Problems

These problems will be discussed at seminar 3. Some of the problems might be replaced after the lecture on Wednesday. Programs in Haskell are not required to compile. The exercises in Nielson have been renumbered. My references are to the 1999 edition.

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1 Show that the statements $S_1; (S_2; S_3)$ and $(S_1; S_2); S_3$ are semantically equivalent using the natural semantics for **While**.

2 With the semantics of problem 7 from Seminar 2 and $\mathcal{N}$ from the text book use structural induction and induction over the shape of derivation trees to prove that $\langle n \rangle \to z$ if and only if $\mathcal{N}[\![n]\!] = z$.

3 Use the natural semantics for the while statement to prove that `while true do skip` will always loop.

4 Extend the language **While** with an if-statement without an else part and define natural semantics inference rules for it. The rules must not rely on the existence of the standard if-statement.

5 Extend the language **While** with a Java-like `do` S `while` b statement and define structural operational semantics inference rules for the statement. The rules must not rely on the existence of the standard while-statement.

6 Will the semantics of **While** change if we replace $[\text{if}^{\text{tt}}_{\text{ns}}]$ and $[\text{if}^{\text{ff}}_{\text{ns}}]$ by

$$\frac{\langle S_0, \sigma \rangle \to \sigma'}{\langle \texttt{if true then } S_0 \texttt{ else } S_1, \sigma \rangle \to \sigma'} \qquad \frac{\langle S_1, \sigma \rangle \to \sigma'}{\langle \texttt{if false then } S_0 \texttt{ else } S_1, \sigma \rangle \to \sigma'}$$

How about

$$\frac{\langle S_0, \sigma \rangle \to \sigma'_0 \qquad \langle S_1, \sigma \rangle \to \sigma'_1}{\langle \texttt{if b then } S_0 \texttt{ else } S_1, \sigma \rangle \to \sigma'}$$

where $\sigma' = \sigma'_0$ if $\mathcal{B}[\![\texttt{b}]\!]\sigma = \text{tt}$ and $\sigma' = \sigma'_1$ if $\mathcal{B}[\![\texttt{b}]\!]\sigma = \text{ff}$.

7 (Exercise 2.21) Prove that

$$\langle S_1, \sigma \rangle \Rightarrow^k \sigma' \text{ implies } \langle S_1; S_2, \sigma \rangle \Rightarrow^k \langle S_2, \sigma' \rangle$$

8 (Exercise 2.20) Find statements S_1 and S_2 and states σ and σ' such that

$$\langle S_1; S_2, \sigma \rangle \Rightarrow^* \langle S_2, \sigma' \rangle$$

holds while $\langle S_1, \sigma \rangle \Rightarrow^* \sigma'$ does not.