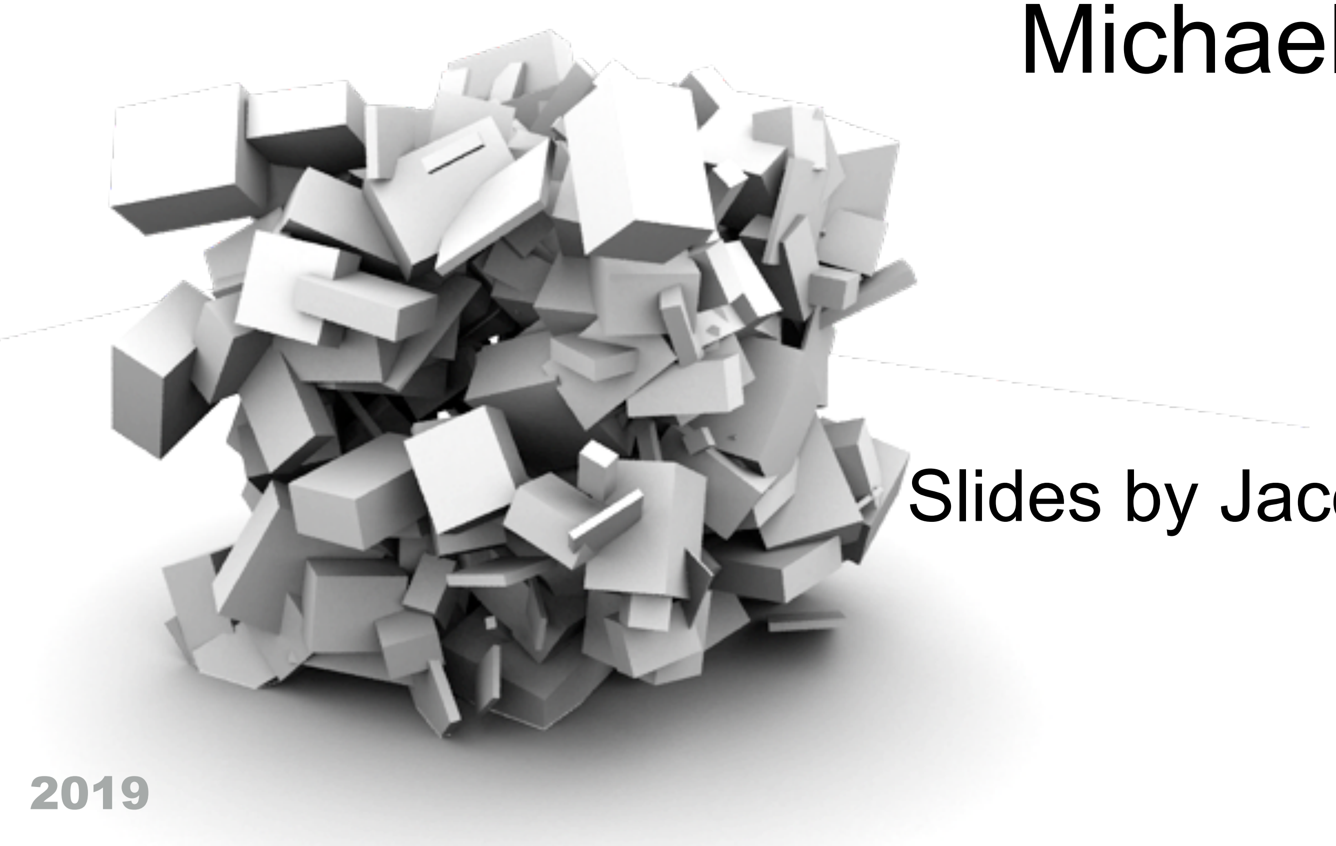
Computer Graphics Introduction to 3D

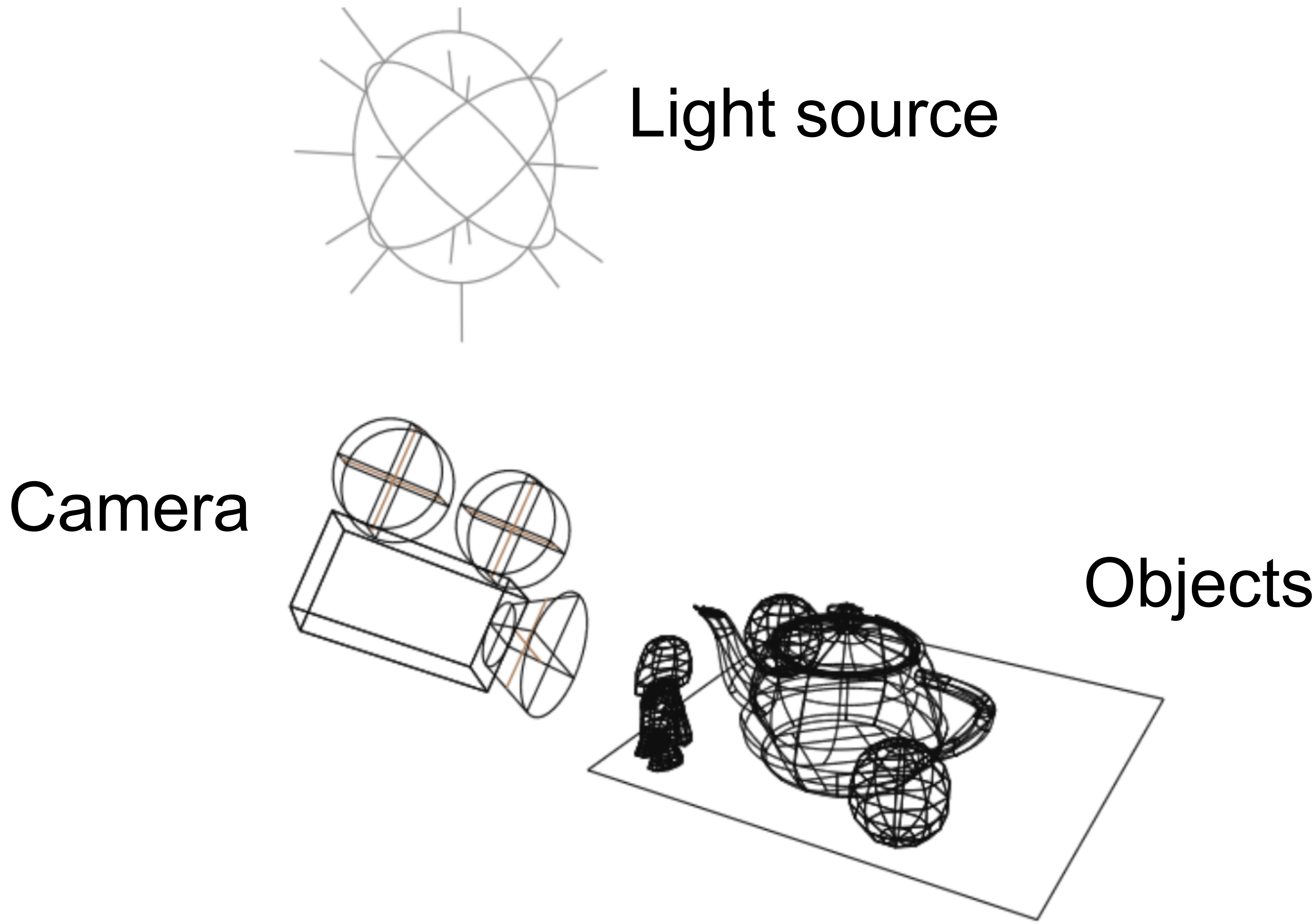
EDAF80
Michael Doggett



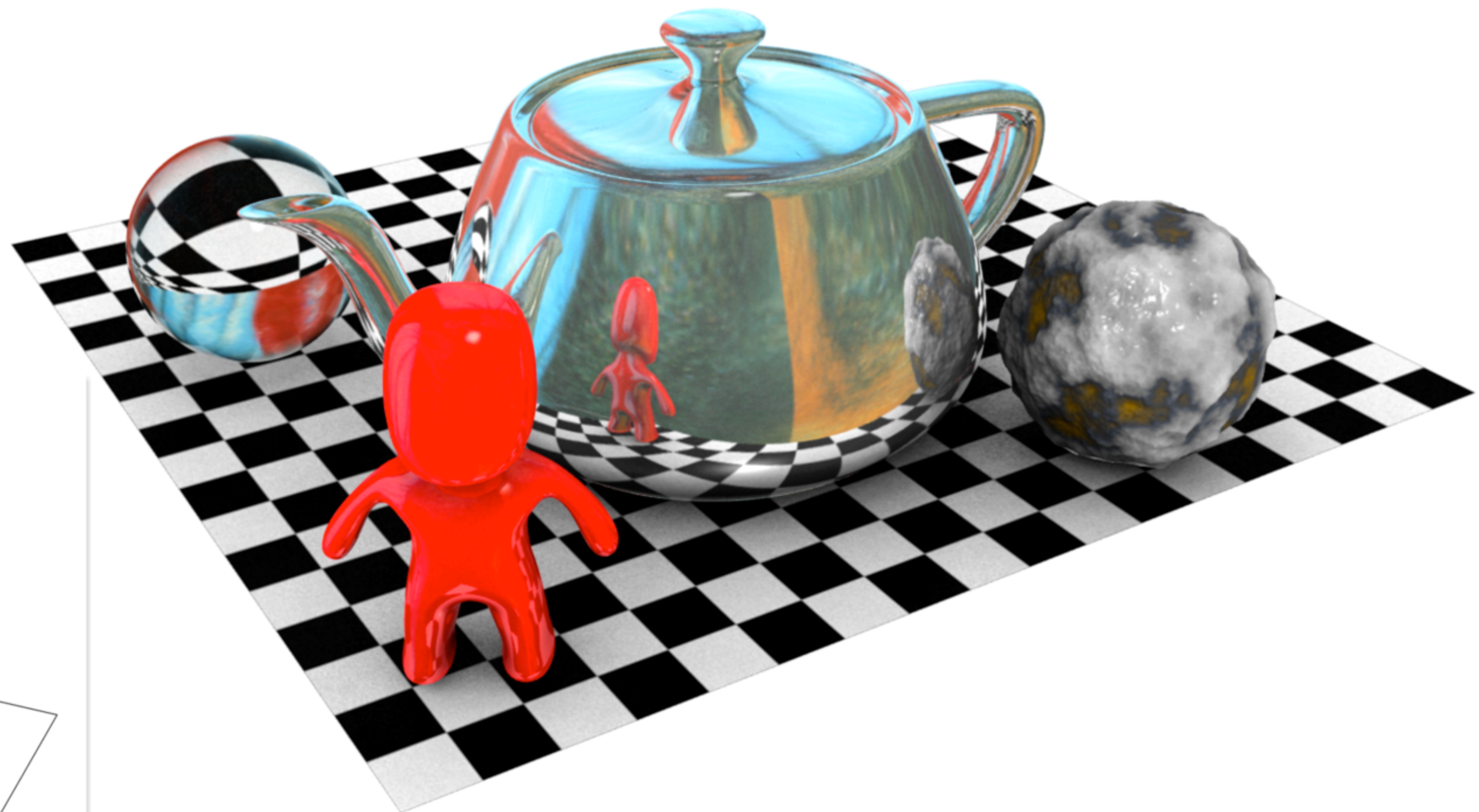
Slides by Jacob Munkberg 2012-13

2019

Create Virtual Scenes



Rendered Image

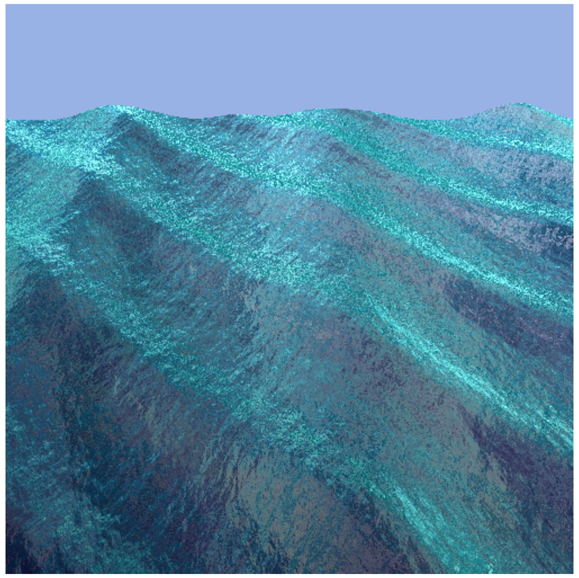


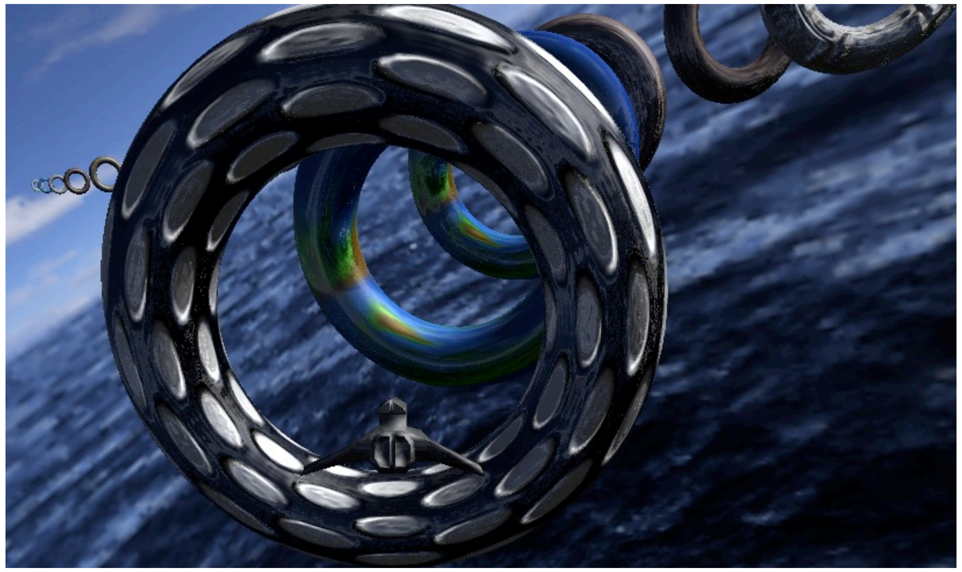
Course Goals

- Introduction to Computer Graphics
- Create and **position** objects in 3D
- Create materials - write **shaders**
- Visualize 3D scenes - **render** images
- Introduction to **OpenGL**
  - Create interactive graphics

