Examination in Programming language theory

Solutions

1

$$\begin{aligned}
&(S\ (K\ (S\ I))\ K)\ x\ y = S\ (K\ (S\ I))\ K\ x\ y = \\
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&I\ y\ ((\ K\ x)\ y) = I\ y\ (\ K\ x\ y) = \\
&y\ x
\end{aligned}$$

Every second step just removes all unnecessary parentheses.

2 Let SWITCH denote the switch statement. Then for $i = 0, \ldots n$

$$\frac{\langle\, Si, \sigma\,\rangle \to \sigma'}{\langle\, \mathrm{SWITCH}, \sigma\,\rangle \to \sigma'} \qquad \text{provided that } \mathcal{A}[\![a]\!]\sigma = i.$$

If the value of a is outside the range the statement will loop (according to the terminology of the book).

3 Prove that

$$\text{if } \langle\, S_1, \sigma\,\rangle \Rightarrow^k \sigma' \text{ then } \langle\, S_1; S_2, \sigma\,\rangle \Rightarrow^k \langle\, S_2, \sigma'\,\rangle \qquad (*)$$

The proof is by induction over k.

The implication (*) is true for $k = 0$ since $\langle\, S_1, \sigma\,\rangle$ and σ' cannot be the same configuration.

For the inductive step assume that (*) is true for $k = k_0$ and that $\langle\, S_1, \sigma\,\rangle \Rightarrow^{k_0+1} \sigma'$.

Then there is a configuration $\langle\, S_1', \sigma_1\,\rangle$ such that $\langle\, S_1, \sigma\,\rangle \Rightarrow \langle\, S_1', \sigma_1\,\rangle \Rightarrow^{k_0} \sigma'$.

From $[\mathrm{comp}^1_{\mathrm{sos}}]$ and $\langle\, S_1, \sigma\,\rangle \Rightarrow \langle\, S_1', \sigma_1\,\rangle$ it follows that $\langle\, S_1; S_2, \sigma\,\rangle \Rightarrow \langle\, S_1'; S_2, \sigma_1\,\rangle$.

From the induction assumption and $\langle\, S_1', \sigma_1\,\rangle \Rightarrow^{k_0} \sigma'$ it follows that $\langle\, S_1'; S_2, \sigma_1\,\rangle \Rightarrow^{k_0} \langle\, S_2, \sigma'\,\rangle$

Combining these to conclusions we get $\langle\, S_1; S_2, \sigma\,\rangle \Rightarrow^{k_0+1} \langle\, S_2, \sigma'\,\rangle$.

4 FIX $f = \bigsqcup\{f^i\ \bot \mid i \in \mathbb{N}\}$.

We show that $f^i\ \bot \sqsubseteq d$ for all i by induction:

$f^0\ \bot = \bot \sqsubseteq d$.

Assume that $f^k\ \bot \sqsubseteq d$. Then $f^{k+1}\ \bot = f(f^k\ \bot) \sqsubseteq f\ d \sqsubseteq d$ since f is monontone.

We now have that d is an upper bound for $\{f^i\ \bot \mid i \in \mathbb{N}\}$ and it must be greater or equal to the least upper bound.

5

$$\begin{aligned}
\mathcal{S}_{cs}[\![\texttt{break}]\!]c'\,c\,\sigma &= c'\ \sigma \\
\mathcal{S}_{cs}[\![\texttt{begin}\ S\ \texttt{end}]\!]c'\,c\,\sigma &= \mathcal{S}_{cs}[\![S]\!]c\ c\ \sigma
\end{aligned}$$

If there is no enclosing block then c' will be the continuation for the complete program. The reasonable value for c' in this case is the identity continuation. This will mean that program halts in the current state.

6 Let S be `x:=x-1; y:=y+1`, b be `x=0`, $P = (x = 2 \wedge y = 0)$, and $Q = (y = 1)$. Then $\{P\}S\{Q\}$ while $\{P\}$ `S; while` $\neg b$ `do` $S\{Q \wedge b\}$ cannot be proved.