

## Examination in Programming language theory

The following texts may be used during the exam:  
Nielson, Nielson, Semantics with Applications.  
Andersson, Programming language theory, Lecture notes.

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1 Let

$$\begin{aligned}K &= \lambda x.\lambda y.x \\S &= \lambda x.\lambda y.\lambda z.x z (y z) \\I &= \lambda x.x\end{aligned}$$

Reduce

$$(S (K (S I)) K) x y$$

to normal form.

2 Extend **While** with a **switch** statement

```
switch a of
0: S0
1: S1
...
n: Sn
```

where  $n$  is a natural number. If the value of  $a$  is a natural number less than or equal to  $n$  then the corresponding statement is executed. Specify a natural operational semantics for this statement and describe what will happen according to your semantics if the value of  $a$  is outside the range.

3 (Exercise 2.21) Prove that

$$\text{if } \langle S_1, \sigma \rangle \Rightarrow^k \sigma' \text{ then } \langle S_1; S_2, \sigma \rangle \Rightarrow^k \langle S_2, \sigma' \rangle$$

4 Let  $f \in D \rightarrow D$  be a continuous function on a ccpo  $(D, \sqsubseteq)$  and let  $d \in D$  satisfy  $f d \sqsubseteq d$ . Show that  $\text{FIX } f \sqsubseteq d$ .

5 Extend **While** with a **break** statement and a block statement

$$S ::= \dots \mid \text{break} \mid \text{begin } S \text{ end}$$

If the **break** statement is executed inside a block then the execution should continue after the smallest enclosing block.

Define a continuation style semantics for the new statements of the extended language and describe what will happen, according to your definition, if there is no block enclosing the **break** statement.

6 The statement **repeat S until b** is assumed to be equivalent to **S; while ¬b do S**

It has been suggested that the axiomatic semantics **repeat S until b** should be

$$\frac{\{P\}S\{Q\}}{\{P\} \text{ repeat } S \text{ until } b \{Q \wedge b\}}$$

Provide an example with suitable  $b$ ,  $S$ ,  $P$  and  $Q$ , such that  $\{P\} \text{ repeat } S \text{ until } b \{Q \wedge b\}$  can be proved but not  $\{P\} S; \text{ while } \neg b \text{ do } S\{Q \wedge b\}$ .