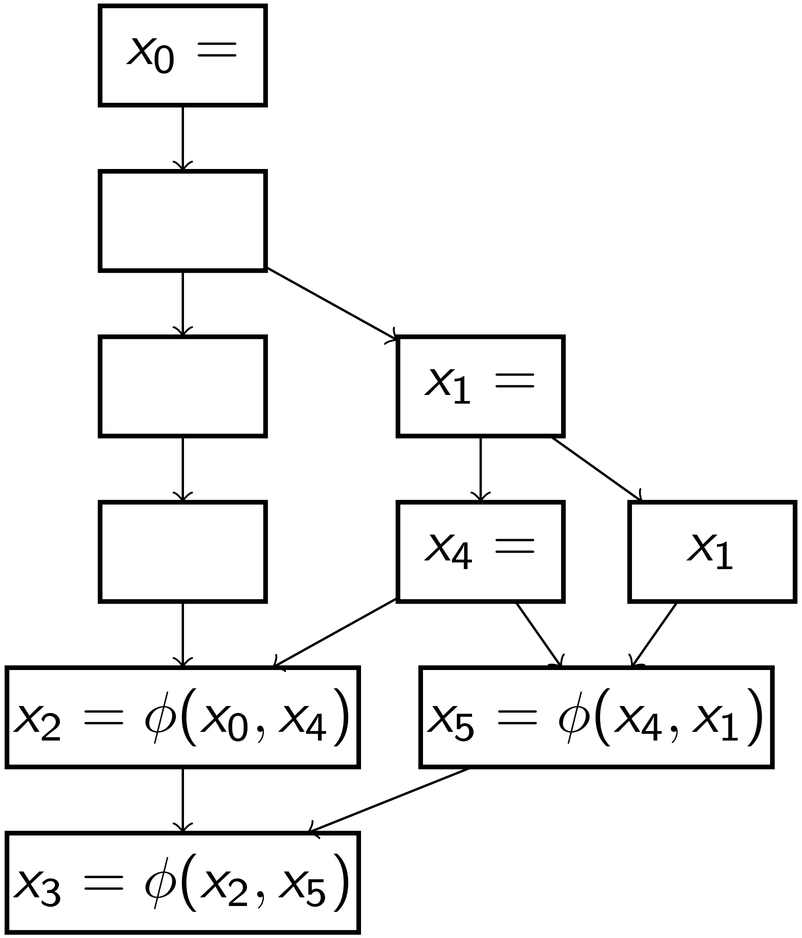
- Informally, we need to insert a ϕ-function in every vertex which is ”just outside” what is dominated by a vertex with an assignment.
- ”Just outside” is called the **dominance frontier** of a vertex u.
- It is written $DF(u)$.
- $DF(u) = \{\ v \mid \exists\ p\ \in\ pred(v),\ u \geqq p,\ u\ \not\gg\ v\ \}$.
- In words: if u dominates a predecessor of v but does not dominate v strictly, then v is in the dominance frontier of u.
- After the dominator tree is found, the dominance frontier for each vertex is computed.
- Each local variable and compiler-generated temporary is inspected: for each vertex u with an assignment to the variable, a ϕ-function is inserted in $DF(u)$.
- Note that a ϕ-function is an assignment — which also needs ϕ-functions in the dominance frontier of its vertex.

Multiple assignments



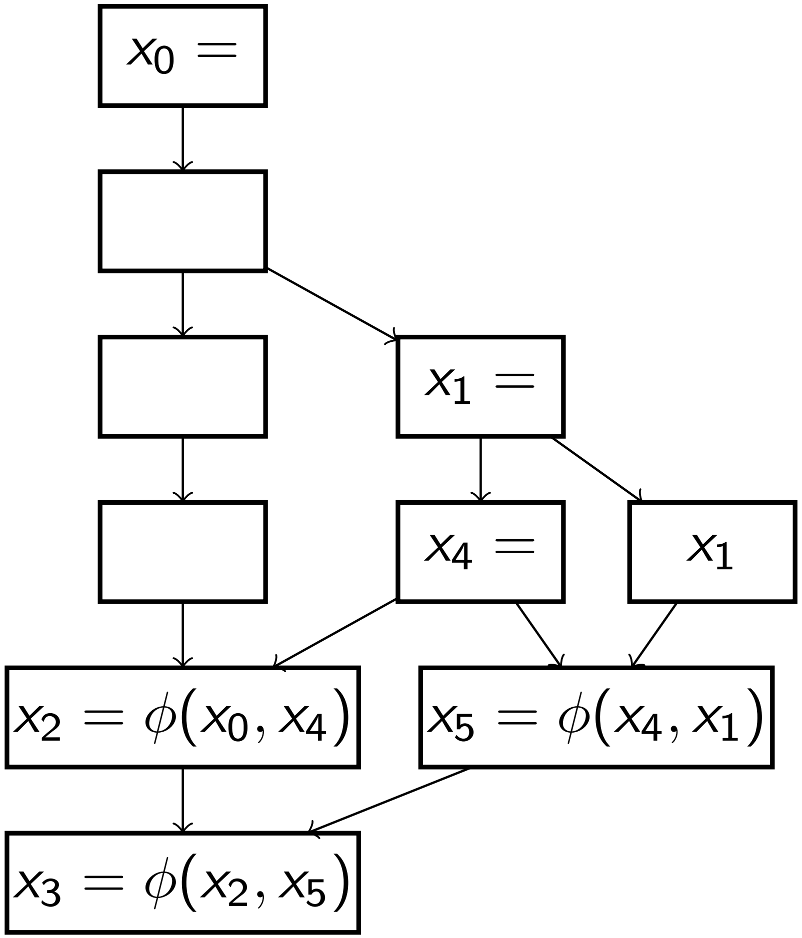
- The assignment to x_4 means that that vertex is dominated by two different assignments.
- Therefore we must rename the variables in a certain order so that after a later assignment the up-to-date version is used during the renaming.
- Obviously it is x_4 that should be the ϕ-operand and not x_1.
- This is achieved by using a stack of variable versions.