Assignments

- Five mandatory programming assignments
- Done in pairs - book a lab!
  - Both students must be present at marking, and answer questions (including lab 5)
- Seminars: Wednesdays 15-17 in E:1406





Organization

- Lectures : Theory and concepts
- Seminars
  - Applied theory & hands-on examples
- Assignments
  - Computer Graphics in practice, C++, OpenGL & GLSL
- Examination
  - All five assignments approved
  - Written exam

Website

- cs.lth.se/edaf80
- Lectures will be posted online
- Online discussion forum
- Booking system for assignment approval sessions
- Code & assignments - GitHub
- Links

Course Material

- Literature
  - Edward Angel, Dave Shreiner,
    Interactive Computer Graphics: A Top-Down Approach with Shader-Based OpenGL,
    Pearson Education, 6th edition
  - Lectures & Seminars
  - Source Code Handouts for Lab 4
- Prerequisites
  - Programming (first course) & Linear Algebra

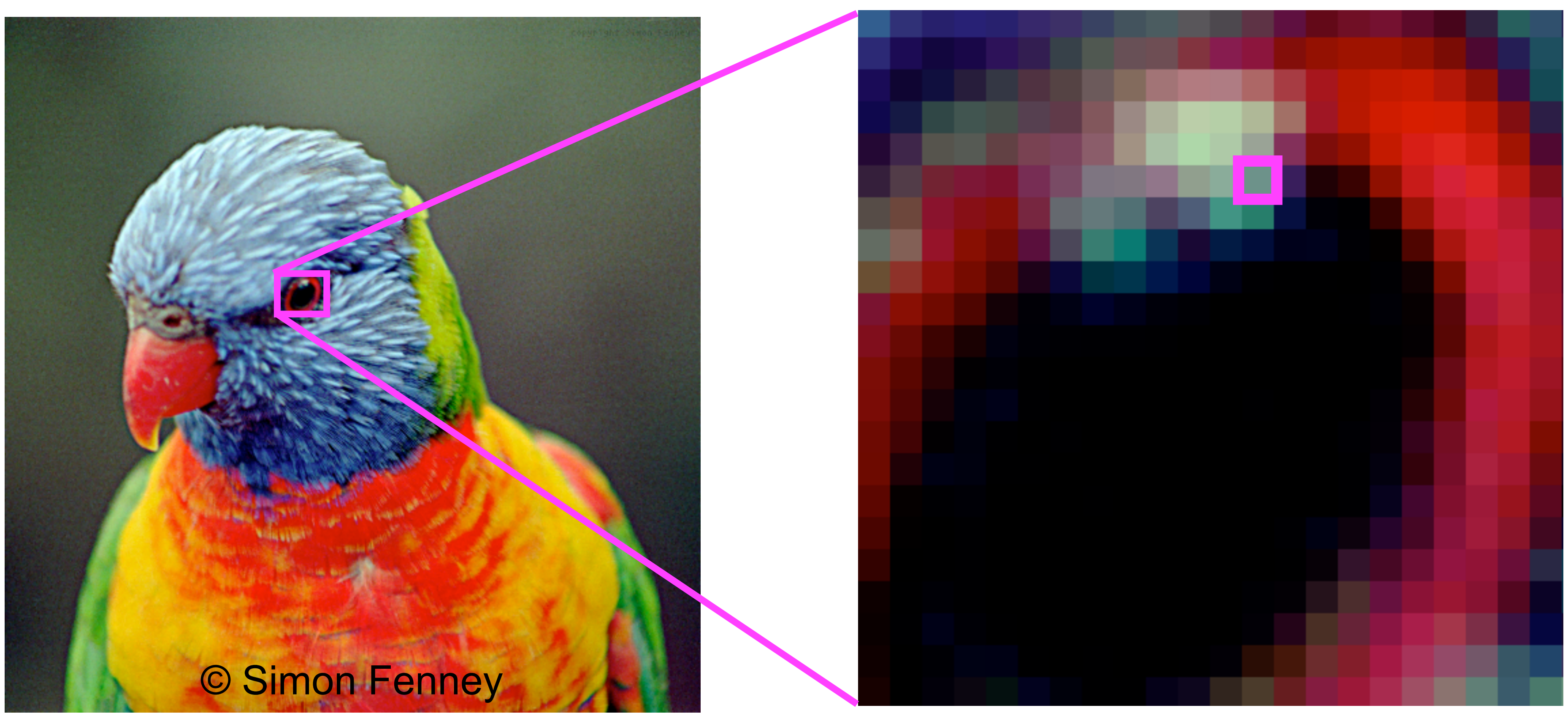
Staff

- Michael Doggett - Lectures, Seminars, Exam
  - Research Scientist at Facebook Reality Labs, Seattle, Docent, GPU architect at ATI/AMD, Post-doc in Germany, PhD in Australia
- Pierre Moreau, Marcus Klang, Tom Hansson
  - PhD students in Computer Graphics/NLP

Definitions

Pixel

- Pixel - Picture element
- Our task - compute color of each pixel



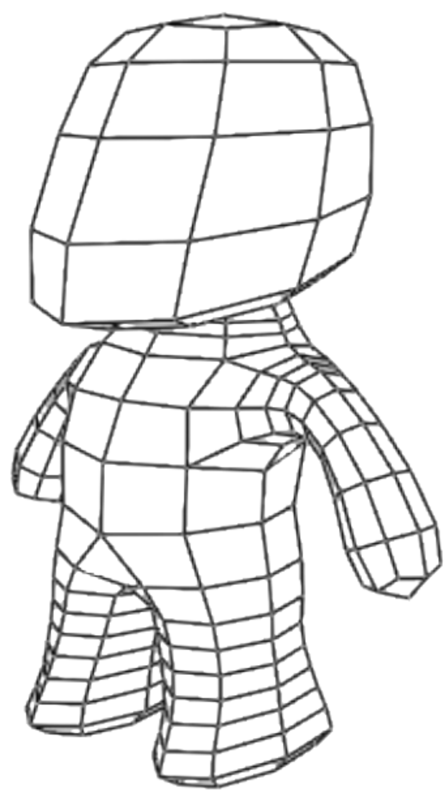
512x512 pixels

22x22 pixels

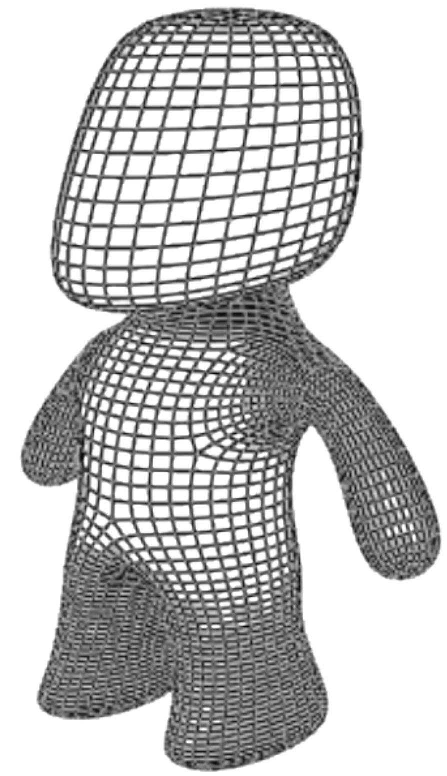
Number of Pixels

- Smartphone (1920 x 1080)
- iPad retina (2048 x1536)
- MacBook Pro retina (2880 x1800)
- 4K TV “Ultra HD” (3840 x 2160)
- Apple iMac (5120 x 2880)
- Hasselblad H4D-60: 60 Mpixel

Image Formation



Mesh



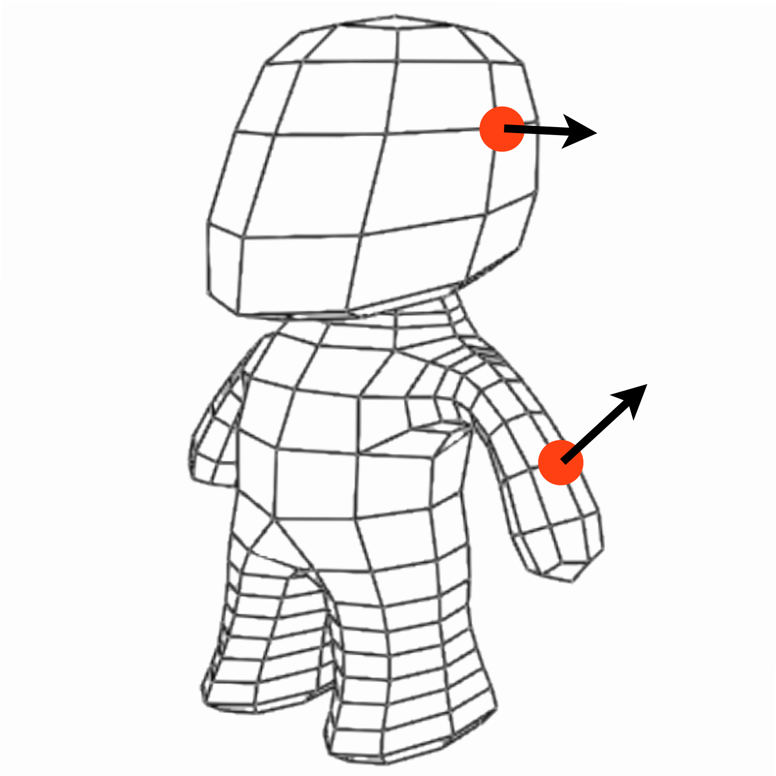
Tessellated Mesh

Lit & Shaded Mesh

We need ways to represent geometry and move objects in three-dimensional coordinate systems

Vertex

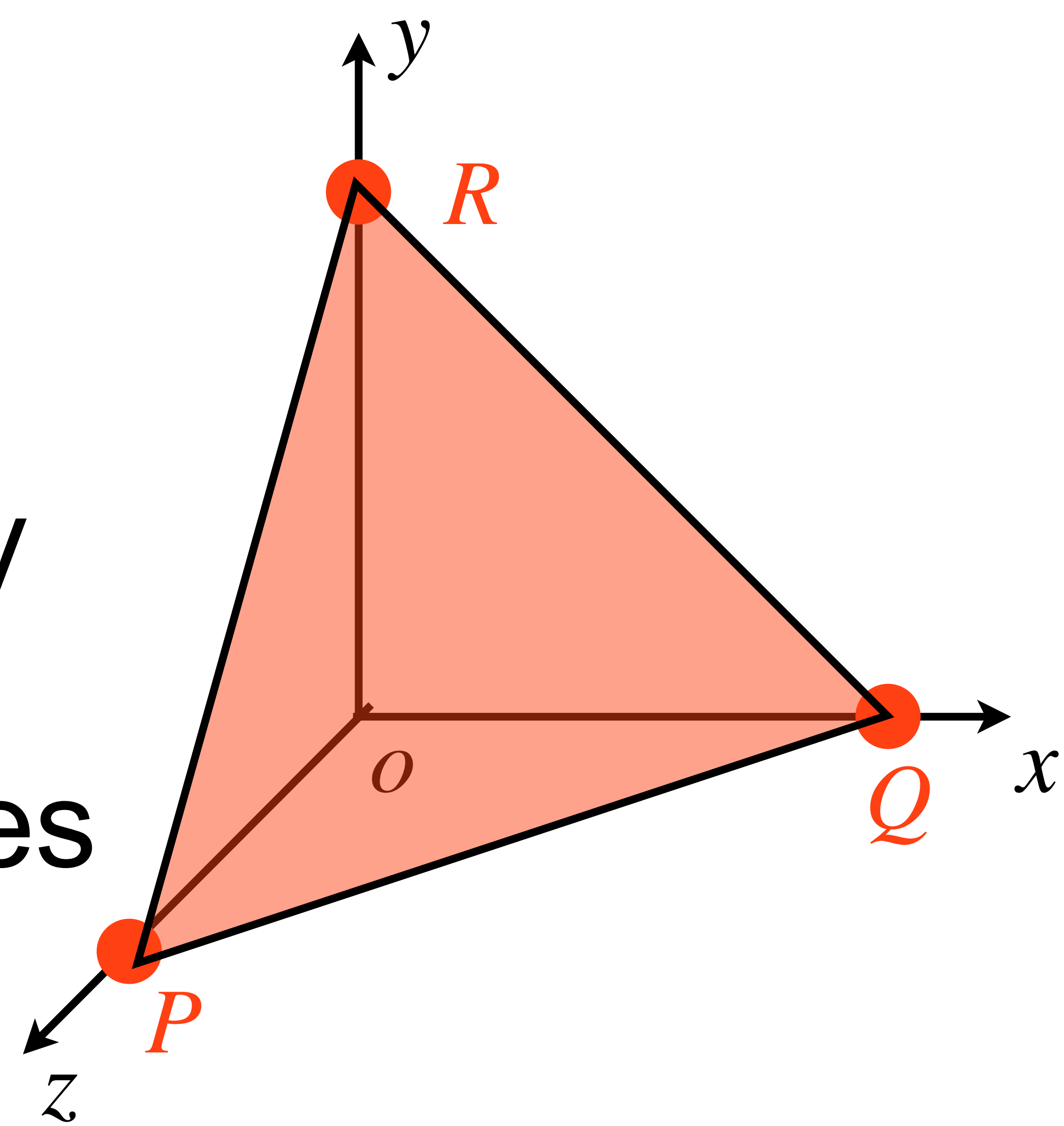
- A set of attributes describing a point in space



```
struct Vertex
{
    float x,y,z;        // pos
    float nx, ny, nz;   // normal
    float r,g,b;        // color
};
```

