Lund University and Institute of Technology
Computer Science
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DAT130, EDA145
Examination

Examination in Programming language theory

This exam has 6 problems, each worth 5 marks. For passing the exam at most 15 marks will be required.

The following texts may be used during the exam:
Nielson, Nielson, Semantics with Applications.
Andersson, Programming language theory, Lecture notes.

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1. Solve Exercise 1.13 in Nielson when the grammar for an aritmetic expression is

$$a ::= n \mid x \mid a_1 + a_2$$

2. Define natural operational semantics for **While** extended with `if` b `then` S without relying on the semantics for the standard two-branch `if`-statement.

3. Provide all steps in the derivation sequence $\langle S, \sigma \rangle \Rightarrow^* \sigma'$, where $S =$ `while` $1 \leq$ `x do (s:=s+y; x:=x-1)`, $\sigma = [s \mapsto 0, x \mapsto 1, y \mapsto 5]$, and σ' is some state using the structural operational semantics for **While**. Each step should be verified by a derivation tree.

4. The lecture notes describe one way to represent natural numbers by lambda terms. This is another:

   Let n be a natural number and $\overline{n} = \lambda x \,.\, \lambda y \,.\, x^n \, y$ where $x^n \, y$ is a short hand for $x \, (x \, (x \, \ldots \, (x \, y)))$ with n applications of x.

   Show that $(\lambda z \,.\, \lambda x \,.\, \lambda y \,.\, x \, (z \, x \, y)) \, \overline{n} = \overline{n+1}$

5. Extend **While** with a `halt` statement that terminates the execution of the program in the current state. Define $\mathcal{S}_{cs}[\![\texttt{halt}]\!]$ using continuation style semantics.

6. Let $P = (D, \sqsubseteq)$ be a ccpo and assume that $F, G \in D \to D$ are continous and that $F(d) \sqsubseteq G(d)$ for all $d \in D$. Show that $\mathrm{FIX}\, F \sqsubseteq \mathrm{FIX}\, G$ where FIX is the least fixed point operator of Theorem 4.37.