Using the Dominator Tree and a Stack of Variable Versions



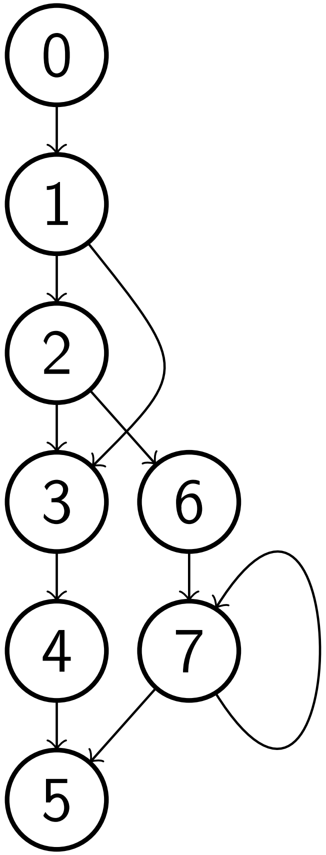
- After ϕ-functions have been inserted (more details below) the dominator tree is traversed during variable renaming.
- Each variable has its own stack of variable versions.
- At a use of a variable in a statement, the variable is replaced in the statement by the top of variable's stack.
- At an assignment a new variable version is pushed on the variable's stack, and the variable is replaced in the statement by the new version.

Illustration of what happens near the assignment to x_1

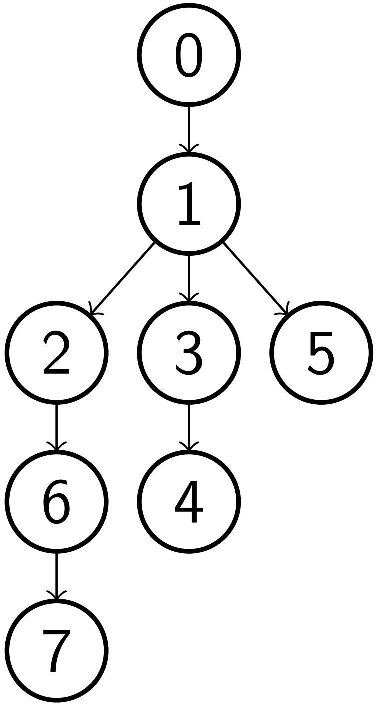


- The new version x_1 is pushed on the stack of x.
- The vertex with x_4 is a child in the DT and is inspected next.
- The new version x_4 is pushed on the stack of x.
- The ϕ-function in the successor vertex gets one of its operands replaced to x_4 from the current top of the stack.
- The vertex with x_4 has no child in the DT and x_4 is popped from the stack.
- x_1 becomes the top of the stack and is used next.

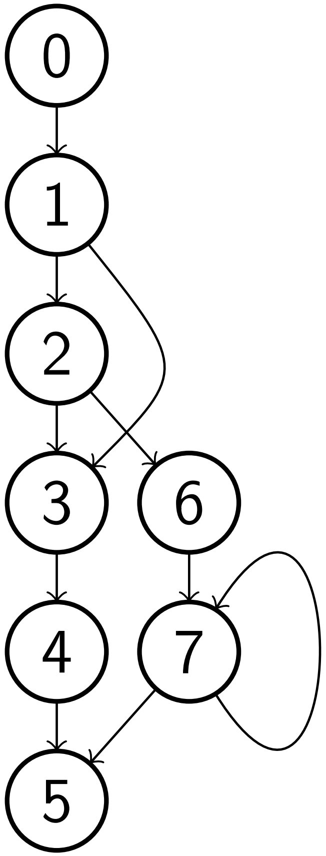
Strict Dominance in the Definition of Dominance Frontier