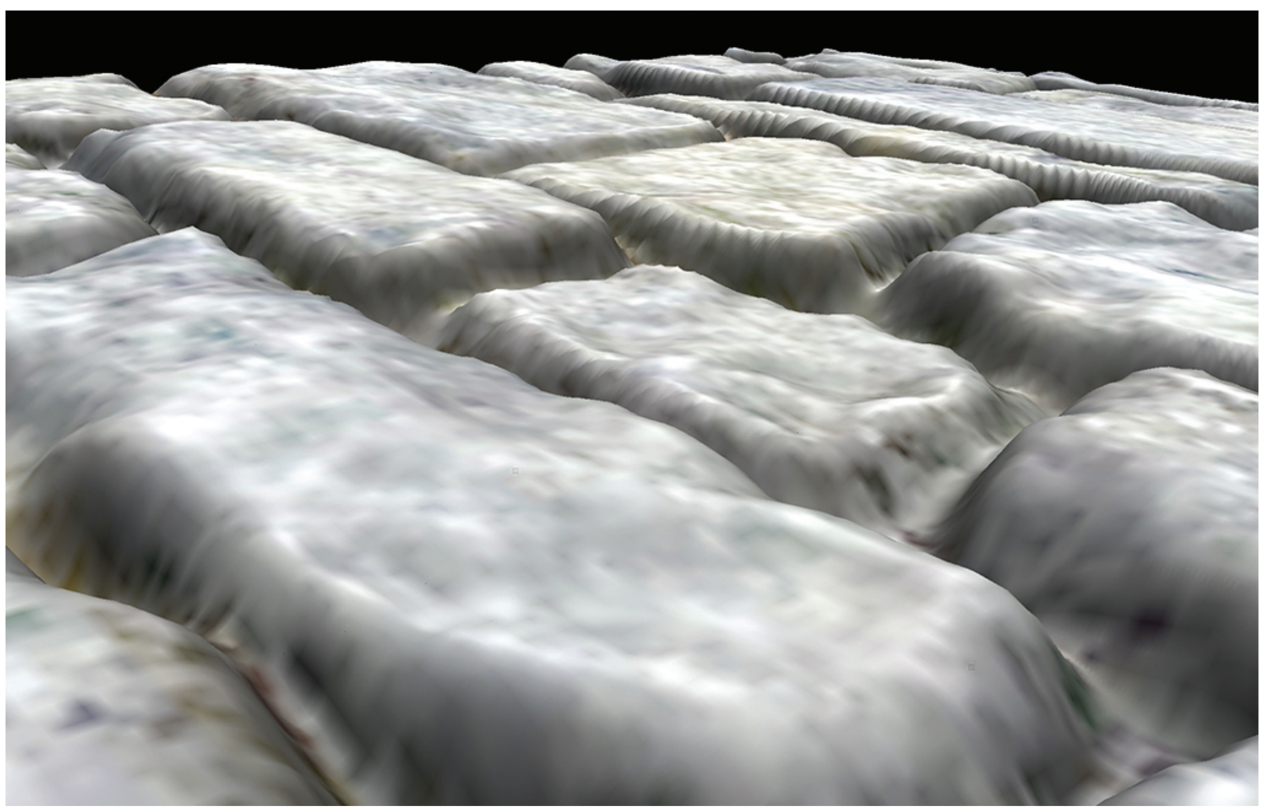
Triangle in 3D

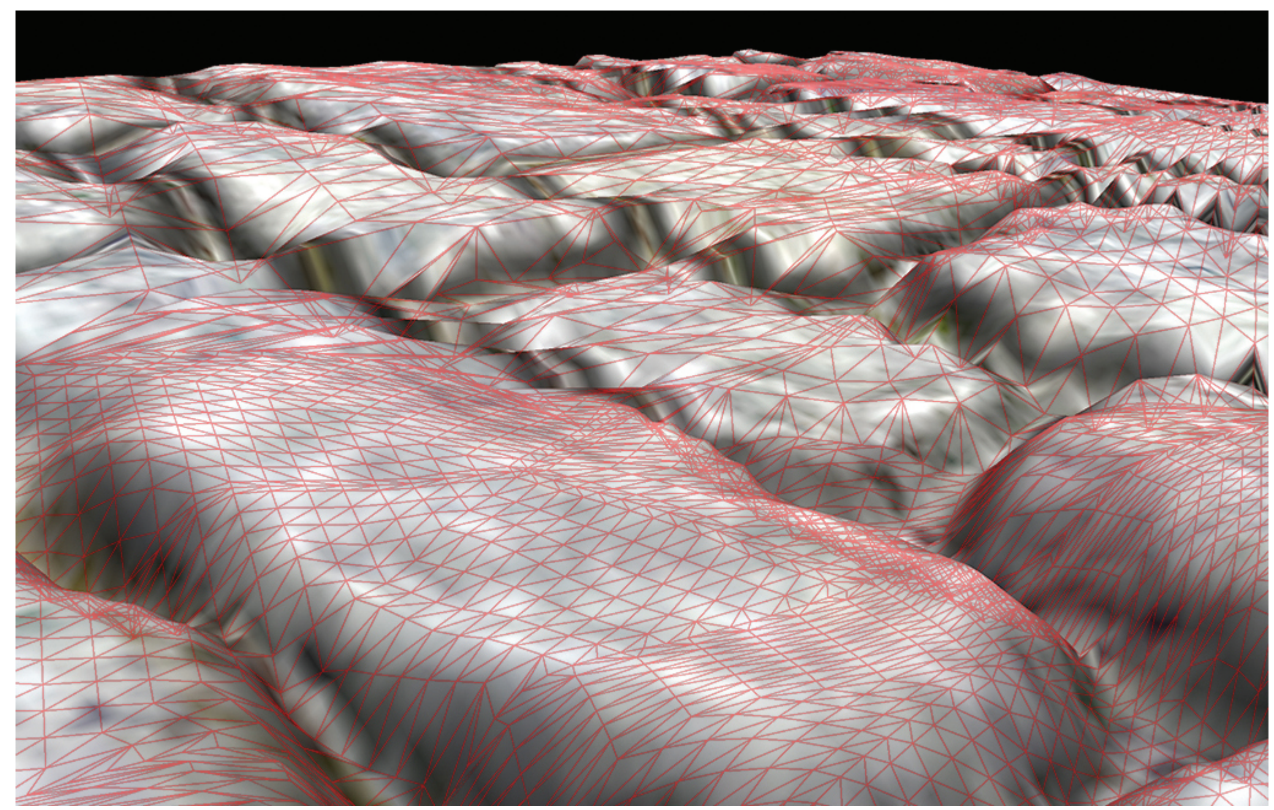
- Defined by three connected vertices
  - Here: specified in the Cartesian system
- Models are typically built from a large collection of triangles



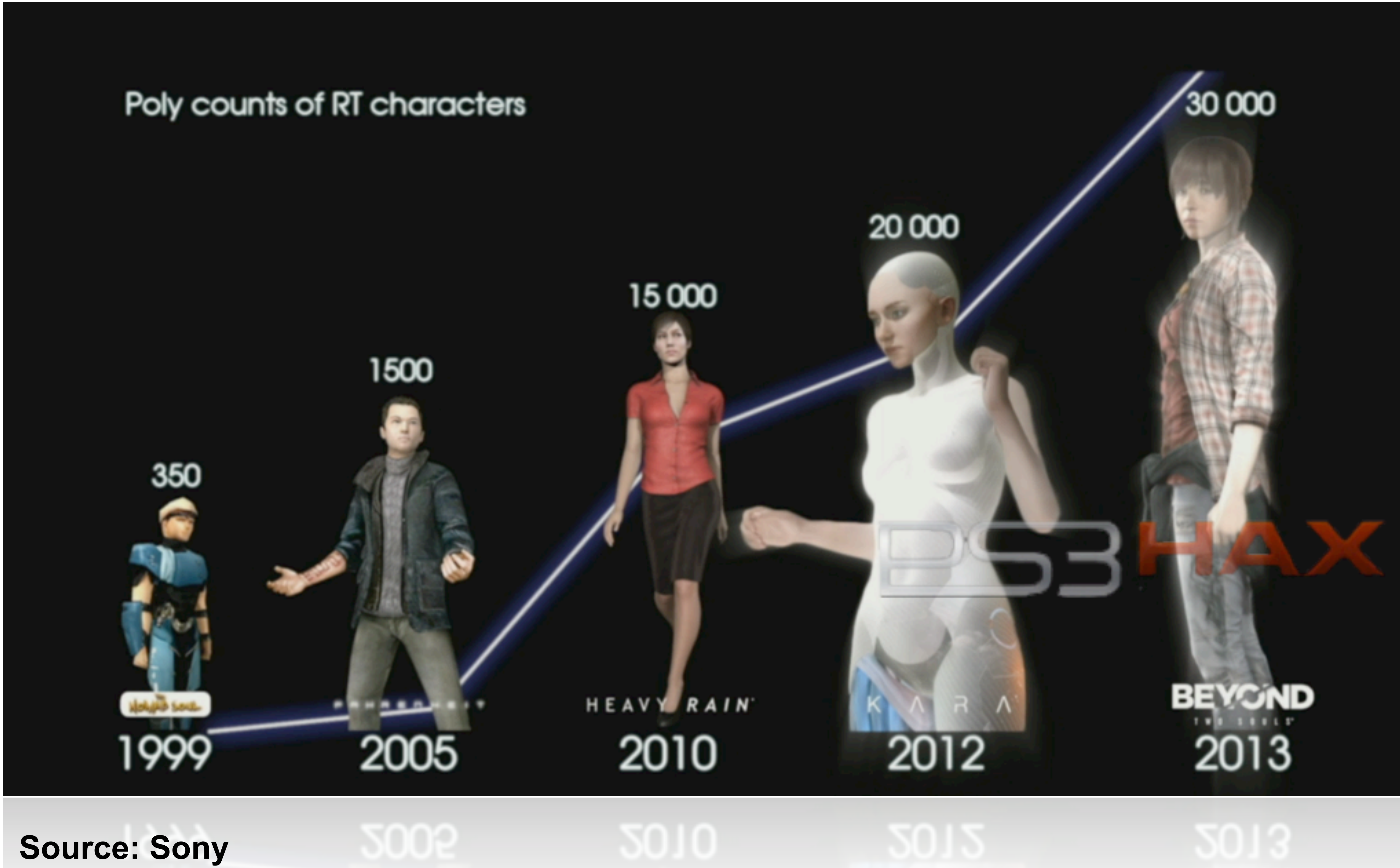
Geometry Meshes

- Model complex shapes from triangles



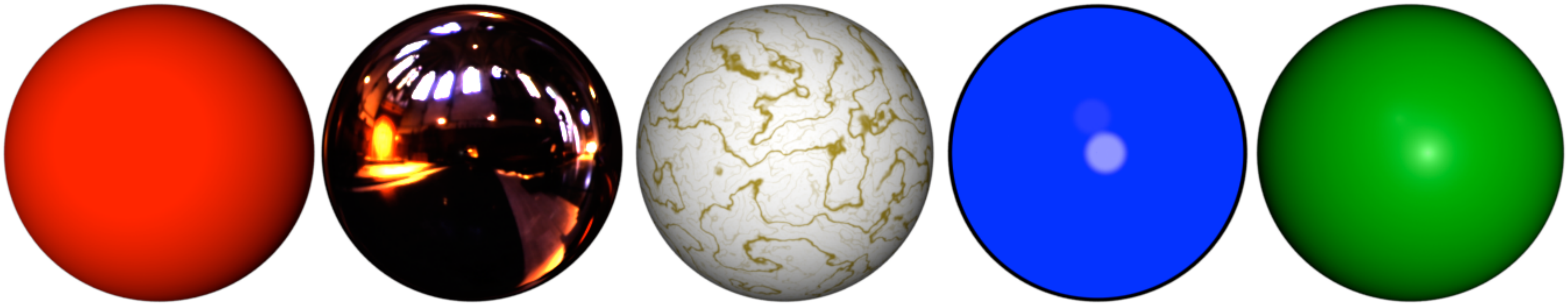


Poly counts of RT characters



Source: Sony

Materials



- Determine the appearance of objects
  - How objects interact with light
- Specified as “small” programs called **shaders**

Simple Pixel Shader

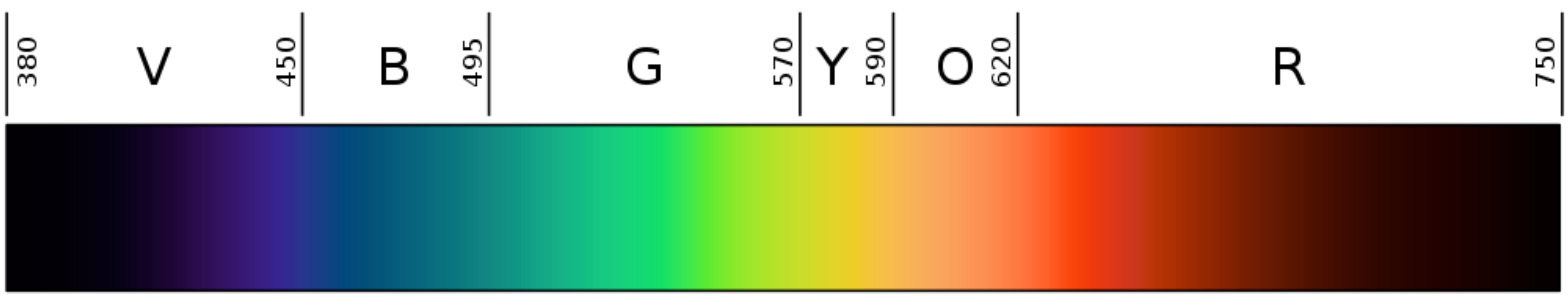
- Set pixel color to red

```
out vec4 fColor;

void main()
{
    fColor = vec4(1,0,0,1);
}
```

