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Computer Science
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Solutions

Examination in Programming language theory

Solutions

1. $$\cfrac{T_1 \qquad \cfrac{T_2 \qquad \langle\, S\,, \sigma_4 \,\rangle \to \sigma_4}{\langle\, S\,, \sigma_2 \,\rangle \to \sigma_4}}{\langle\, S\,, \sigma_0 \,\rangle \to \sigma_4}$$

   where
   $$\begin{aligned} S_1 &= s := s + y \\ S_2 &= x := x - 1 \\ S_3 &= S_1; S_2 \end{aligned}$$

   $$T_1 = \frac{\langle\, S_1\,, \sigma_0 \,\rangle \to \sigma_1 \qquad \langle\, S_2\,, \sigma_1 \,\rangle \to \sigma_2}{\langle\, S_3\,, \sigma_0 \,\rangle \to \sigma_2}$$

   $$\begin{aligned} \sigma_0 &= [s \mapsto 0, x \mapsto 2, y \mapsto 5] \\ \sigma_1 &= [s \mapsto 5, x \mapsto 2, y \mapsto 5] \\ \sigma_2 &= [s \mapsto 5, x \mapsto 1, y \mapsto 5] \\ \sigma_3 &= [s \mapsto 10, x \mapsto 1, y \mapsto 5] \\ \sigma_4 &= [s \mapsto 10, x \mapsto 0, y \mapsto 5] \end{aligned}$$

   $$T_2 = \frac{\langle\, S_1\,, \sigma_2 \,\rangle \to \sigma_3 \qquad \langle\, S_2\,, \sigma_3 \,\rangle \to \sigma_4}{\langle\, S_3\,, \sigma_2 \,\rangle \to \sigma_4}$$

2. $\langle\, \texttt{if}\ b\ \texttt{then}\ S, \sigma \,\rangle \Rightarrow \langle\, S, \sigma \,\rangle$ if $\mathcal{B}[\![b]\!] = \mathbf{tt}$.

   $\langle\, \texttt{if}\ b\ \texttt{then}\ S, \sigma \,\rangle \Rightarrow \sigma$ if $\mathcal{B}[\![b]\!] = \mathbf{ff}$.

3. Let $A = \lambda x\,.\,\lambda y\,.\,y(x\ x\ y)$. $A\ A\ F = (\lambda x\,.\,\lambda y\,.\,y(x\ x\ y))\ A\ F = (\lambda y\,.\,y(A\ A\ y))\ F = F(A\ A\ F)$.

4. Assume that $\mathcal{S}'_{cs}[\![S_i]\!]\ c\ \sigma = c(\mathcal{S}_{ds}[\![S_i]\!]\ \sigma)$ for all c, σ and $i = 1, 2$. $\mathcal{S}'_{cs}[\![S_1; S_2]\!]\ c\ \sigma = (\mathcal{S}'_{cs}[\![S_1]\!] \circ \mathcal{S}'_{cs}[\![S_2]\!])\ c\ \sigma = (\mathcal{S}'_{cs}[\![S_1]\!](\mathcal{S}'_{cs}[\![S_2]\!]\ c))\ \sigma = \mathcal{S}'_{cs}[\![S_1]\!](\mathcal{S}'_{cs}[\![S_2]\!]\ c)\ \sigma = (\mathcal{S}'_{cs}[\![S_2]\!]\ c)(\mathcal{S}_{ds}[\![S_1]\!]\ \sigma) = c(\mathcal{S}_{ds}[\![S_2]\!](\mathcal{S}_{ds}[\![S_1]\!]\sigma)) = c(\mathcal{S}_{ds}[\![S_1; S_2]\!]\sigma)$

5.
```
data Formula = Formula Stm State State
data Rule = Rule Formula [Formula] (Bexp -> State -> Bool)
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6. There are four monotone functions.

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
a	A	A	A	B
b	A	A	B	B
c	A	B	B	B

   We show that f_1 is continuous. Every subset of $\{\, a, b, c \,\}$ is a chain. Thus there are 8 chains. For each chain C we must show that $f_1(\bigsqcup C) = \bigsqcup\{\, f_1(x) \mid x \in C \,\}$. En each case both members are equal to A.

   (It is easy to show that if all chains are finite then monotonicity implies continuity.)