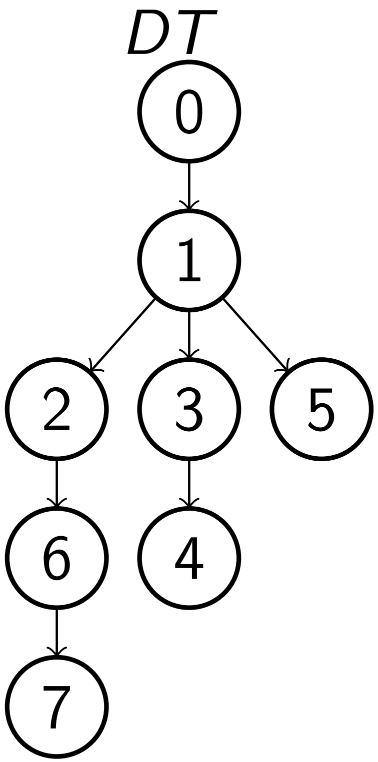


DT

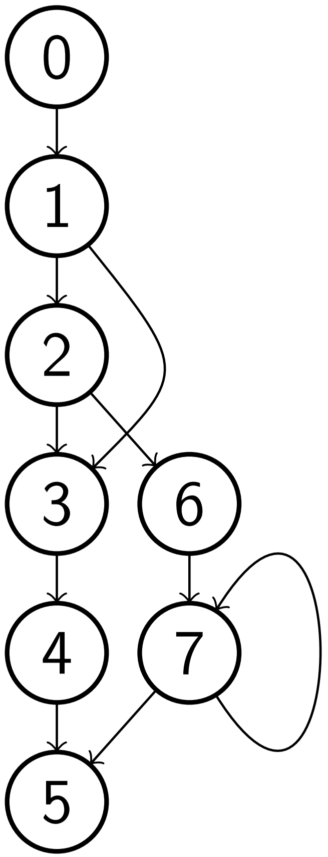


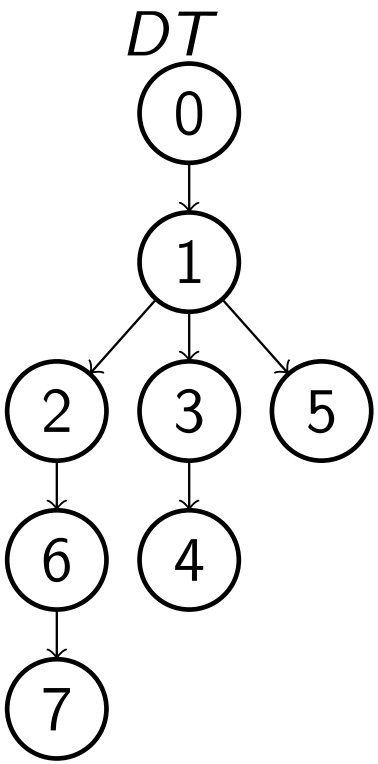
- $DF(u) = \{v | \exists p \in pred(v), u \underline{\gg} p, u \not\gg v\}$.
- Consider 7 and suppose it contains `++i`.
- It then needs $i = \phi(i, i)$.
- $DF(7) = \{5, 7\}$.
- When 7 is added to its own DF it is u, p, and v in the definition.
- This situation is the reason for using not strict dominance in the definition.





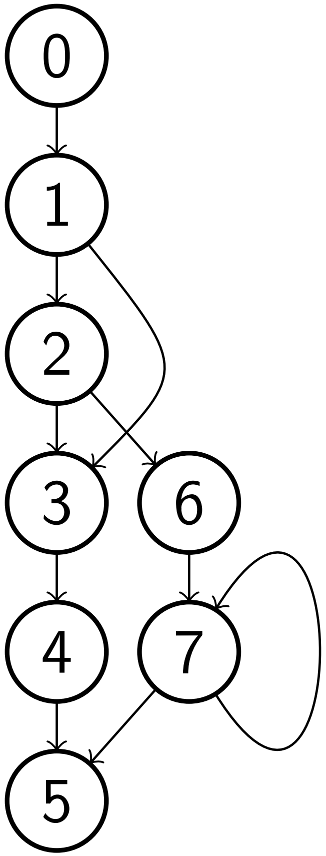
- $DF(u) = \{v | \exists p \in pred(v), u \geqq p, u \not\gg v\}$.
- Below $children(u)$ is the set of children of u in the dominator tree.
- The dominance frontier is computed bottom up in the dominance tree using:
- $$DF(u) = DF_{local}(u) \cup \bigcup_{c \in children(u)} DF_{up}(c)$$
- $DF_{local}(u) \stackrel{\text{def}}{=} \{v \in succ(u) | \ u \not\gg v\}$.
- $DF_{up}(c) \stackrel{\text{def}}{=} \{v \in DF(c) \mid idom(c) \not\gg v\}$.
- These formulas can be simplified further as we will see, but first we will build intuition into why they are correct.