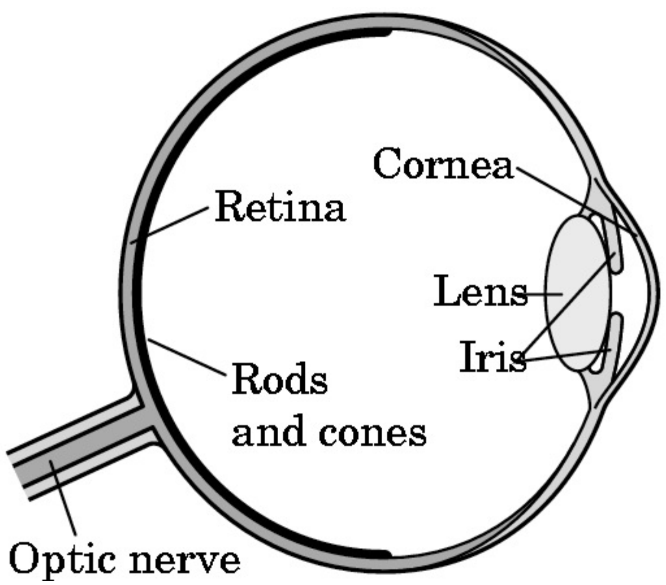
Light

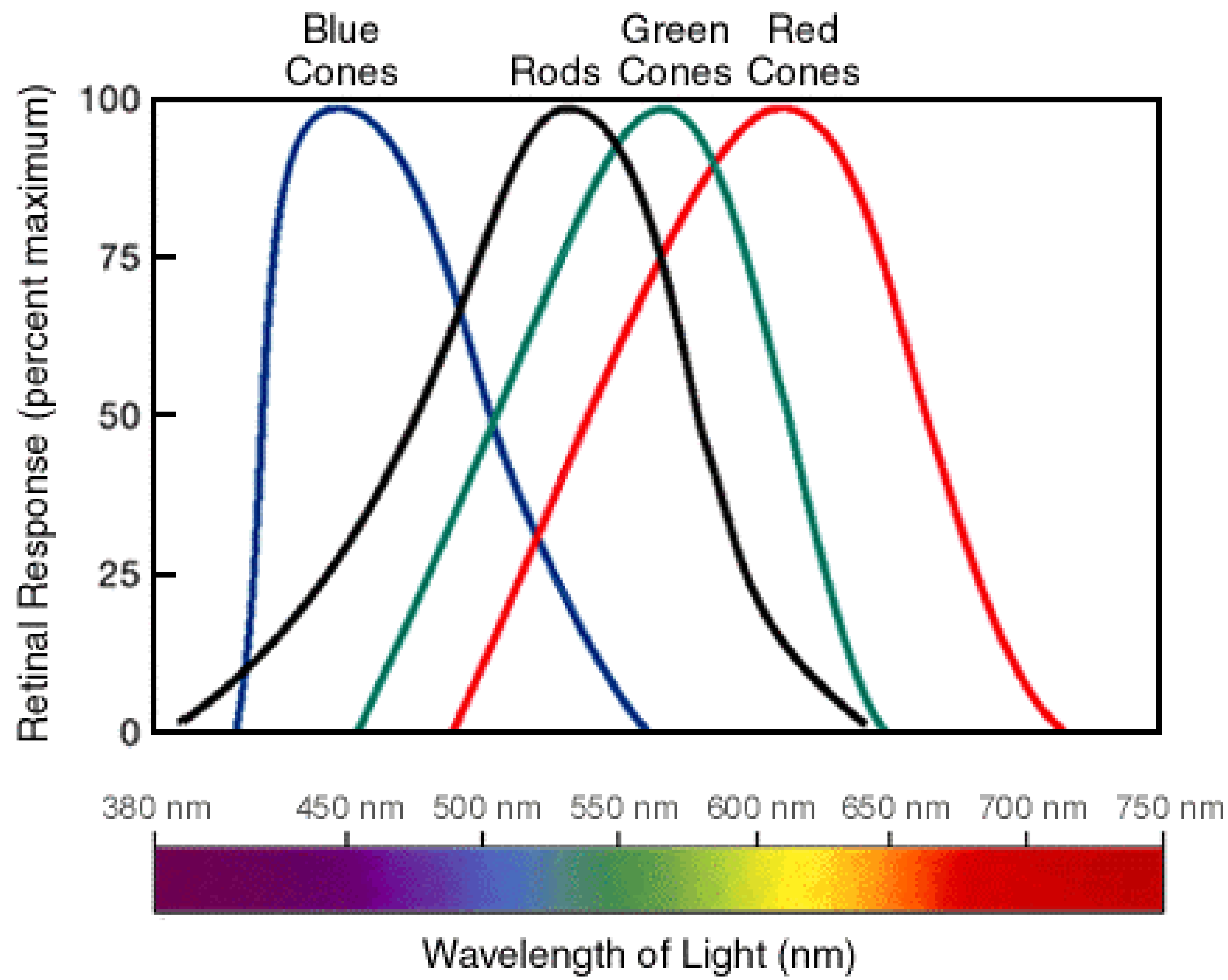
- Light is the part of the electromagnetic spectrum that causes a reaction in our visual systems
- Wavelengths in 380-750 nanometers
- Long wavelengths appear as reds and short wavelengths as blues



Color Perception

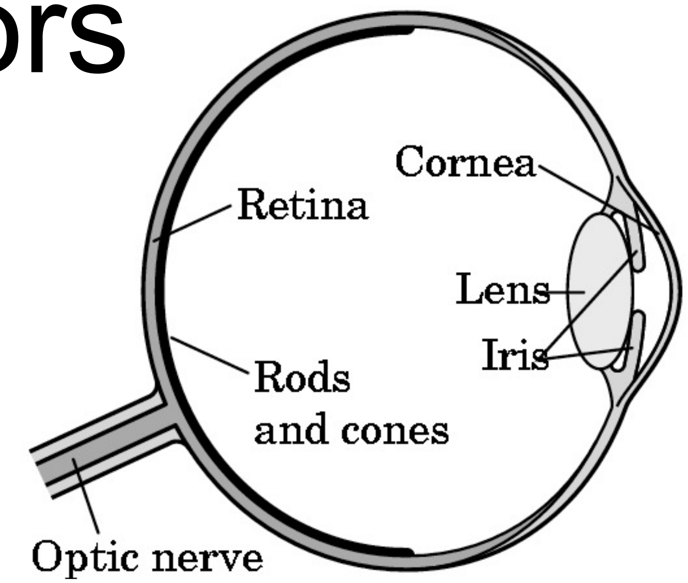
- Color stimulates cones in the retina
- Three different kinds of cones





Human Visual System (HVS)

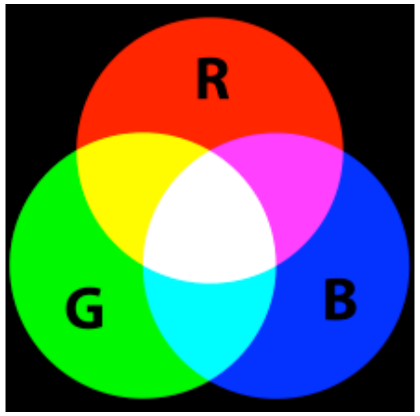
- HVS has two types of sensors
  - **Rods**: monochromatic, night vision
  - **Cones**: Color sensitive, three types of cones
  - Only three values (the *tristimulus* values) are sent to the brain



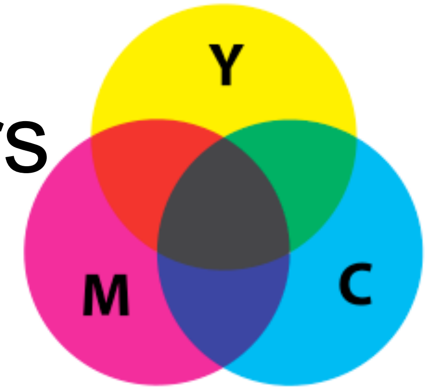
- Need only match these three values
- Need only three primary colors: RGB

Color

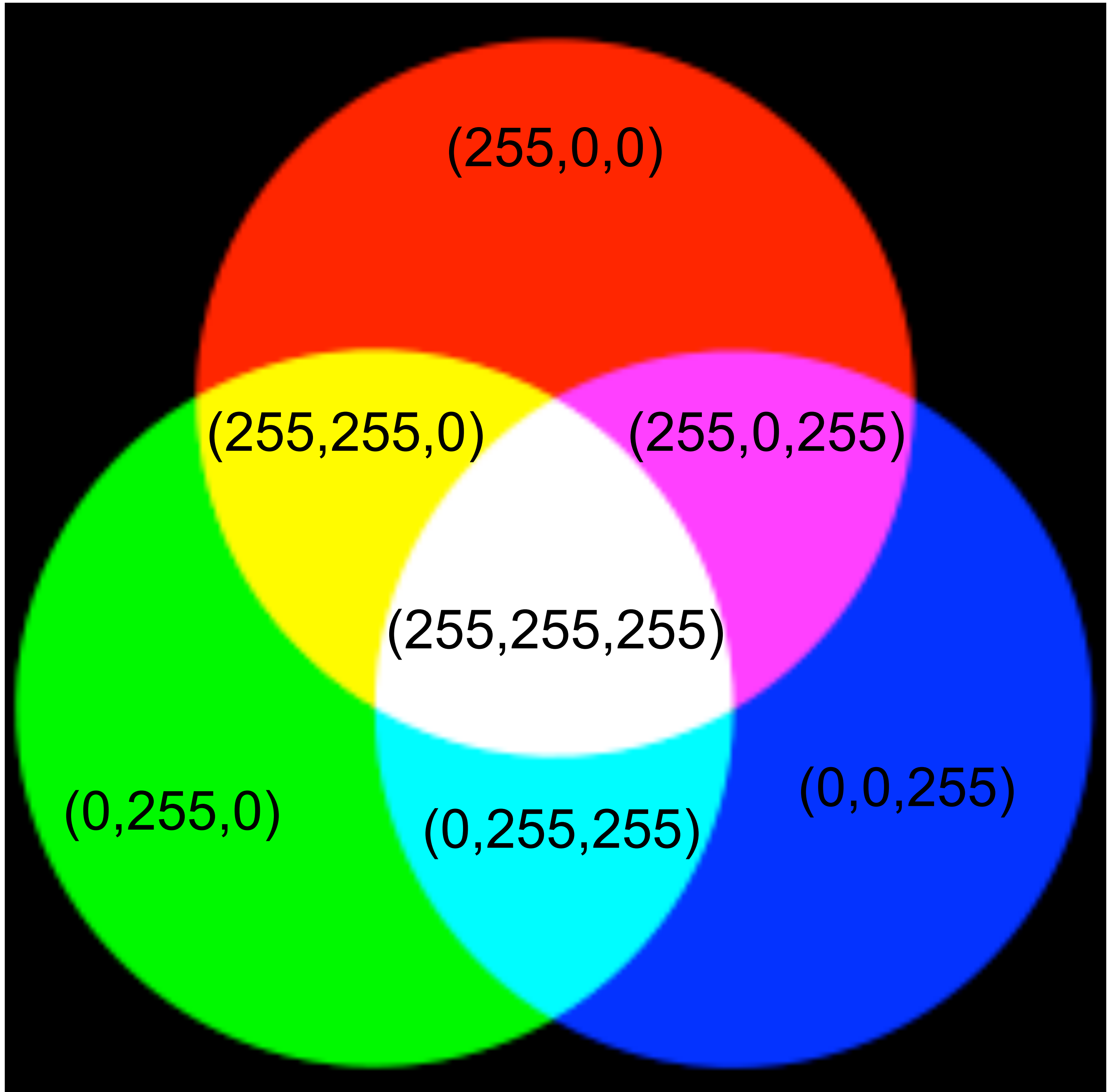
- Additive color
  - Form a color by adding amounts of three primaries: Red (R), Green (G), Blue (B)
  - Often stored using 8 bits per primary which gives $(2^8)^3$ = 16.8M unique colors



