Examination in Programming language theory

The following texts may be used during the exam:
Nielson, Nielson, Semantics with Applications.
Andersson, Programming language theory, Lecture notes.

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1 Extend the **While** language with integer division,

$a \;::=\; n \mid x \mid a_1 + a_2 \mid a_1 \star a_2 \mid a_1 - a_2 \mid a_1/a_2$

Define the meaning function $\mathcal{A}$ for these arithmetic expressions. The meaning of an expression containing a division by 0 should be $\perp$.

2 In a *guarded command*

$$\begin{array}{llll} \texttt{if} & b_1 & \rightarrow & S_1 \\ \mid & b_2 & \rightarrow & S_2 \\ \texttt{fi} & & & \end{array}$$

the conditions b_1 and b_2 may be true or false independently. If just one of them is true the corresponding statement is executed, if both are true one of the statements is chosen nondeterministicly for execution, and if both are false the execution should abort, i.e. should not terminate properly in a state. Extend the **While** language with this statement giving it meaning using natural semantics.

3 Extend **While** with a non-deterministic statement S_1 `or` S_1 as in Nielson 2.4. It is necessary to change the type of the meaning function $\mathcal{S}_{ds}$. Specify the new type and define $\mathcal{S}_{ds}[\![x := a]\!]$, $\mathcal{S}_{ds}[\![S_1 \texttt{ or } S_2]\!]$ and $\mathcal{S}_{ds}[\![S_1; S_2]\!]$.

4 The lecture notes on Natural deduction define introduction and elimination rules for $\wedge$, $\vee$, $\rightarrow$, and $\neg$. Add an equivalence operator, $\leftrightarrow$, such that $p \leftrightarrow q$ if and only if p and q are both false or both true. Define introduction and elimination rules for this operator for use in natural deduction.

5 The lecture notes define a representation for natural numbers using lambda expressions. Define a lambda expression, Leq, that represents the $\leq$ operation such that

$$\mathrm{Leq}\ \ulcorner n \urcorner\ \ulcorner m \urcorner = \begin{cases} \mathsf{T} & \text{if } n \leq m \\ \mathsf{F} & \text{if } n > m \end{cases}$$

Hint:

$$n \leq m \triangleq \begin{cases} \mathbf{tt} & \text{if } n = 0 \\ \mathbf{ff} & \text{if } n \neq 0 \text{ and } m = 0 \\ n-1 \leq m-1 & \text{otherwise} \end{cases}$$

6 Let $D \triangleq \langle 2^{\mathbb{N}}, \sqsubseteq \rangle$ be a poset where $A \sqsubseteq B$ if and only if $B \subseteq A$ (observe the order of the operands).

a. Which is the least element in D?

b. Compute $\bigsqcup\{\{0,1\},\{1,2\}\}$

c. Construct an infinite chain in D where the least upper bound is an infinite set.

d. Let $D' \triangleq \langle 2^{\mathbb{N}}, \subseteq \rangle$. Let compl $\in D \rightarrow D'$ be defined by $\mathrm{compl}(A) \triangleq \mathbb{N} \setminus A$ where $\setminus$ is set difference as defined in Appendix A of the text book. Show that compl is monotone.

e. D is a chain complete poset. It remains to be so even if some elements are removed from $2^{\mathbb{N}}$. Give an example of an element that can be removed and one that cannot.