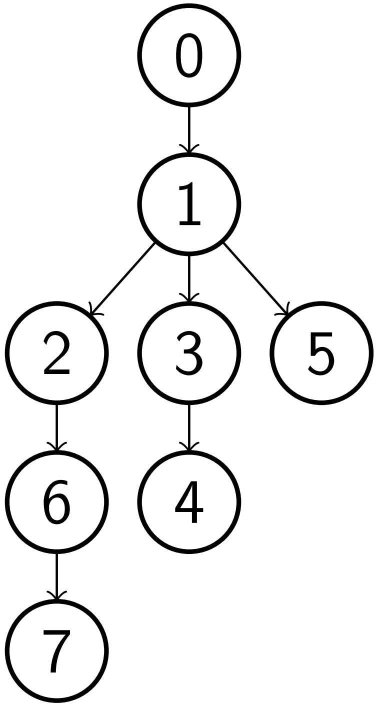


- $DF_{local}(u) \stackrel{\text{def}}{=} \{v \in succ(u) | \; u \not\gg v\}$.
- The set $DF_{local}(u)$ is the contribution to $DF(u)$ which can be determined by only looking at the successors of u in the CFG.
- Since u does not dominate v strictly, but clearly it dominates a predecessor of v (namely itself), $v \in DF(u)$.
- For example, $3 \in DF(2)$ and $7 \in DF(7)$
- But e.g. $3 \notin DF(1)$ since $1 \gg 3$.

$DF_{up}(c)$

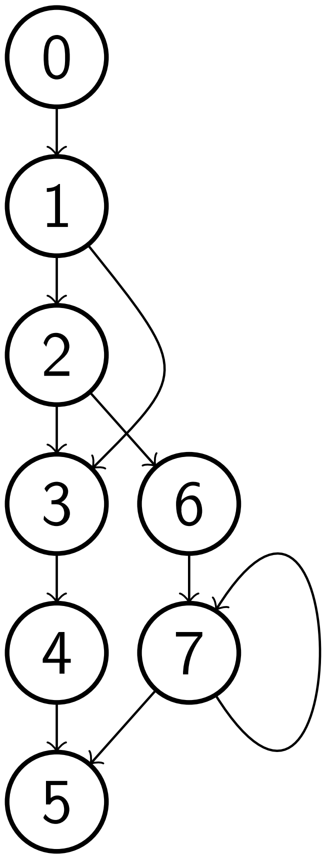


DT

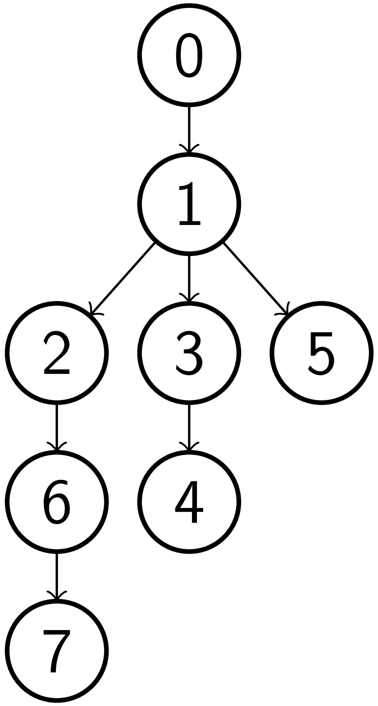


- $DF_{up}(c) \stackrel{\text{def}}{=} \{v \in DF(c) \mid idom(c) \not\gg v\}$.
- The set $DF_{up}(c)$ is the contribution from a vertex c to the DF of $idom(c)$.
- Example: $DF_{up}(4) = \{5\}$.
- To see that $DF_{up}(c) \subseteq DF(idom(c))$, consider any vertex $v \in DF(c)$.
- From the definition of $DF(c)$ there must be a $p \in pred(v)$ such that $c \underline{\gg} p$.
- Since dominance is transitive and obviously $idom(c) \underline{\gg} c$ we must have $idom(c) \underline{\gg} p$.
- Thus the vertices in $DF(c)$ which are not strictly dominated by $idom(c)$ should be added to $DF(idom(c))$ and this is what $DF_{up}(c)$ achieves.

