Extending Knowledge Base Update into First-Order Knowledge Bases

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Plan of presentation

- Belief update operator
- Classical approach of Winslett (PMA)
- Modified approach of Doherty, Łukaszewicz and Madalińska (MPMA)
- The MPMA in first-order logic.

Belief update operator (Katsuno & Mendelzon, Winslett)

Given a knowledge base KB, representing the reasoner's belief set, and a piece of new information α, representing the *effect of a performed action*, determine the new reasoner's knowledge base $KB * \alpha$.

Note: belief update deals with dynamic environments in which new information reflects changes brought about by actions that have occurred.

Postulates (Katsuno & Mendelon, 1995)

(1) $KB \star \alpha$ implies α.

(2) If KB implies α, then $KB \star \alpha$ is equivalent to KB.

(3) If both KB and α are satisfiable, then $KB \star \alpha$ is also satisfiable.

(4) If $KB_1 \equiv KB_2$ and $\alpha_1 \equiv \alpha_2$, then $KB_1 \star \alpha_1 \equiv KB_2 \star \alpha_2$.

(5) $(KB \star \alpha_1) \wedge \alpha_2$ implies $KB \star (\alpha_1 \wedge \alpha_2)$.

(6) If $KB \star \alpha_1$ implies α_2 and $KB \star \alpha_2$ implies α_1, then $KB \star \alpha_1 \equiv KB \star \alpha_2$.

(7) If KB is complete, *i.e.* has at most one model, then $(KB \star \alpha_1) \wedge (KB \star \alpha_2)$ implies $KB \star (\alpha_1 \vee \alpha_2)$.

(8) $(KB_1 \vee KB_2) \star \alpha \equiv (KB_1 \star \alpha) \vee (KB_2 \star \alpha)$.

Example 1 Let $KB = p$ and $\alpha = p \vee q$.
Since $KB \models \alpha$, $KB * \alpha = KB$.

$p = heads$ and $q = tails$.

A belief update formalism, called PMA (*possible model approach*), satisfying postulates of Katsuno-Mendelzon was introduced by Winslett (1991).

Language of PMA

We start with a language $\mathcal{L}_{pma}$ of classical propositional logic based on a finite fixed set $ATM = \{p, q, r, \ldots\}$ of atoms and two truth constants $\top$ (truth) and $\bot$ (falsity).

If α and β are formulas and p is an atom, then we write $\alpha[p \leftarrow \beta]$ to denote the formula which is obtained from α by simultaneously replacing all occurrences of p by β.

A *literal* is an atom or its negation.

Interpretations are maximal consistent sets of literals.

For any formula α, we write $|\alpha|$ to denote the set of all *models* of α.

Modified PMA (MPMA)

PMA — Minimal change with respect to all atoms

MPMA — Minimal change with respect to a subset of atoms.

Question: Which atoms should be released from the process of minimization?

Answer: All non-redundant atoms of the update formula α.

Definition 1 Let α be a formula. An atom p occurring in α is said to be *redundant* for α iff $\alpha[p \leftarrow \top] \equiv \alpha[p \leftarrow \bot]$. ∎

An atom is redundant for a formula iff the logical value of the formula does not depend on the logical value of the atom.

Eliminants

Let p be an atom and suppose that α is a formula. We write $\exists p.\alpha$ to denote the formula $\alpha[p \leftarrow \top] \vee \alpha[p \leftarrow \bot]$. If $P = \{p_1, \ldots, p_n\}$ is a set of atoms and α is a formula, then $\exists P.\alpha$ stands for $\exists p_1 \cdots \exists p_n.\alpha$.

A formula $\exists P.\alpha$, where $P = \{p_1, \ldots, p_n\}$, is called an *eliminant of* $\{p_1, \ldots, p_n\}$ *in* α.

Intuitively, such an eliminant can be viewed as a formula representing the same knowledge as α about all atoms not in P and providing no information about the atoms in P.

Formal definition of MPMA

Definition 2 Let KB be a knowledge base, α be an update formula and suppose that P is the set of all non-redundant atoms occurring in α. Then

$$KB * \alpha \equiv \alpha \wedge \exists P.KB.$$

∎

Definition 2 shows that the MPMA works in three steps. First, we select the atoms that may vary their values when the action corresponding to the update formula α is performed. Next, we weaken the knowledge base KB by eliminating all those variable atoms. Finally, we strengthen $\exists P.KB$ by combining it with the update formula.

Relation between MPMA and logics for reasoning about action and change

- MPMA is a very simple form of Sandewall's temporal logic PMON, constructed to reason about action an change.

- MPMA can be reformulated in Dijkstra's semantics, originally developed to reason about programs, but also used to reason about action and change

First-order belief update

We deal with a first-order language with equality. Formulae are constructed in the usual way using sentential connectives $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$, quantifiers $\forall, \exists$, Boolean constants $\top$ (true), $\bot$ (false) and the equality sign $=$.

A sentence is a formula containing no free variables.

An *atom* is a formula of the form $P(\overrightarrow{t})$, where P is an n-ary predicate symbol and $\overrightarrow{t}$ is an n-tuple of terms. A *literal* is an atom or its negation. A literal is said to be *ground* if it contains no variables.

