Solutions

1. $X(X\,X) = (X\,X)\,K\,S\,K = X\,X\,K\,S\,K = (X\,K\,S\,K)\,K\,S\,K = X\,K\,S\,K\,K\,S\,K =$
   $K\,K\,S\,K\,S\,K\,K\,S\,K = K\,K\,S\,K\,K\,S\,K = K\,K\,K\,S\,K = K\,S\,K = S$

2. $M = M\ I\ S\ S = (\lambda m.m\ I\ S\ S)M$ so $M = Y(\lambda m.m\ I\ S\ S)$ has the requested property.

3. There are 26 abstractions in A and 27 identifiers inside the parentheses. Observe that r is the last abstraction and that it occurs first and last in the body. Informally using A^n for $A\ \cdots A$ with n occurences of A we get
   $BF = A\,A^{25}F = F(A^{26}F) = F(BF)$

4. The new alternative will process the declarations in reverse order so we will have different meanings for `var x=1; var x=2`.

$$\dfrac{\dfrac{\langle\,\epsilon,\sigma\,\rangle\to\sigma}{\langle\,\texttt{var x:=2};\epsilon,\sigma\,\rangle\to\sigma[\texttt{x}\mapsto 2]}}{\langle\,\texttt{var x:=1; var x:=2};\epsilon,\sigma\,\rangle\to\sigma[\texttt{x}\mapsto 2][\texttt{x}\mapsto 1]}$$

5.

$$\langle\texttt{random(x)}\ ,\sigma\rangle\to\sigma[x\mapsto n]\qquad\text{where } n\in N$$

$$\langle\texttt{random(x)}\ ,\sigma\rangle\Rightarrow\sigma[x\mapsto n]\qquad\text{where } n\in N$$

   It can be done by replacing `random(x)` by
   `x:=0; y:=0; while (y<1) do (x:=x+1 or y=1)` where `y` is a new variable. It is not possible to implement this so that all values are equally probable. On the other hand, the operational rules do not specify anything about probability.

6.

$$\frac{\langle S,\sigma\rangle\Rightarrow\langle S',\sigma'\rangle\qquad\langle\texttt{protect}\ S'\ \texttt{end},\sigma'\rangle\Rightarrow\sigma''}{\langle\texttt{protect}\ S\ \texttt{end},\sigma\rangle\Rightarrow\sigma''}$$

   and

$$\frac{\langle S,\sigma\rangle\Rightarrow\sigma'}{\langle\texttt{protect}\ S\ \texttt{end},\sigma\rangle\Rightarrow\sigma'}$$

   This solution is not in the spirit of structural operational semantics since the first rule describes a "big step". I don't know any better solution.

7. We have to redefine the upd_P fuction:

$$\text{upd}_P(\texttt{proc}\ \ p(x_1,x_2)\ \ \texttt{is}\ \ S;D_P,env_V,env_P) =$$
$$\qquad\text{upd}_P(D_P,env_V,env_P[p\mapsto(x_1,x_2,S,env_V,env_P)])$$
$$\text{upd}_P(\epsilon,env_V,env_P) = env_P$$

   This is then used in the rule for a block without change.

$$\frac{\langle\,D_V,env_V,sto\,\rangle\to_D(env'_V,sto')\qquad env'_V,env'_P\vdash\langle\,S,sto'\,\rangle\to sto''}{env_V,env_P\vdash\langle\,\texttt{begin}\ \ D_V\ D_P\ S\ \ \texttt{end}\ \ ,sto\,\rangle\to sto''}$$

   where $env'_P = upd_P(D_P,env'_V,env_P)$
   The call statement obeys the rule

$$\frac{env'_V,env'_P\vdash\langle\,\texttt{begin var}\ \ x_1=n_1;\texttt{var}\ \ x_2=n_2;S\ \ \texttt{end},sto'\,\rangle\to sto''}{env_V,env_P\vdash\langle\,\texttt{call}\ \ p(a_1,a_2),sto\,\rangle\to sto''}$$

   where $env_P\ p = (x_1,x_2,S,env'_V,env'_P)$ and
   $\mathcal{N}[\![n_1]\!] = \mathcal{A}[\![a_1]\!](sto\circ env_V)$
   $\mathcal{N}[\![n_2]\!] = \mathcal{A}[\![a_2]\!](sto\circ env_V)$