More about dominance frontiers

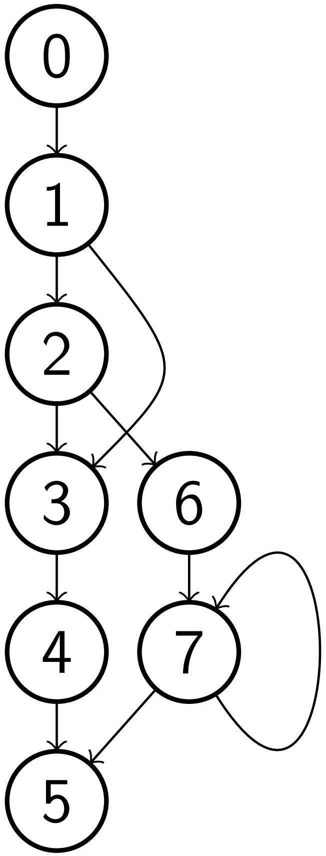


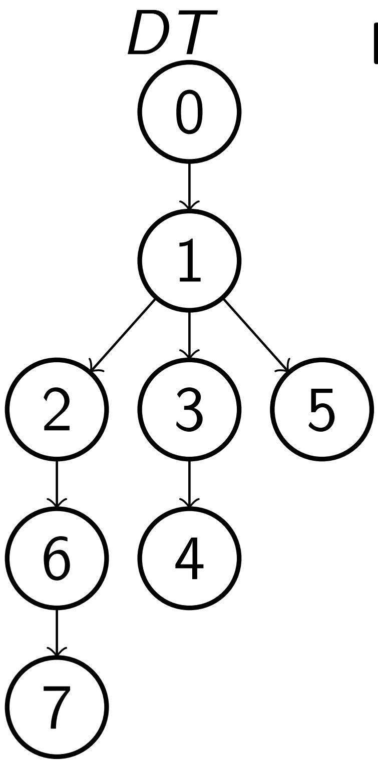
DT



- In the book is also shown that every vertex in $DF(v)$ is accounted for in either $DF_{local}(v)$ or $DF_{up}(c)$ where $idom(c) = v$.
- One can also show that instead of:
- $DF_{local}(u) \stackrel{\text{def}}{=} \{v \in succ(u) | \; u \not\gg v\}$, we can use:
- $DF_{local}(u) \stackrel{\text{def}}{=} \{v \in succ(u) | \; u \neq idom(v) \}$, and
- $DF_{up}(c) \stackrel{\text{def}}{=} \{v \in DF(c) | idom(c) \neq idom(v)\}$.

Computing the Dominance Frontiers of a CFG

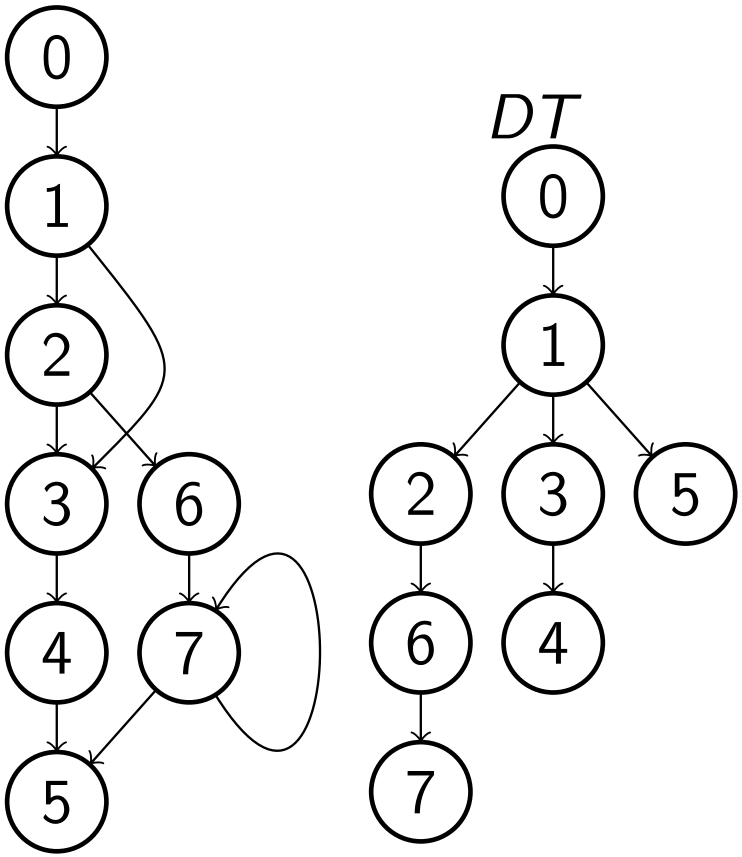




procedure *df*(G, DT)
    for each u in a postorder traversal of DT **do**
        $DF(u) \leftarrow \emptyset$
        for each $v \in succ(u)$ **do**
            if $(idom(v) \neq u)$
                add v to $DF(u)$

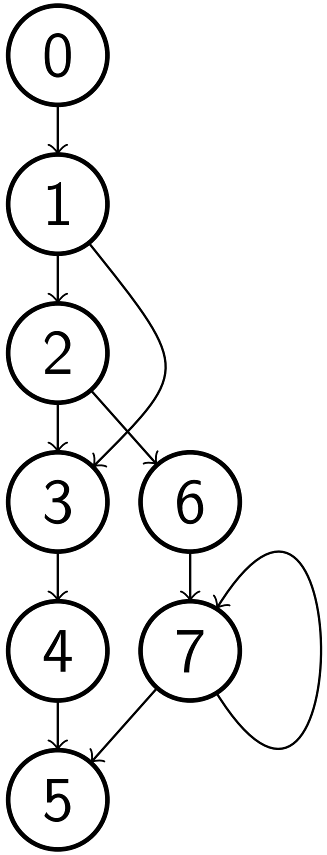
        for each $w \in children(u)$ **do**
            for each $v \in DF(w)$ **do**
                if $(idom(v) \neq u)$
                    add v to $DF(u)$

Computing the Dominance Frontiers of a CFG



- By postorder traversal is meant that when we visit vertex u, we first compute the dominance frontier of each child c of u in DT before we compute $DF(u)$.
- You will implement this function in Lab 2.
- Recursively walk through the dominator tree.
- The first computed set will be $DF_{local}(7) = \{5, 7\}$.
- $DF_{up}(c)$ is never explicitly stored but computed by inspecting $DF(c)$
- The first complete computed dominance frontier will be $DF(7) = \{5, 7\}$.
- Then $DF(6), DF(2), DF(4), DF(3)$ etc...