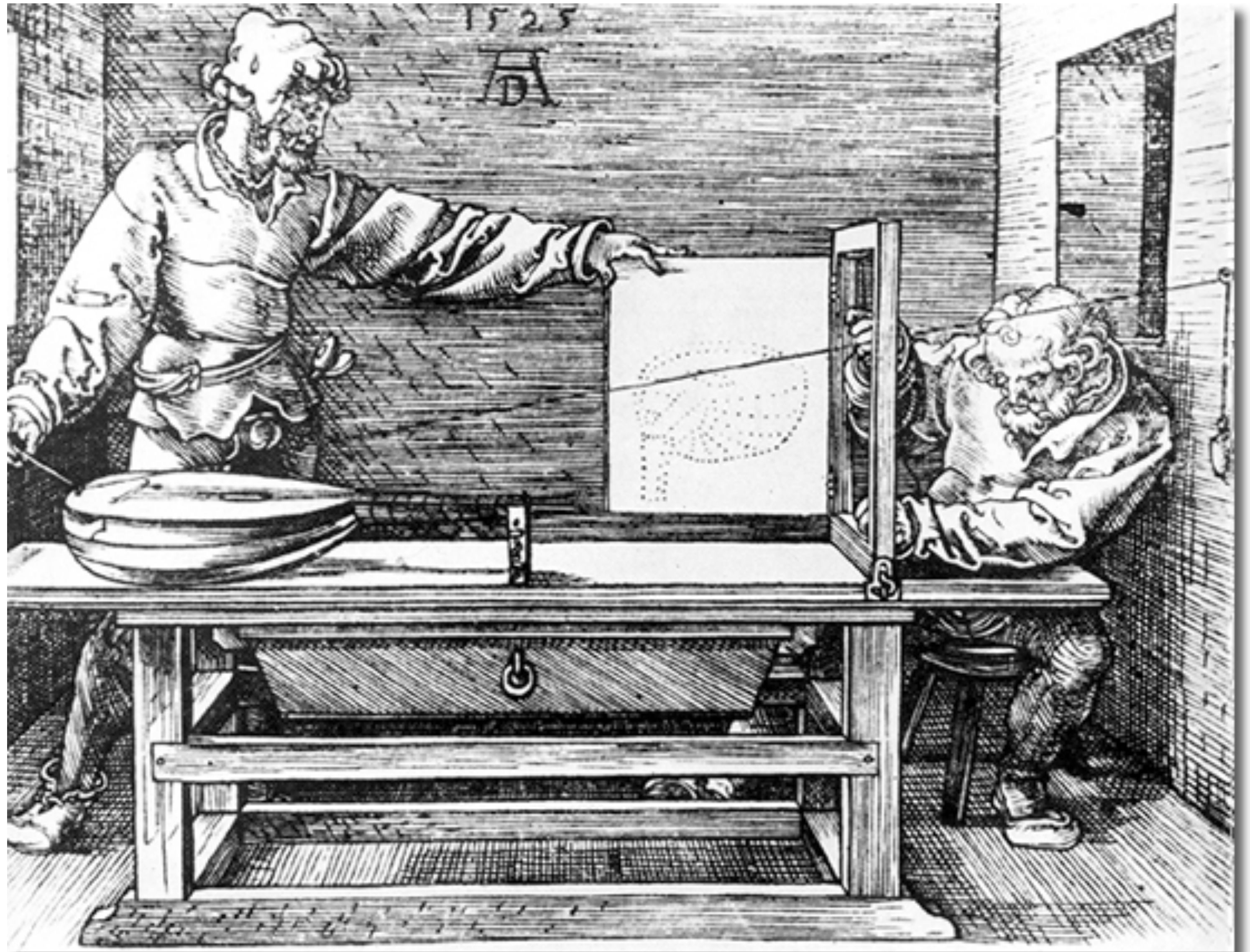
- Subtractive color
  - Form a color by filtering white light with Cyan (C), Magenta (M), and Yellow (Y) filters
  - Light-material interactions, printing



RGB Color

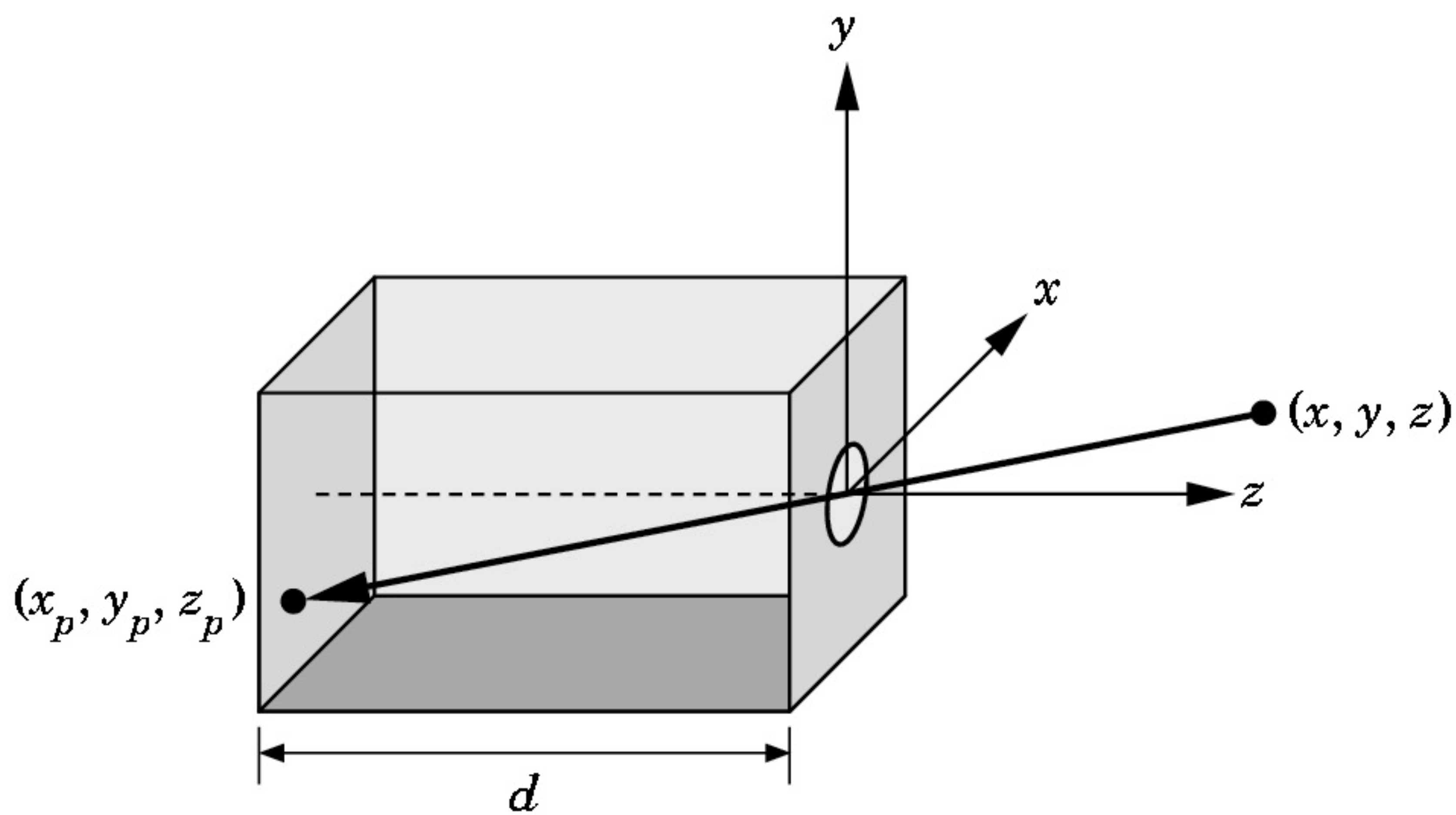


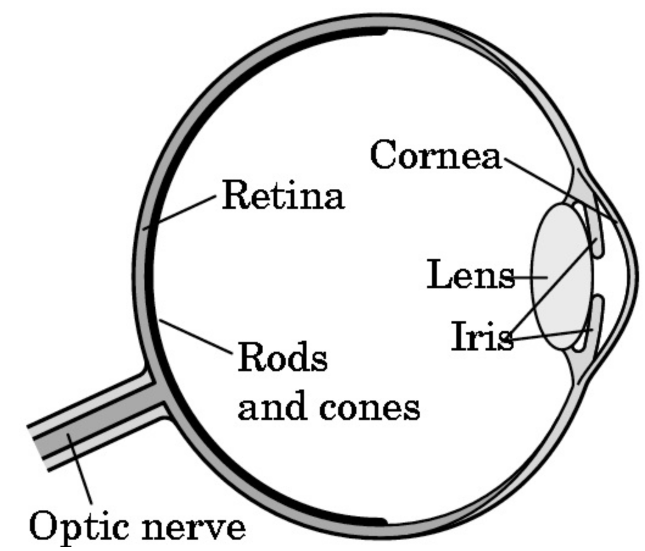
Rendering a 2D image



Perspective drawing in the Renaissance: “Man drawing a lute” by Albrecht Dürer, 1525

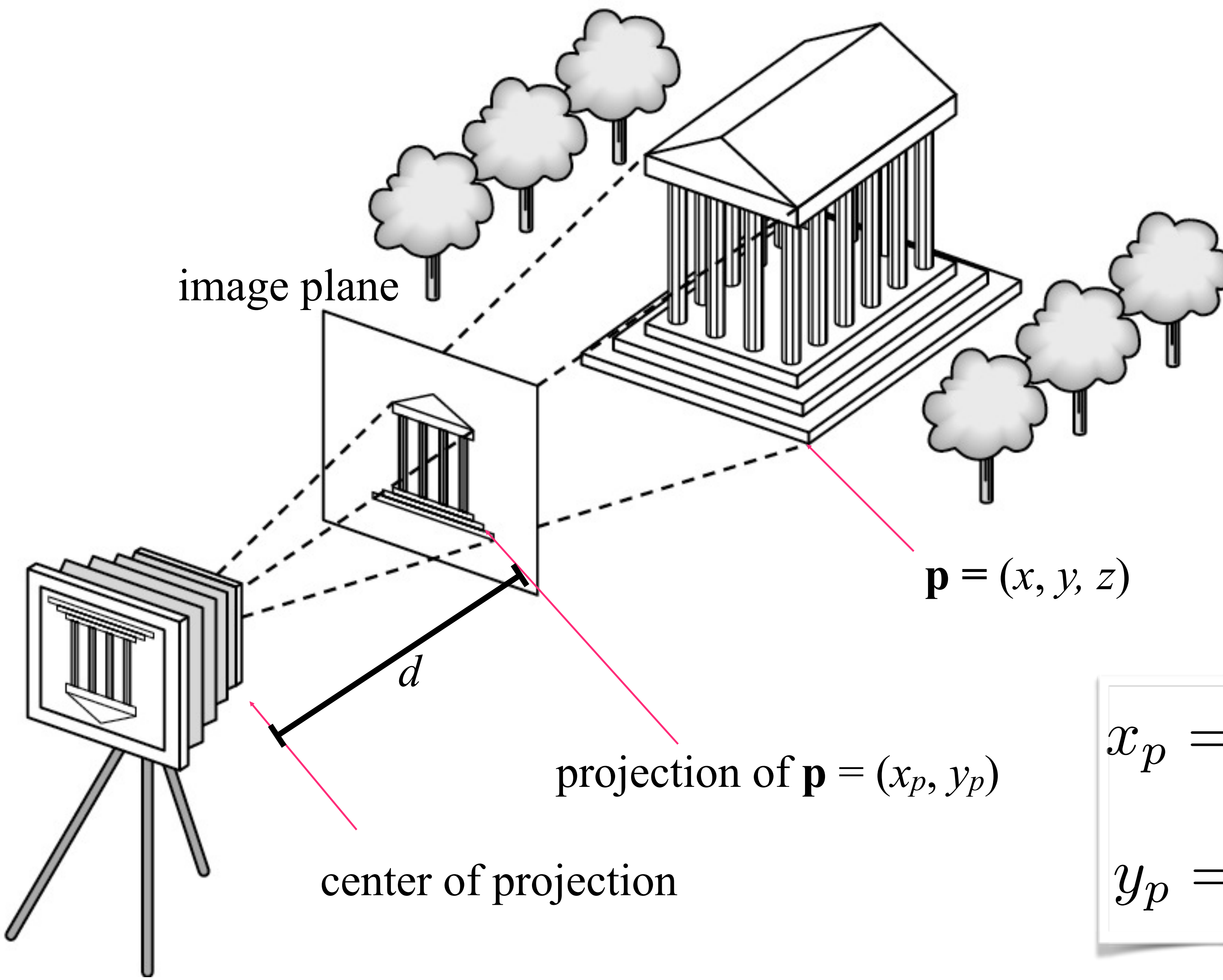
Pinhole Camera





- Projection of a 3D point (x,y,z) on image plane: $x_p = -d\frac{x}{z}, \quad y_p = -d\frac{y}{z}$
  - Equal triangles: $\frac{x}{z} = \frac{x_p}{z_p} \Leftrightarrow x_p = z_p\frac{x}{z} = -d\frac{x}{z}$

Synthetic Camera Model



$$x_p = d\frac{x}{z}$$

$$y_p = d\frac{y}{z}$$

Image Formation

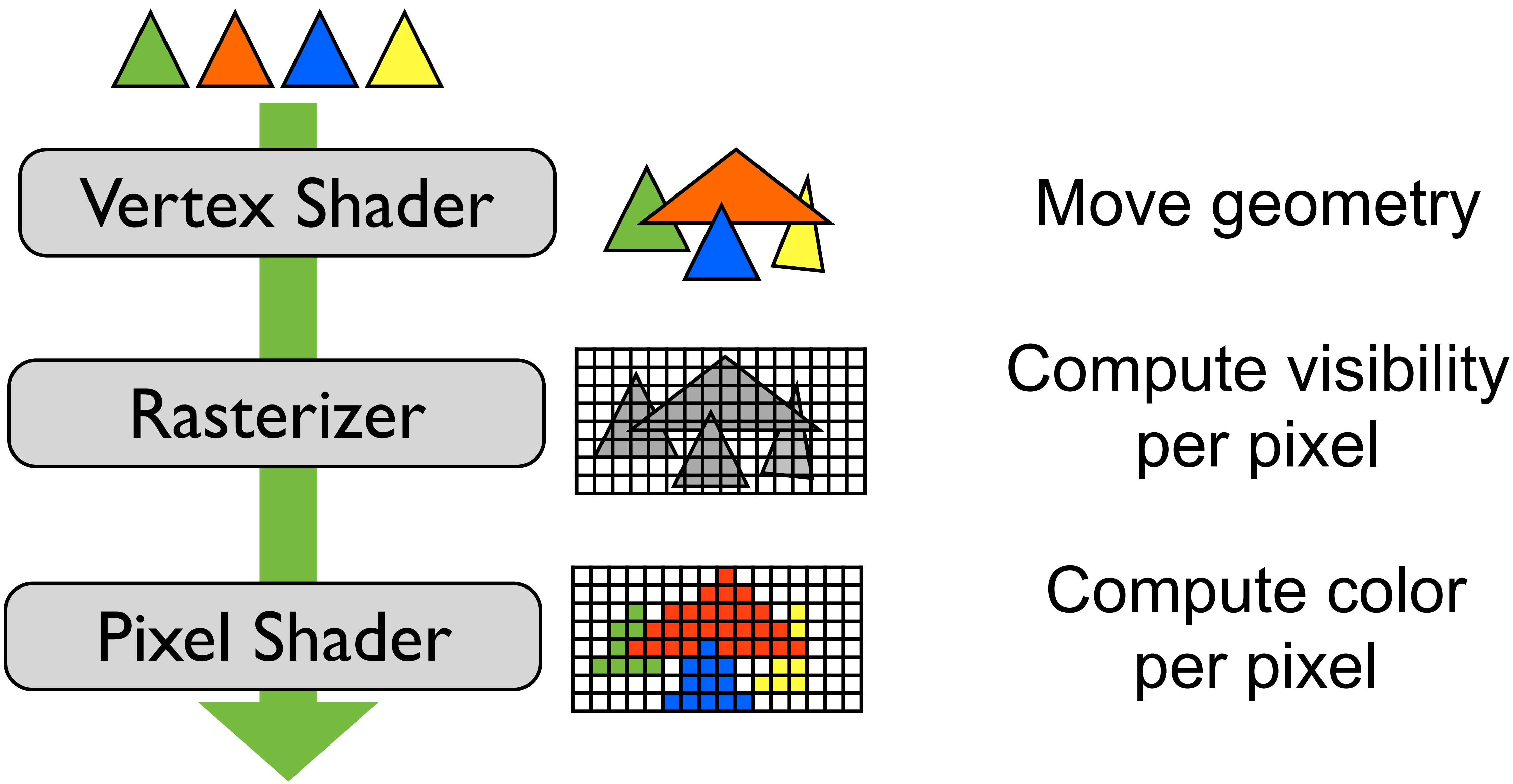
- Create geometric models
  - Position the models in a 3D scene
  - Assign materials to each model
  - Add lights & position a virtual camera
- For each pixel - find visible object
- Compute color of pixel based on the visible object's material and light

