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Computer Science
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Problems

These problems will be discussed on seminar in week 3.

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1 (Exercise 4.7) **AM** refers to variables by their names rather than by their addresses. The abstract machine $\mathbf{AM}_1$ differs from **AM** in that

- the configurations have the form $\langle c, e, m \rangle$ where c and e are as in **AM** and m, the memory, is a (finite) list of values, that is $m \in \mathrm{Z}^*$, and
- the instructions FETCH-x and STORE-x are replaced by GET-n and PUT-n where n is a natural number (an address).

Specify the operational semantics of the machine. You may write $m[n]$ to select the n'th value in the list m (when n is positive and less or equal to the length of m). What happens if we reference an address that is outside the memory?

2 Find all chains in $< 2^{\{0,1\}}, \subseteq >$ and the least upper bound of each chain.

3 Let A be a set, $A_0 \subseteq A$ and $F \in 2^A \to 2^A$ where $F(S) \triangleq S \cup A_0$. Show that F is continuous when we use $\subseteq$ as the ordering relation. If you wish you may assume that $A = \mathrm{N}$ and $A_0 = \{0, 2\}$. There are two things to prove:

a. F is monotone.

b. If C is a chain in 2^A then $F(\bigcup C) = \bigcup F(C)$.

4 Let D be an infinite set of sets such that $< D, \subseteq >$ is a ccpo. The powerset operator $\mathcal{P}$ takes a set $A \in D$ as an argument and returns its powerset, 2^A. Thus $\mathcal{P} \in D \to 2^D$. Show that $\mathcal{P}$ is monotone but not continuous when we use $\subseteq$ as the ordering relation in both sets. You may assume that $D = 2^{\mathbb{N}}$.

5 Consider a statement with denotational semantics

$$\mathcal{S}_{ds}[\![\texttt{try S establish b}]\!]\sigma = \begin{cases} \mathcal{S}_{ds}[\![S]\!]\sigma, & \texttt{if } \mathcal{B}[\![\texttt{b}]\!](\mathcal{S}_{ds}[\![S]\!]\sigma) = \texttt{tt} \\ \sigma, & \texttt{if } \mathcal{B}[\![\texttt{b}]\!](\mathcal{S}_{ds}[\![S]\!]\sigma) = \texttt{ff} \end{cases} \tag{1}$$

Explain informally what the statement does.

6 Add a "swap" statement, $x_0 :=: x_1$, to While. The execution of the statement should exchange the values of the variables. Define the direct style denotational semantics for this statement.

7 In most programming languages the evaluation of an expression may change the state. As an example `n++` in Java is an expression that returns the initial value of `n` and then increments the value of `n`. Define a denotational semantics for Aexp extended with x`++`. The semantic function must now return a pair with the value and the new state.

8 (5.50) Show that $\mathcal{S}_{ds}[\![\texttt{while true do skip}]\!]$ is the totally undefined function by computing FIX F, where F is taken from table 5.1 or $F\,g \triangleq \lambda\sigma.(\mathcal{B}[\![\texttt{b}]\!]\sigma\ ?\ (g \circ \mathcal{S}_{ds}[\![S]\!])\sigma\ :\ \sigma)$. It suffices to compute the first two values of $F^n\ \bot$.