Inserting ϕ-functions



- ϕ-functions are inserted for one variable at a time.
- A counter **iteration** is incremented when the next variable is processed — i.e. gets its ϕ-functions inserted into the CFG.
- Each vertex has two attributes for the ϕ-function insertion which keeps track of for which iteration (value of **counter**) it was processed:
  - **has_already** – used to determine whether a ϕ-function for a certain variable has already been inserted in that vertex.
  - **work** – used to determine whether that vertex has been put in a worklist called **W**.
- These variables are all set to zero initially.

Insert ϕ-functions

```
procedure insert-φ
    W ← ∅
    for each variable x do
        iteration ← iteration + 1
        for each u ∈ vertex_with_assignment(x) do
            work[u] ← iteration
            add u to W

        while (W ≠ ∅) do
            take u from W
            for each v ∈ DF(u) do
                if (has_already[v] < iteration)
                    place x ← φ(x, ..., x) at v
                    has_already[v] ← iteration
                    if (work[v] < iteration)
                        work[v] ← iteration
                        add v to W
```

Remarks on previous slide

- The use of an explicit counter and the attributes **work** and **has_already** is how the algorithm was originally described by researchers from IBM.
- This is more efficient than using lookup-functions to determine whether a vertex has a certain ϕ-function or a vertex is in the worklist.
- For optimizing compilers research the speed of the compiler at normal optimization levels, e.g. `-O2` is extremely important.
- However, some optimizations which analyze the whole program is sometimes allowed to take hours.

Rename

- Rename performs a traversal of the dominator tree.
- In a vertex u the sequence of three-address statements is examined one statement at a time:
  - First the source operands (right hand side, or RHS) are renamed by replacing the operand with the version of the variable on the top of the variable's rename stack.
  - Then the destination operand (left hand side, or LHS) is renamed by creating a new variable version, pushing it on the rename stack, and replacing the operand with the new version of the variable.
- Then the ϕ-functions of each successor vertex v in the CFG is inspected and the operand corresponding to the edge (u, v) is renamed.
- Then each child c in the DT is processed.
- Finally every new version created and pushed on a rename stack in u is popped from its rename stack.

Rename Algorithm

procedure *rename*(u)

    for each statement t in u **do**

        for each variable $x \in RHS(t)$

            replace use of x by use of x_i where $i = top(S(x))$

        for each variable $x \in LHS(t)$ **do**

            $i \leftarrow C(x)$

            replace x by x_i

            push i onto $S(x)$

            $C(x) \leftarrow C(x) + 1$

    for each $v \in succ(u)$ **do**

        $j \leftarrow which\_pred(u, v)$

        for each ϕ-function in v **do**

            replace the j-th operand in $RHS(\phi)$ by x_i where $i = top(S(x))$

    for each $v \in children(u)$ **do**

        rename(v)

    pop every variable version pushed in u

Unnecessary ϕ-functions

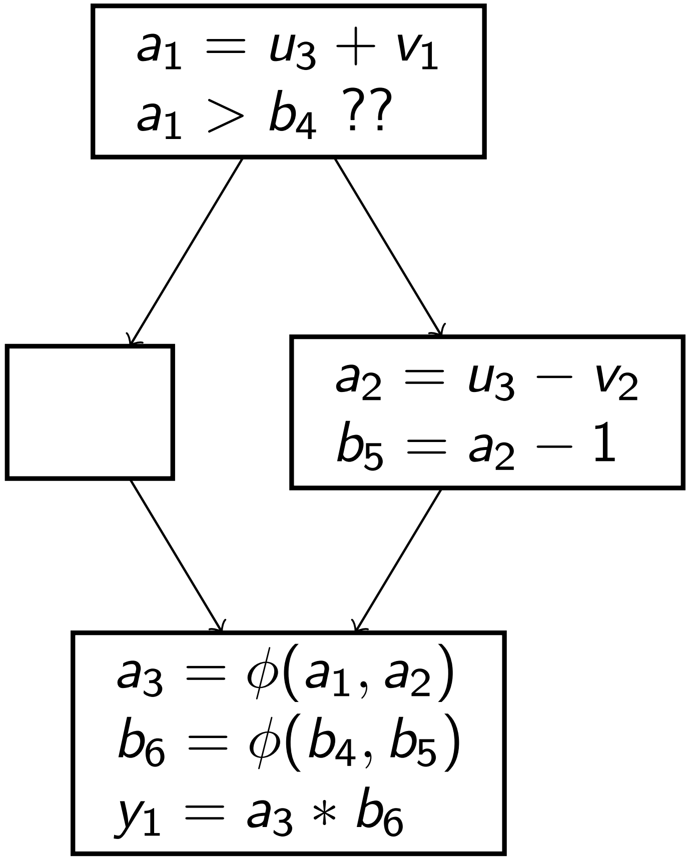
- It's unnecessary to insert a ϕ-function if its value is never used:

  ```
  if (a > 0) {
          a = a + 1;
          f(a);
  }
  return b;
  ```
- Before the return, there will be a ϕ-function due to the assignment to a.
- In general the cost to determine whether the value will be used is not worth the effort.
- It's not uncommon that a ϕ-function is inserted in a vertex where the value is overwritten before being used. This special case can be easy to determine and may be worth the effort of avoiding inserting an unnecessary ϕ-function.

