Lund University and Institute of Technology
Computer Science
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2003–12–16, 8.00–13.00
DAT130, EDA145
Examination

Examination in Programming language theory

This exam has 6 problems, each worth 5 points. For passing the exam at most 15 points will be required.

The following texts may be used during the exam:
Nielson, Nielson, Semantics with Applications.
Andersson, Programming language theory, Lecture notes.

1. Let $\mathrm{B} \triangleq \lambda f \,.\, \lambda g \,.\, \lambda x \,.\, f(g\ x)$. Show that $\mathrm{B}\ f\ (\mathrm{B}\ g\ h) = \mathrm{B}\ (\mathrm{B}\ f\ g)\ h$. Show the result after each single reduction.

2. Extend **While** with a Java like `do` S `while` b statement, where S is executed at least once. Define natural semantic rules for this statement. The rules must not rely on the standard `while` statement.

3. Extend **Aexp** with the construct `let` D_V `in` a, where D_V is a list of initializations,

   $$D_V ::= \epsilon \mid x = a\ :\ D_V$$

   The informal meaning is that the value of the expression after `in` is evaluated in a state where all the initializations have been made. Each initialization is made in a state where all previous initializations have been made. Define $\mathcal{A}[\![\texttt{let}\ D_V\ \texttt{in}\ a]\!] \in \textbf{State} \to \mathrm{Z}$.

   You may need to define a function $\mathcal{D}_V \in \mathbf{D_V} \to \textbf{State} \to \textbf{State}$, where $\mathbf{D_V}$ is the set of all initialization lists.

4. Show by structural induction that $\mathcal{S}_{\mathrm{wp}}[\![S]\!](P \wedge Q) = \mathcal{S}_{\mathrm{wp}}[\![S]\!]P \wedge \mathcal{S}_{\mathrm{wp}}[\![S]\!]Q$ for all $S \in \textbf{Stm}$.

5. Extend **While** with a nondeterministic choice statement S_1 `or` S_2. Define the semantics for this statement by

   (a) partial correctness rules.

   (b) weakest precondition semantics.

6. It is an axiom of set theory that there is a set D such that $\emptyset \in D$ and if $d \in D$ then $d \cup \{ d \} \in D$. It is called the Axiom of infinity and informally it states that there is at least one infinite set. It can be proved that there is a least (using $\subseteq$) set with this property. Call this set N. Let $F(d) \triangleq d \cup \{ d \}$ be a function on N.

   (a) Find five elements in N.

   (b) Is F monotone on $(\mathrm{N}, \subseteq)$? Explain why.

   (c) Is N a ccpo? If not, add elements to make it one.

   (d) Is F continuous on the ccpo in (c) and if so which is the least fixed point? Explain why.

   (e) There is a good reason for naming the set N and there is a better name for F. Which?