Lund University
Computer Science
Lennart Andersson

Problems

These problems will be discussed at seminar 5. Programs in Haskell are not required to compile. The exercises in Nielson have been renumbered. My references are to the 1999 edition.

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1 Let

$$\begin{aligned} K &\triangleq \lambda x\,.\,\lambda y\,.\,x \\ S &\triangleq \lambda x\,.\,\lambda y\,.\,\lambda z\,.\,xz(yz) \\ X &\triangleq \lambda t\,.\,t\,K\,S\,K \end{aligned}$$

Show that $X\,(X\,X) = S$.

2 Let

$$A \triangleq \lambda abcdefghijklmnopqstuvwxyzr\,.\,r(thisisafixedpointcombinator)$$
$$B \triangleq A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A\,A$$

Show that B is a fixed point combinator, i.e. $BF = F(BF)$ for any F.

3 (Exercise 3.7) **AM** refers to variables by their names rather than by their addresses. The abstract machine $\mathbf{AM}_1$ differs from **AM** in that

- the configurations have the form $\langle\, c, e, m\,\rangle$ where c and e are as in **AM** and m, the memory, is a (finite) list of values, that is $m \in \mathrm{Z}^*$, and
- the instructions FETCH-x and STORE-x are replaced by GET-n and PUT-n where n is a natural number (an address).

Specify the operational semantics of the machine. You may write $m[n]$ to select the n'th value in the list m (when n is positive and less or equal to the length of m). What happens if we reference an address that is outside the memory?

4 Find all chains in $(2^{\{\,0,1\,\}}, \subseteq)$ and the least upper bound of each chain.

5 Let A be a set, $A_0 \subseteq A$ and $F \in 2^A \to 2^A$ where $F(S) \triangleq S \cup A_0$. Show that F is continuous when we use $\subseteq$ as the ordering relation. If you wish you may assume that $A = \mathrm{N}$ and $A_0 = \{\,0, 2\,\}$. There are two things to prove:

a. F is monotone.

b. If C is a chain in 2^A then $F(\bigcup C) = \bigcup F(C)$.

6 Let D be an infinite set of sets such that $(D, \subseteq)$ is a ccpo. The powerset operator $\mathcal{P}$ takes a set $A \in D$ as an argument and returns its powerset, 2^A. Thus $\mathcal{P} \in D \to 2^D$. Show that $\mathcal{P}$ is monontone but not continuous when we use $\subseteq$ as the ordering relation in both sets. You may assume that $D = 2^{\mathrm{N}}$.