Variable versions are almost only for illustration

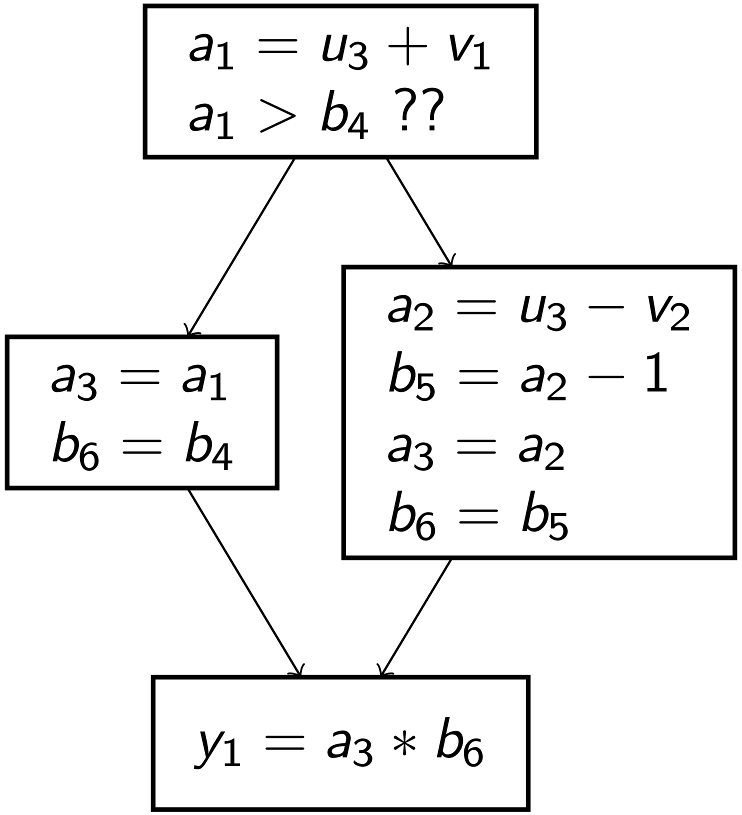
- Most optimization algorithms ignore the variable version number and treat for instance a_i and a_j as completely different variables which have no more in common than a_i and b_k have.
- Therefore no counter is usually needed: it's sufficient to simply create a new temporary variable.
- However, Partial Redundancy Elimination, SSAPRE, needs to know from which original variable such a temporary comes.

Translation from SSA Form



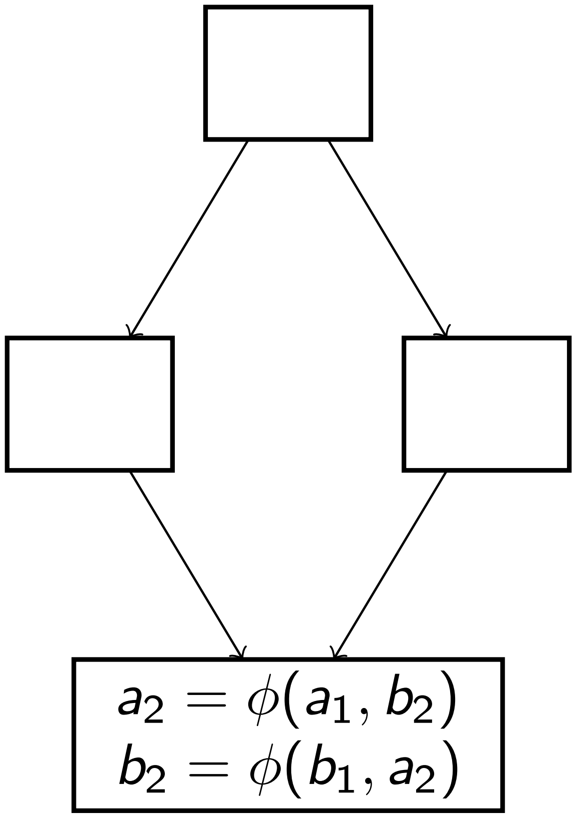
- The basic idea when translating from SSA Form is to replace the ϕ-functions with copy statements in the predecessor vertices.

