Problems

These problems will be discussed at the seminar in week 7.

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1 In the section on implementation of functional languages, lambda calculus was extended with natural numbers and addition. Now add a conditional expression $e?e_1 : e_2$ that has the value of e_1 if the value of e is zero and e_2 otherwise. (Ignore the fact that some kind of subtraction is needed to make it useful.) Introduce an instruction that can be used to handle such expressions and extend the compiling function.

2 Define the transition relation $\triangleright$ for the instruction of the previous problem.

3 Add a read statement, `read` x, to While. Defining the denotational semantics for this language requires a semantic function $\mathcal{S}_{ds}[\![S]\!] \in (State \times Z^*) \hookrightarrow (State \times Z^*)$ where the Z^* is the set of integer lists used to represent numbers to be read by `read` statements. Define $\mathcal{S}_{ds}[\![\texttt{read } x]\!]$ and $\mathcal{S}_{ds}[\![S_1; S_2]\!]$.

4 Assume that the semantics for `while` b `do` S was erroneously defined by $\mathcal{S}_{ds}[\![\texttt{while } b \texttt{ do } S]\!] =$ FIX F, where $F\ g\ \sigma = \mathcal{B}[\![\texttt{b}]\!]\sigma\ ?\ (\mathcal{S}_{ds}[\![S]\!] \circ g)\sigma : \sigma$. Use the definition of FIX to derive the meaning and explain it informally.

5 Extend Aexp with a `let` expression,

`let` $x = a_0$ `in` a_1

and define the semantic function $\mathcal{A} \in \mathbf{Aexp} \rightarrow State \rightarrow \mathbb{Z}$ for it.

6 It is an axiom of set theory that there is a set D such that $\emptyset \in D$ and if $d \in D$ then $d \cup \{d\} \in D$. It is called the Axiom of infinity and informally it states that there is at least one infinite set. It can be proved that there is a least (using $\subseteq$) set with this property. Call this set N. Let $F(d) \triangleq d \cup \{d\}$ be a function on N.

a. Find five elements in N.

b. Is F monotone on $(\mathrm{N}, \subseteq)$? Explain why.

c. Is N a ccpo? If not, add elements to make it one.

d. Is F continuous on the ccpo in (c) and if so which is the least fixed point? Explain why.

e. There is a good reason for naming the set N and there is a better name for F. Which?