Challenges

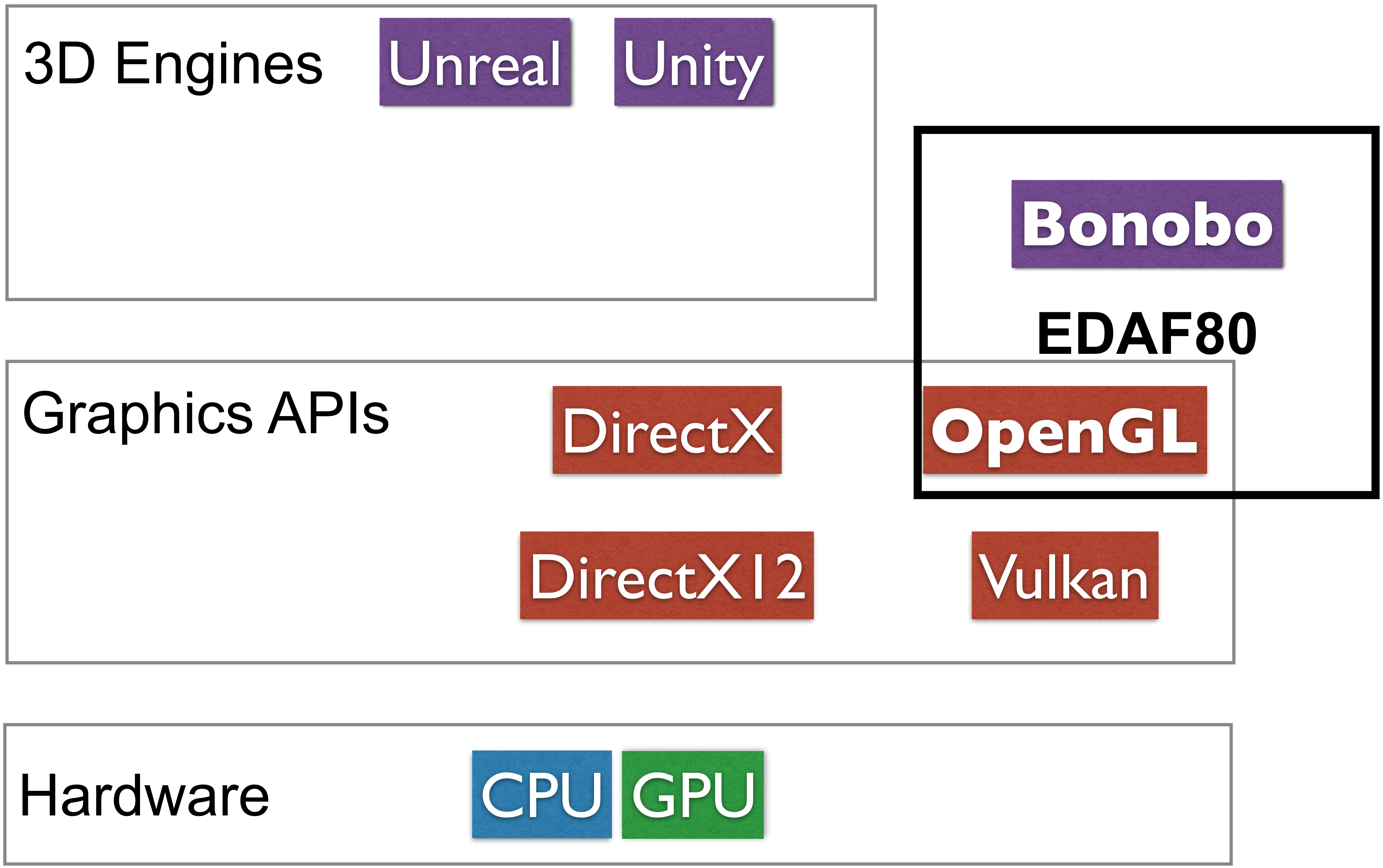
- Create geometric models 1M triangles
  - Position the models in a 3D scene (Move 1M tris)
  - Assign materials to each model
  - Add lights & position a virtual camera
- For each pixel (5M), compute which triangle (of the 1M tris) that is visible
- Compute color of pixel (5M) based from the object's material and light

Graphics Hardware

- **Pipeline** that accelerates the costly tasks of rendering



Graphics layers



Real-time vs Offline

- Real-time
  - Render image in ~16 ms
  - Instant feedback
  - User interactions
- Offline (feature films)
  - Each image may take hours or days
  - Photorealism
  - No user interaction

Real-Time 2019

- RTX Quake
- Control

Offline

- Alita

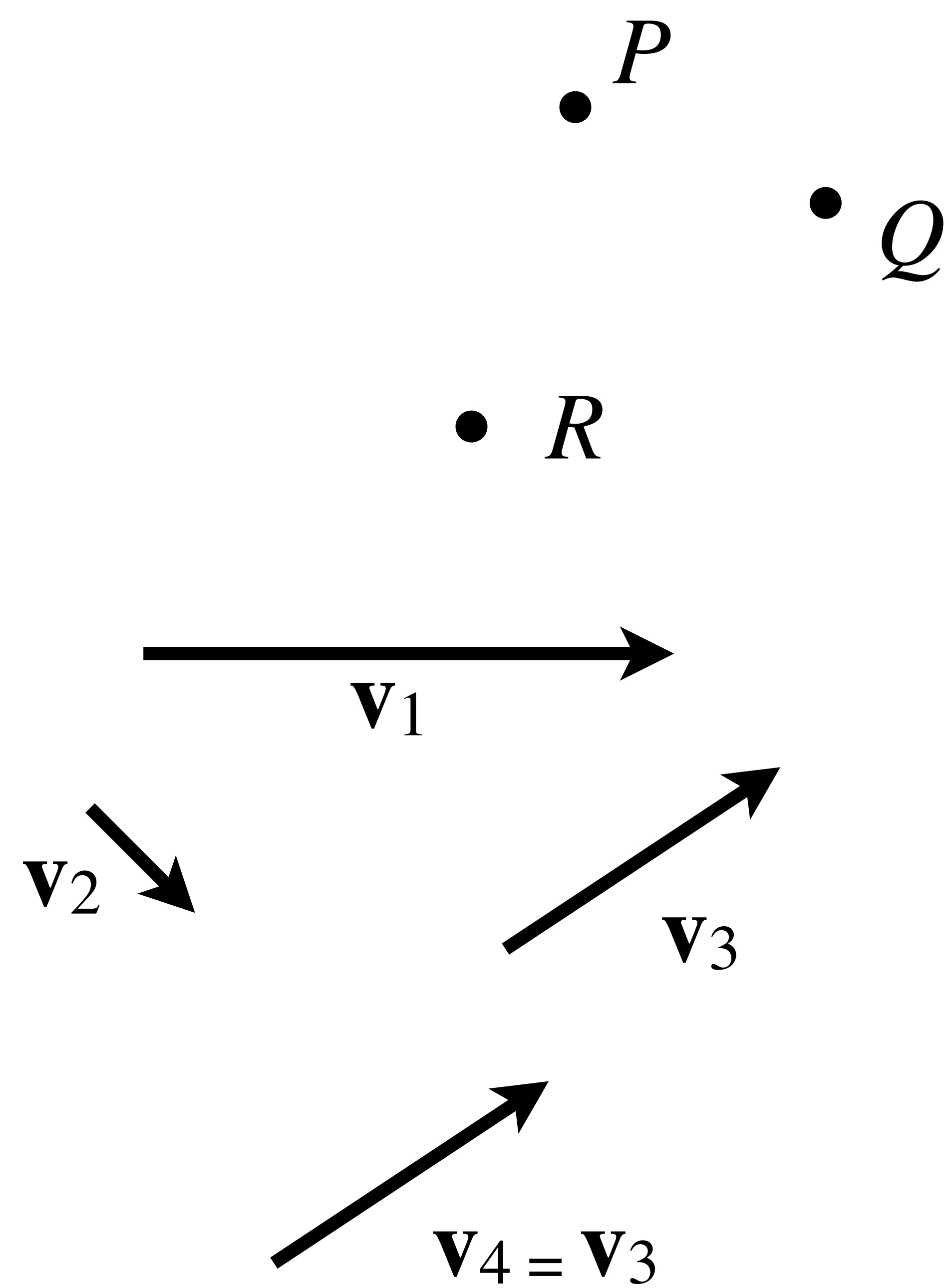
Linear Algebra

Linear Algebra

- Points vs Vectors
- Angles between vectors
- Dot (scalar) product
- Cross product
- Coordinate system

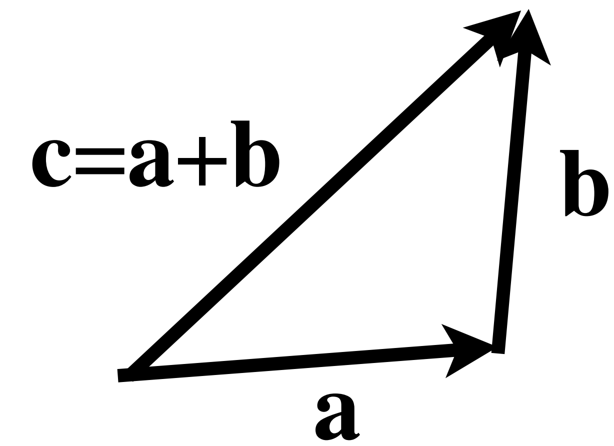
Points & Vectors

- A **point** is a location in space
- A **vector** represents a direction and magnitude

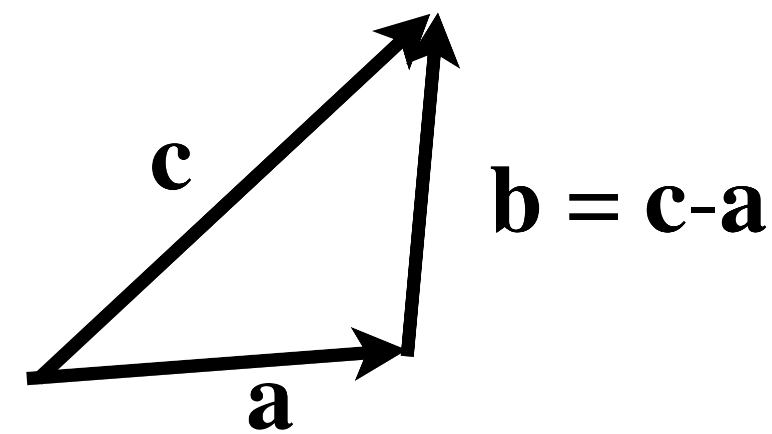


Basic Operations

- Vector-vector addition

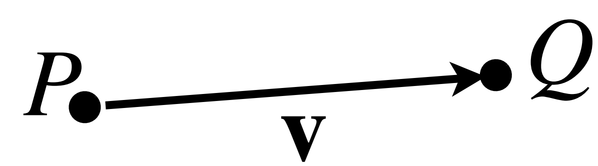


- Vector-vector subtraction



- Point-point subtraction

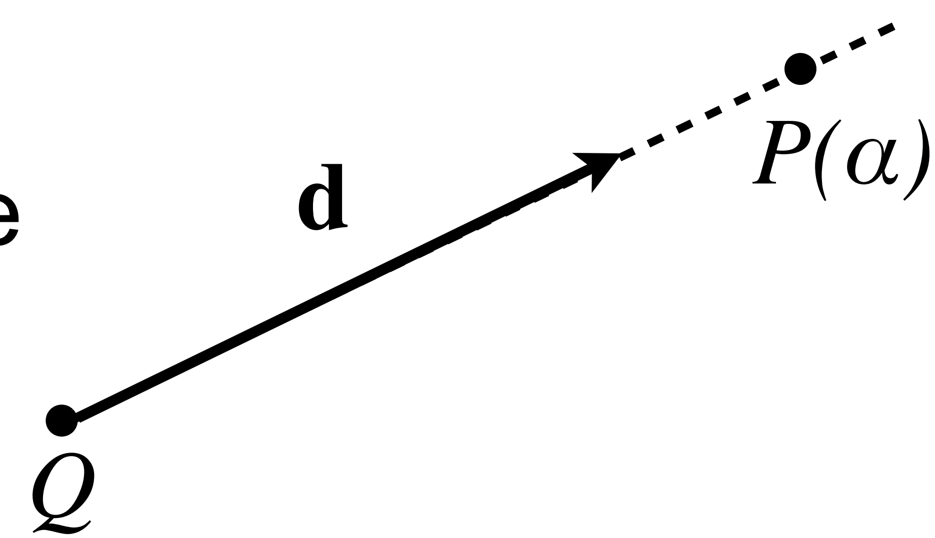
$$\mathbf{v} = Q - P \Leftrightarrow Q = P + \mathbf{v}$$



