F6

Natural semantics

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Operational semantics

- *Natural semantics* describes the *overall* results of the computation; the way humans think.
- *Structural operational semantics* describes the individual steps in the computation as taken by the computer.

Configuration – Intuition

- $\langle S, \sigma \rangle$ is a configuration where the statement S is to be executed in the state σ.
- σ is a configuration where no more statements are to be executed and σ is the *terminal* state.

Configuration – Implementation

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data Configuration = Terminal State
                   | NonTerminal Stm State
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Axioms and rules

- $\langle x := a, \sigma \rangle \to \sigma[x \mapsto \mathcal{A}[\![a]\!]\sigma]$
- $\langle \texttt{skip}, \sigma \rangle \to \sigma$
- $$\frac{\langle S_1, \sigma \rangle \to \sigma' \qquad \langle S_2, \sigma' \rangle \to \sigma''}{\langle S_1; S_2, \sigma \rangle \to \sigma''}$$

Axioms and rules ...

- $$\frac{\langle S_1, \sigma \rangle \to \sigma'}{\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2, \sigma \rangle \to \sigma'} \qquad \text{if } \mathcal{B}[\![b]\!]\sigma = \mathbf{tt}$$
- $$\frac{\langle S_2, \sigma \rangle \to \sigma'}{\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2, \sigma \rangle \to \sigma'} \qquad \text{if } \mathcal{B}[\![b]\!]\sigma = \mathbf{ff}$$

Axioms and rules ...

- $$\frac{\langle S, \sigma \rangle \to \sigma' \qquad \langle \texttt{while}\ b\ \texttt{do}\ S, \sigma' \rangle \to \sigma''}{\langle \texttt{while}\ b\ \texttt{do}\ S, \sigma \rangle \to \sigma''} \qquad \text{if } \mathcal{B}[\![b]\!]\sigma = \mathbf{tt}$$
- $$\langle \texttt{while}\ b\ \texttt{do}\ S, \sigma \rangle \to \sigma \qquad \text{if } \mathcal{B}[\![b]\!]\sigma = \mathbf{ff}$$

Structural operational semantics

- $\langle x := a, \sigma \rangle \Rightarrow \sigma[x \mapsto \mathcal{A}[\![a]\!]\sigma]$
- $\langle \texttt{skip}, \sigma \rangle \Rightarrow \sigma$

Axioms and rules ...

- $$\frac{\langle S_1, \sigma \rangle \Rightarrow \langle S_1', \sigma' \rangle}{\langle S_1; S_2, \sigma \rangle \Rightarrow \langle S_1'; S_2, \sigma' \rangle}$$
- $$\frac{\langle S_1, \sigma \rangle \Rightarrow \sigma'}{\langle S_1; S_2, \sigma \rangle \Rightarrow \langle S_2, \sigma' \rangle}$$

Axioms and rules ...

- $$\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2, \sigma \rangle \Rightarrow \langle S_1, \sigma \rangle \qquad \text{if } \mathcal{B}[\![b]\!]\sigma = \mathbf{tt}$$
- $$\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2, \sigma \rangle \Rightarrow \langle S_2, \sigma \rangle \qquad \text{if } \mathcal{B}[\![b]\!]\sigma = \mathbf{ff}$$

Axioms and rules ...

$$\langle \texttt{while}\ b\ \texttt{do}\ S, \sigma \rangle \Rightarrow \langle \texttt{if}\ b\ \texttt{then}\ (S; \texttt{while}\ b\ \texttt{do}\ S)\ \texttt{else skip}, \sigma \rangle$$

The intermediate state

Lemma. *If* $\langle S_1; S_2, \sigma \rangle \Rightarrow^k \sigma''$ *then there is a state* σ' *and natural numbers* k_1 *and* k_2 *such that* $\langle S_1, \sigma \rangle \Rightarrow^{k_1} \sigma'$ *and* $\langle S_2, \sigma' \rangle \Rightarrow^{k_2} \sigma''$ *where* $k = k_1 + k_2$. ■

Exercise 2.21

Lemma. *If* $\langle S_1, \sigma \rangle \Rightarrow^k \sigma'$ *then* $\langle S_1; S_2, \sigma \rangle \Rightarrow^k \langle S_2, \sigma' \rangle$. ■

Exercise 2.20

Lemma. *Assume that* $\langle S_1; S_2, \sigma \rangle \Rightarrow^* \langle S, \sigma' \rangle$. *Then it is not necessarily the case that* $\langle S_1, \sigma \rangle \Rightarrow^* \sigma'$. ■

Equivalence

Theorem. *For every statement* S *in* **While** *we have* $\mathcal{S}_{ns}[\![S]\!] = \mathcal{S}_{sos}[\![S]\!]$. ■

From natural to structural

Lemma. *For every statement* S *in* **While** *and states* σ *and* σ' *we have that*
$\langle S, \sigma \rangle \rightarrow \sigma'$ *implies* $\langle S, \sigma \rangle \Rightarrow^* \sigma'$. ■

From structural to natural

Lemma. *For every statement S in* **While** *and states σ and σ' and natural number k we have that $\langle S, \sigma \rangle \Rightarrow^k \sigma'$ implies $\langle S, \sigma \rangle \rightarrow \sigma'$.* ■