If $\overrightarrow{t} = (t_1, \ldots, t_n)$ and $\overrightarrow{t'} = (t'_1, \ldots, t'_n)$ are tuples of terms then $\overrightarrow{t} = \overrightarrow{t'}$ is an abbreviation for $t_1 = t'_1 \wedge \ldots \wedge t_n = t'_n$.

If A is a formula and $P(\overrightarrow{t})$ is a ground atom, then we write $A(P(\top/\overrightarrow{t}))$ (resp. $A(P(\bot/\overrightarrow{t}))$ to denote the formula obtained from A by replacing all occurrences of $P(\overrightarrow{t})$ by $\top$ (resp. $\bot$).

A **knowledge base:** a finite set of sentences over a fixed first-order language with equality.

We shall never distinguish between a knowledge base KB and the sentence being the conjunction of all its members.

An **update formula:** a Boolean combination of ground literals

Eliminants of ground terms in first-order logic

We write $SEP(A, P(\overrightarrow{t}))$ to denote the result of replacing each occurrence of the form $P(\overrightarrow{t'})$ in A by

$$[\overrightarrow{t} = \overrightarrow{t'} \wedge P(\overrightarrow{t})] \vee [\overrightarrow{t} \neq \overrightarrow{t'} \wedge P(\overrightarrow{t'})].$$

A and $SEP(A, P(\overrightarrow{t}))$ are equivalent.

Definition 3 An *eliminant* of a ground atom $P(\overrightarrow{t})$ in a first-order formula A, denoted by $\exists P(\overrightarrow{t}).A$, is the formula

$$SEP(A, P(\overrightarrow{t}))[\top/P(\overrightarrow{t})] \vee \\ SEP(A, P(\overrightarrow{t}))[\bot/P(\overrightarrow{t})].$$

∎

Definition 4 *Let A be a first-order formula and suppose that $\overrightarrow{P} = (P_1(\overrightarrow{t_1}), \cdots, P_n(\overrightarrow{t_n}))$ is an n-tuple of ground atoms. An eliminant of $P_1(\overrightarrow{t_1}), \cdots,$ $P_n(\overrightarrow{t_n})$ in A, written $\exists\overrightarrow{P}.A$ is*

$$\exists P_1(\overrightarrow{t_1}) \cdots \exists P_n(\overrightarrow{t_n}).A.$$

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Definition 5 Let a knowledge base KB be a closed first order formula and let update formula α be a Boolean combination of ground atoms. Denote by $ATM(\alpha)$ the set of all non-redundant atomic formulae appearing in α.

$$KB \star \alpha \equiv \exists\, ATM(\alpha).KB \wedge \alpha.$$

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Example 2 Suppose that there are at least two green objects in the world and the performed action is to paint a house h into red. That is,

$$KB = \{\exists x.\exists y.x \neq y \land Green(x) \land Green(y)\}$$
$$\alpha = Red(h) \land \neg Green(h).$$

Since Red does not occur in KB, $\exists Red(h).KB$ is equivalent to KB. Thus, there remains to eliminate $Green(h)$. It can be shown that

$$\exists Green(h).KB = \exists y h \neq y \land Green(y).$$

Thus

$$KB * \alpha = \exists y.h \neq y \land Green(y) \land Red(h) \land \neg Green(h).$$

This agrees with our intuition. ∎

Example 3 Assume that all objects in the considered world are blue and suppose that the performed action is to paint a house h into yellow. That is, $KB = \{\forall x.Blue(x)\}$ and $\alpha = Yellow(h) \wedge \neg Blue(h)$. Since $Yellow$ does not occur in KB, $\exists Yellow(h).KB$ is equivalent to KB. Thus, we have to eliminate $Blue(h)$ in KB.

$$\exists Blue(h).KB = \forall x.x \neq h \Rightarrow Blue(x).$$

Thus,

$$KB * \alpha = \forall x.x \neq h \Rightarrow Blue(x) \wedge \\ Yellow(h) \wedge \neg Blue(h).$$

Example 4 Suppose that there are at least two distinct objects: one is red and the other is green. The performed action is to paint a house h into yellow. That is, $KB = \{\exists x.\exists y.x \neq y \wedge Red(x) \wedge Green(y)\}$ and $\alpha = Yellow(h) \wedge \neg Red(h) \wedge \neg Green(h)$. Now we need to eliminate $Red(h)$ and $Green(h)$.

It can be shown that

$$\exists Red(h)\exists Green(h).KB = \exists x\exists y.x \neq y \wedge$$
$$(h = x \vee Red(x)) \wedge [y = h \vee Green(y)].$$

Thus

$$KB * \alpha = \exists x\exists y.x \neq y \wedge$$
$$(h = x \vee Red(x)) \wedge (y = h \vee Green(y)) \wedge$$
$$Yellow(h) \wedge \neg Red(h) \wedge \neg Green(h).$$

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Future work

- Integrity constraints

- Comparing first-order MPMA with reasoning about action paradigm (prerequisites of actions)

Example 5 My initial belief is that either Alice or Jane is in the office (but not both). Now I see Bob going out of the office. What do I believe now? ∎

Example 6 My initial belief is that either Alice or Jane is currently blond (but not both). Now I learn that Alice dyed her hair into red. What do I believe now? ∎