Lines

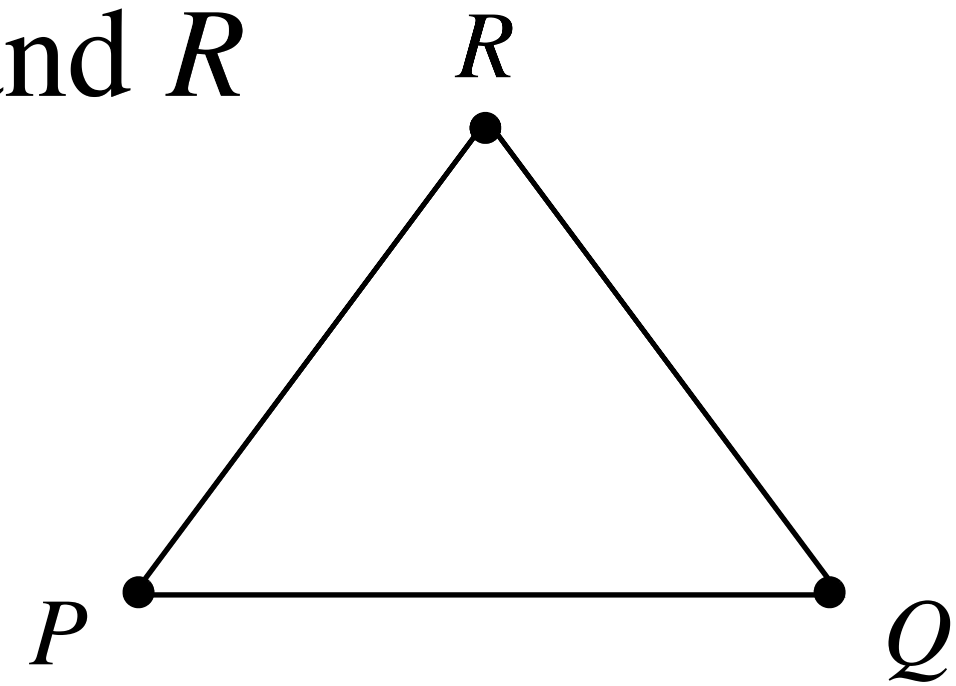
- Parametric form
  - Start with a point Q and a vector $\mathbf{d}$
  - If $\alpha>0$, the line is called the **ray** from Q in the direction $\mathbf{d}$
  - α is a scalar parameter that determines how far we have travelled along the line

$$P(\alpha) = Q + \alpha\mathbf{d}$$



Triangle

- Defined by three points P, Q and R

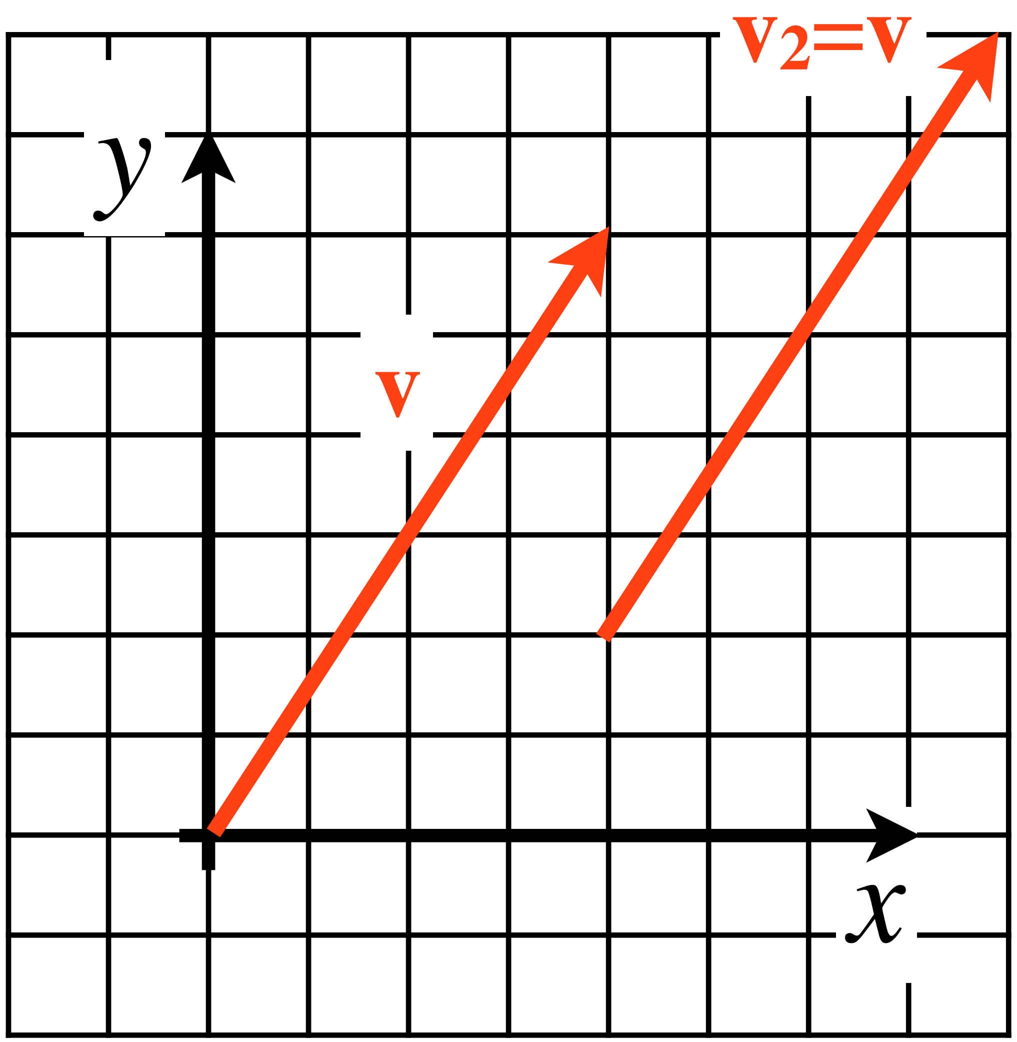


- Points inside triangle: $wP + uQ + vR$
- u,v,w : barycentric coordinates

$$u + v + w = 1$$

$$u, v, w \geq 0$$

Cartesian Coordinates



$$\mathbf{v} = 4\mathbf{x} + 6\mathbf{y}$$

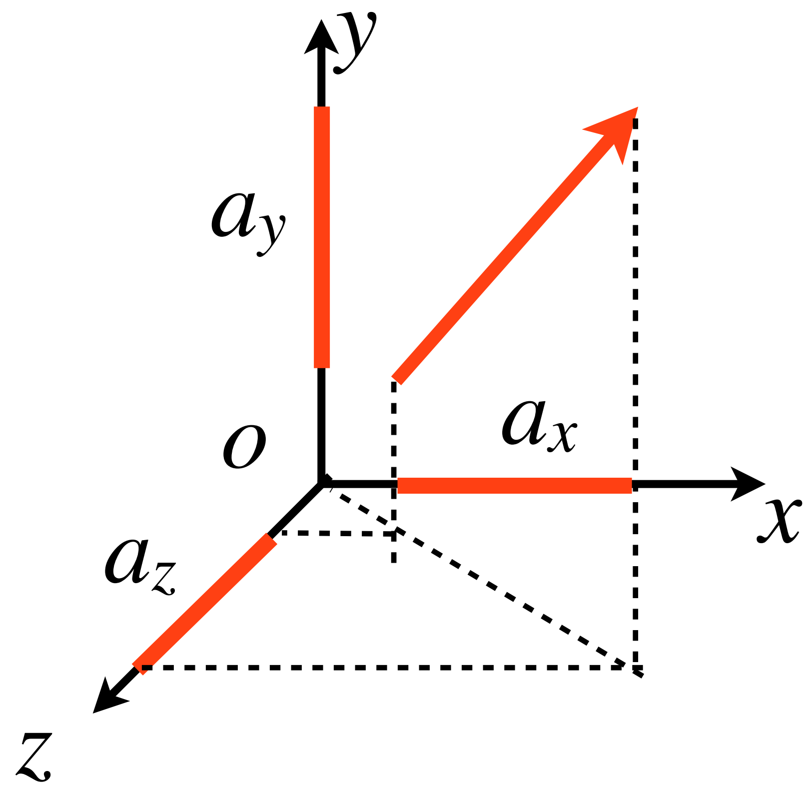
$$= 4\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 6\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|\mathbf{v}| = \sqrt{(4^2 + 6^2)}$$

Cartesian Coordinates in 3D

- Express vector in terms of three orthonormal basis vectors

$$\mathbf{a} = a_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + a_y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + a_z \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{Basis}} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$



In this basis, the vector $\mathbf{a}$ can be expressed as

$$\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Vector notation

- To avoid clutter, we introduce the notation:

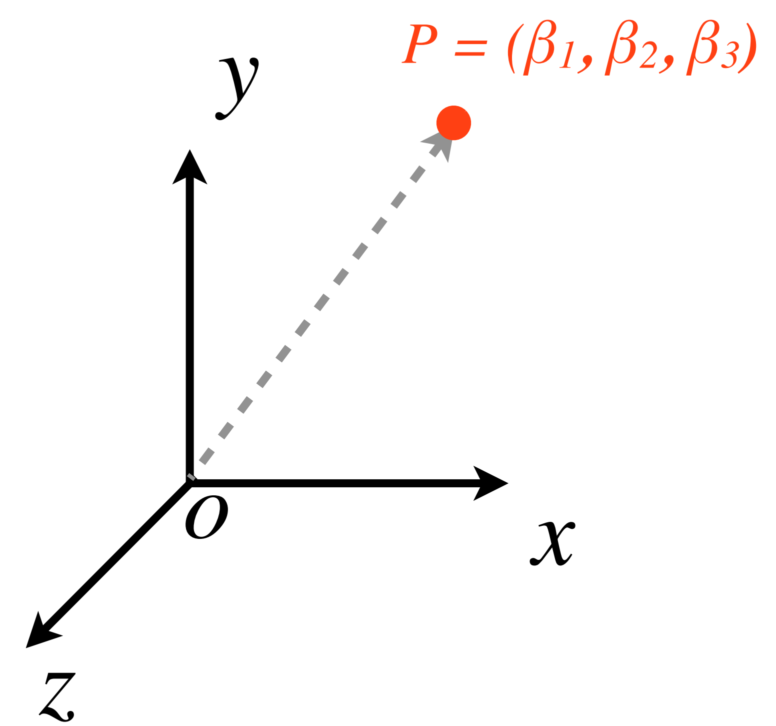
$$\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = (a_x, a_y, a_z)$$

- Note that $\mathbf{a} = (a_x, a_y, a_z)$ is still a **column-vector**, expressed in a certain basis

Points in 3D

- Coordinate **frame**
  - Basis vectors + **origin**

$$\begin{aligned}\mathbf{v}_1 &= (1,0,0)\\ \mathbf{v}_2 &= (0,1,0)\\ \mathbf{v}_3 &= (0,0,1)\\ \mathbf{o} &= (0,0,0)\end{aligned}$$

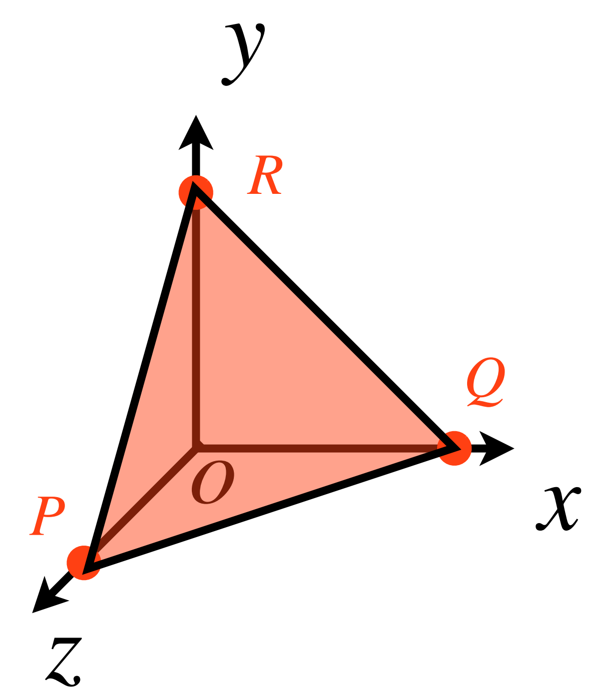


- In this frame, a **point** is given by:

$$\begin{aligned}P &= \mathbf{o} + \beta_1\mathbf{v}_1 + \beta_2\mathbf{v}_2 + \beta_3\mathbf{v}_3\\ &= \beta_1(1,0,0) + \beta_2(0,1,0) + \beta_3(0,0,1)\\ &= (\beta_1, \beta_2, \beta_3)\end{aligned}$$

Example: Triangle in 3D

- Defined by three points,
  $P = (0,0,1)$
  $Q = (1,0,0)$
  $R = (0,1,0)$
  specified in the Cartesian system