Translation from SSA Form



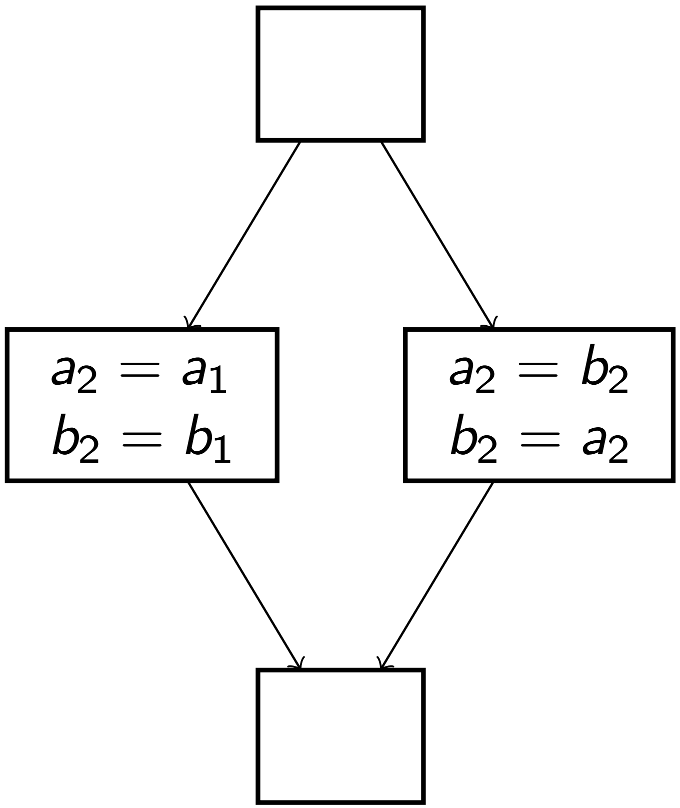
- It's thus necessary to have a vertex to insert the copy statements into!
- Without the leftmost vertex, there is an edge from a vertex with multiple successors to a vertex with multiple predecessors and such an edge is called a **critical edge**.
- Critical edges are removed by inserting an extra empty vertex.
- This is done before dominance analysis.

Translation from SSA Form

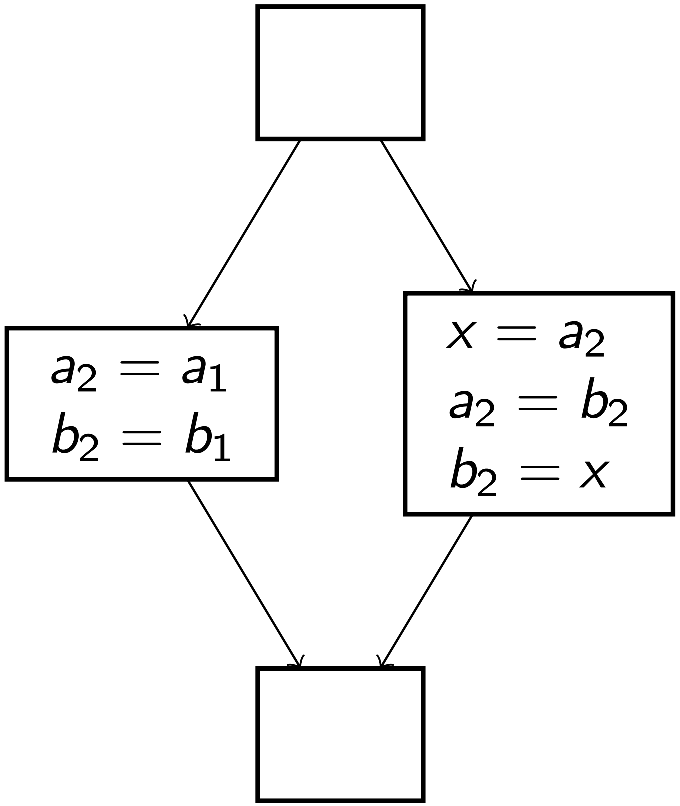


- The ϕ-functions are ***parallel*** copy statements.
- Conceptually all ϕ-functions are executed concurrently by first reading all operands and then writing all destinations.
- So what will go wrong here with a "naive" translation from SSA Form?

Translation from SSA Form



- What is wrong here?



- The value of a_2 must be saved before being overwritten!

Detect Use of Uninitialized Variables

- If version zero is used and there was no explicit initializer for the variable (i.e. no `int a = 1`) it means we might have discovered a buggy program with undefined behavior!
- If the code is executed at runtime, it is undefined behavior, but the code might never be reached.
- It is a good idea to warn about it anyway.

Copy Propagation

- During Translation to SSA Form, a copy statement `a = b` can be optimized as follows:
- The current value of b, i.e. the version on the top of b's rename stack is pushed on a's rename stack and this copy statement (`MOV`) can then be removed.
- You will do this during Lab 2.