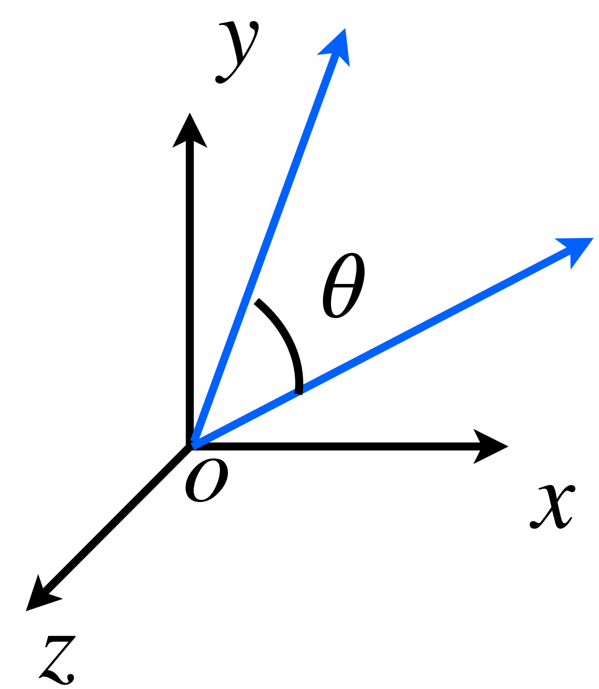


$$P = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Euclidean 3D Space

- Coordinate **frame**
  - Basis vectors + **origin**

$$\begin{aligned} \mathbf{v}_1 &= (1,0,0) \\ \mathbf{v}_2 &= (0,1,0) \\ \mathbf{v}_3 &= (0,0,1) \\ \mathbf{o} &= (0,0,0) \end{aligned}$$

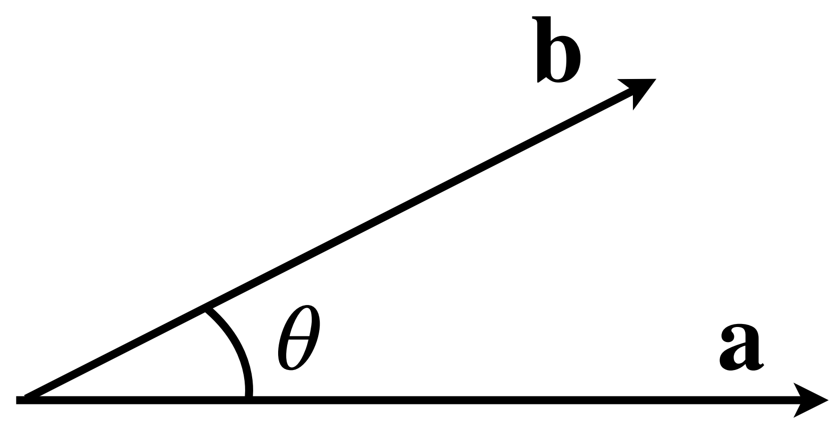


- The Cartesian coordinates express the three basis vectors, origin at (0,0,0)
- How to define the angle between two vectors in this coordinate frame?

Dot Product (scalar product)

- Given two 3D vectors $\mathbf{a} = (a_x, a_y, a_z)$ and $\mathbf{b} = (b_x, b_y, b_z)$, the dot product is given by

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z = |\mathbf{a}||\mathbf{b}| \cos\theta$$



- Angle between $\mathbf{a}$ and $\mathbf{b}$: $\theta = \arccos\left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)$

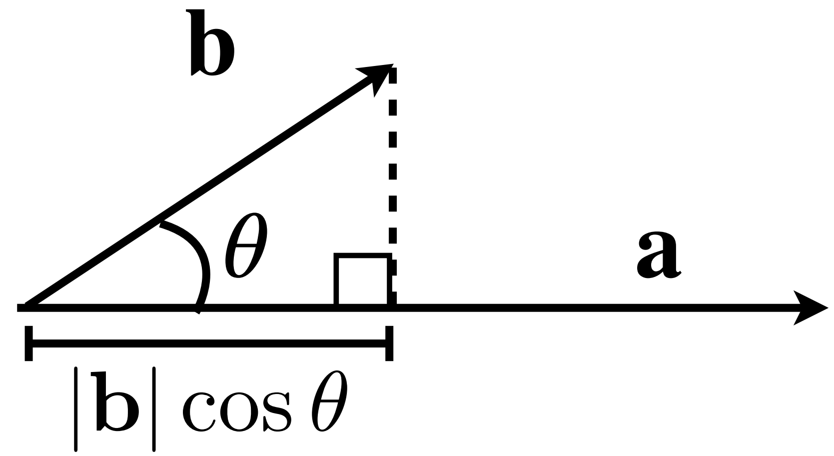
Dot Product (scalar product)

- Use cases

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z = |\mathbf{a}||\mathbf{b}| \cos\theta$$

  - The projection of $\mathbf{b}$ on $\mathbf{a}$

$$|\mathbf{b}| \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$$



  - Square magnitude of vector

$$\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2 \Leftrightarrow |\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$$

  - $\mathbf{a}$ and $\mathbf{b}$ orthogonal if

$$\mathbf{a} \cdot \mathbf{b} = 0$$

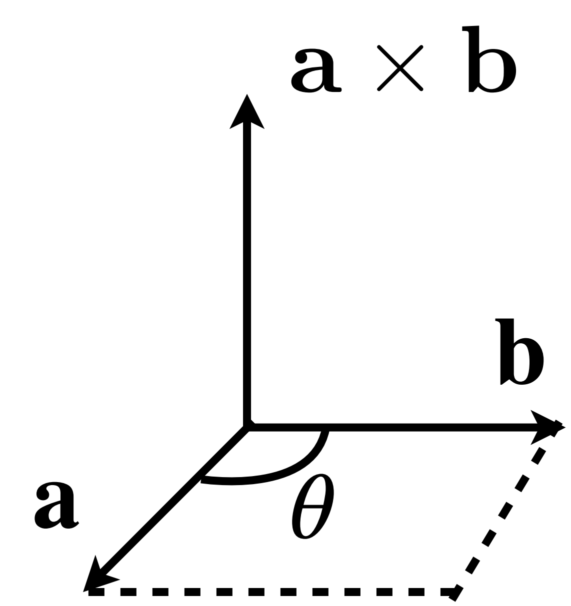
Cross Product

- Given two 3D vectors $\mathbf{a} = (a_\mathrm{x}, a_\mathrm{y}, a_\mathrm{z})$ and $\mathbf{b} = (b_\mathrm{x}, b_\mathrm{y}, b_\mathrm{z})$, the cross product is given by

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)$$

- Signed area of the parallelepiped spanned by vectors $\mathbf{a}$ and $\mathbf{b}$:

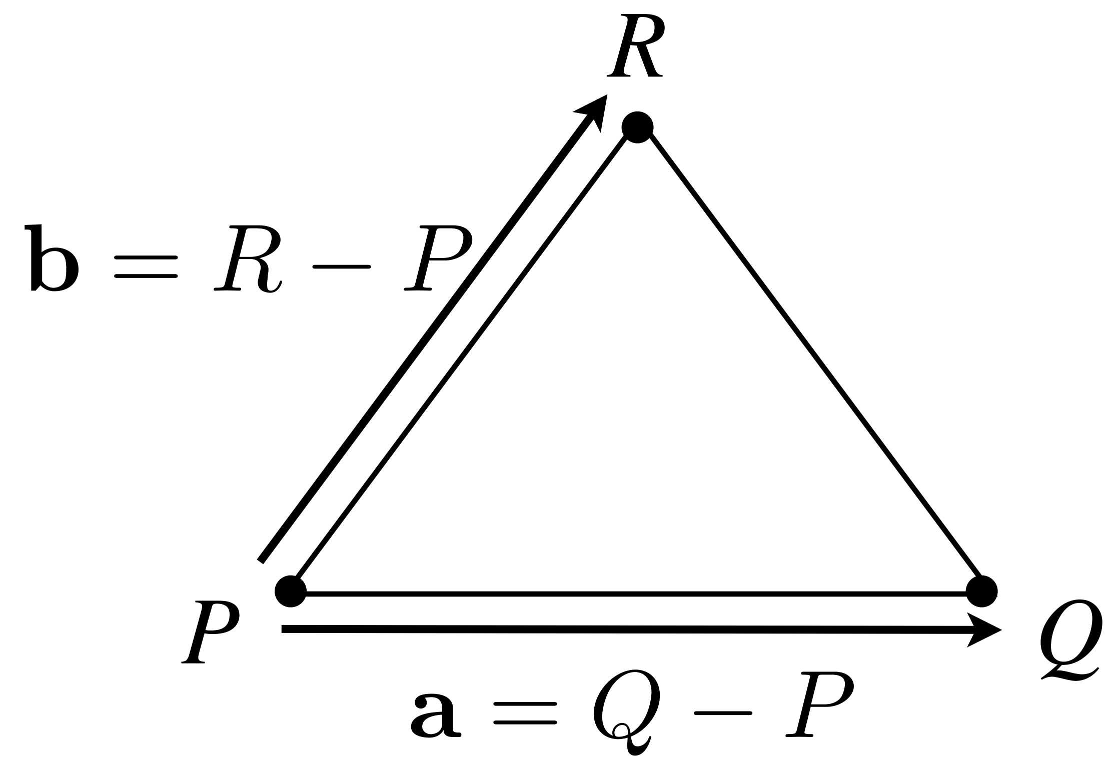
$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$$



- $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$
- $\mathbf{a} \times \mathbf{a} = 0$
- Note: $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

Triangle Normal

- Introduce two edge vectors $\mathbf{a}, \mathbf{b}$

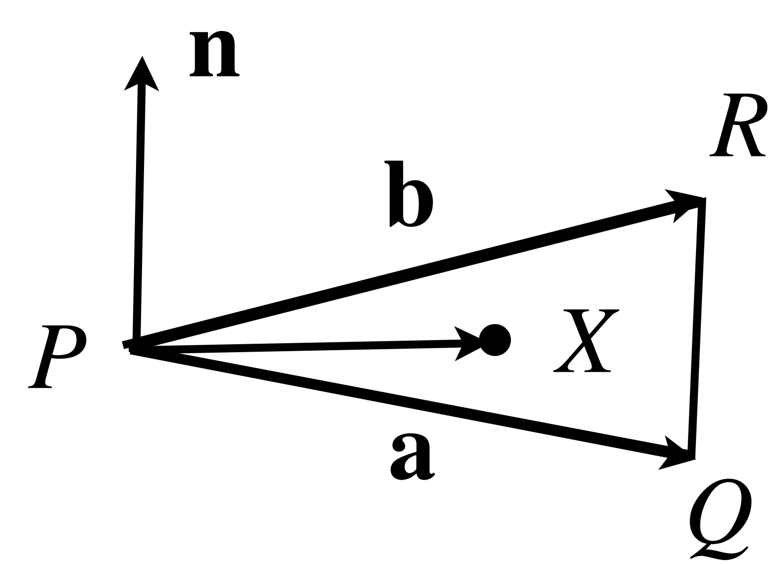


- Triangle face normal

$$\mathbf{n} = \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}$$

- Plane equation:
  - X belongs to the triangle plane if

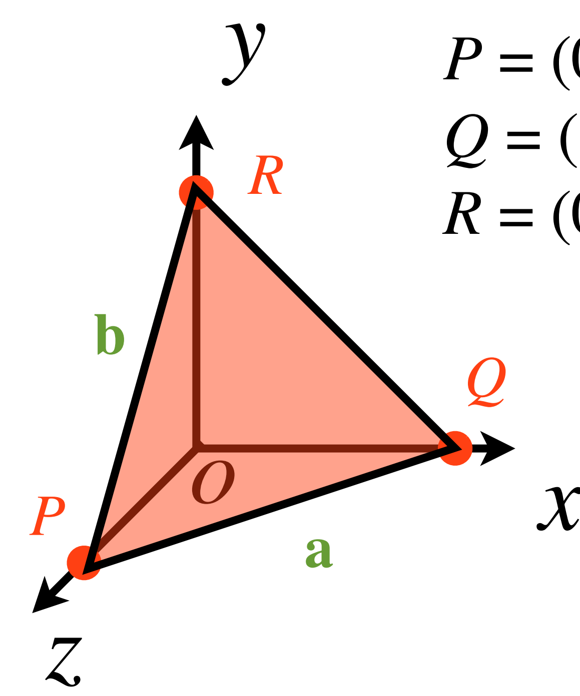
$$\mathbf{n} \cdot (X - P) = 0$$



Example: Triangle in 3D

- Edge vectors

$$\begin{aligned} \mathbf{a} &= Q - P = (1, 0, -1) \\ \mathbf{b} &= R - P = (0, 1, -1) \end{aligned}$$



$P = (0,0,1)$
$Q = (1,0,0)$
$R = (0,1,0)$

- Face normal

$$\mathbf{n} = \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|} = \frac{(1,1,1)}{|(1,1,1)|} = \frac{1}{\sqrt{3}}(1,1,1)$$

- Plane: $\mathbf{n} \cdot (X - P) = 0 \qquad X = (x, y, z)$

$$x + y + z - 1 = 0$$

Matrices

- We will use
  - Matrix-matrix multiplication: $\mathbf{A}\,\mathbf{B} = \mathbf{C}$
    - Concatenate transforms, change basis
  - Matrix-vector multiplication $\mathbf{A}\,\mathbf{x} = \mathbf{y}$
    - Transform vectors and points

Matrix-vector multiplication

$$A\mathbf{x} = \mathbf{y}$$

$$\begin{bmatrix} \boxed{a_{00} \quad a_{01} \quad a_{02}} \\ a_{10} \quad a_{11} \quad a_{12} \\ a_{20} \quad a_{21} \quad a_{22} \end{bmatrix} \begin{bmatrix} \boxed{\begin{matrix} x \\ y \\ z \end{matrix}} \end{bmatrix} = \begin{bmatrix} \boxed{a_{00}x + a_{01}y + a_{02}z} \\ a_{10}x + a_{11}y + a_{12}z \\ a_{20}x + a_{21}y + a_{22}z \end{bmatrix}$$

Matrix-matrix multiplication

$$AB = C$$

$$\begin{bmatrix} \boxed{a_{00} \quad a_{01} \quad a_{02}} \\ a_{10} \quad a_{11} \quad a_{12} \\ a_{20} \quad a_{21} \quad a_{22} \end{bmatrix} \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} \boxed{c_{00}} & c_{01} & c_{02} \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{bmatrix}$$

$$c_{ij} = \sum_{k=0}^{2} a_{ik} b_{kj}$$

Next lecture

- Transforms
- Coordinate Spaces
- Homogeneous Coordinates
- Seminar: OpenGL intro & C++ basics