U-model Based Adaptive IMC for Nonlinear Dynamic Plants

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Abstract

A novel technique, involving U-model based IMC (Internal Model Control), is proposed for the adaptive control of nonlinear dynamic plants. The proposed scheme combines the robustness of the IMC and the ability of Neural Networks to identify arbitrary nonlinear functions, with the control-oriented nature of the U-model to achieve adaptive tracking of stable nonlinear plants. The proposed structure has a more general appeal than many other schemes involving polynomial NARMAX (Nonlinear Autoregressive Moving Average with Exogenous inputs) model and the Hammerstein model, etc. Additionally, the control law is shown to be more simplistic in nature. The effectiveness of the proposed scheme is demonstrated with the help of simulations for the adaptive control of the Hammerstein model.

1. Introduction

Adaptive control of uncertain nonlinear dynamic plants is currently an important area of research. Several adaptive control designs have been developed recently. An extensive discussion on most of these designs can be found in [1]. The adaptive control design generally involves two important choices:
1. Selection of an identification scheme
2. Selection of a control law based on the structure of the identifying scheme.

It is clear that the modelling part plays a very important role in the overall system design and performance. This is because the control law is actually based upon the modelling scheme. Researchers have considered situations where the model function is known but parameters are uncertain or unknown [1], [2], and also the ones where the system function is completely unknown. The latter case is of more interest because of its generality and has motivated the use of neural network approximators [3], [4].

It must be noted that the main difficulty in the design of nonlinear control systems is the lack of a general modelling framework which allows the synthesis of a simple control law [5]. In some instances linearizing structures have been used but these suffer from ”local applicability” and therefore, are not always very attractive. In the area of nonlinear modelling, NARMAX (Nonlinear Autoregressive Moving Average with Exogenous inputs) is the most commonly used structure. It has the ability to represent a broad range of nonlinear systems [6]. However because of its overly complex structure, the NARMAX does not lend itself to easy manoeuvring for controller design [7].

To simplify the control law synthesis part, a new control-oriented model termed as the U-Model has recently been suggested [5]. The U-model has a more general appeal as compared to the polynomial NARMAX model [8] and Hammerstein model. Additionally, this model is control-oriented in nature which makes the control synthesis part easier. Specifically, the control law based on the U-model exhibits a polynomial structure in the current input term. Based on the U-model a pole placement controller [5] and a new IMC (Internal Model Control) structure for dynamic nonlinear plants with known parameters [9] have recently been proposed. In this paper we extend the work in [9], and propose an adaptive IMC structure based on U-model for tracking of nonlinear dynamic plants. The main advantages of the proposed approach are its generality and simplistic control law. The proposed structure is depicted in the block diagram (figure 1), where the dynamic nonlinear plant is modelled as a U-model whose parameters are identified using Gaussian RBF (Radial Basis Filter) neural network and are used to synthesize a simple law for control inputs to the plant.

The rest of the paper has been organized as follows. Section 2 presents a basic discussion on the U-model, the Gaussian RBF and the IMC. A new adaptive IMC based on the U-model is proposed in section 3. To demonstrate the effectiveness of the proposed scheme, Simulink based simulations were carried out for the adaptive tracking of the Hammerstein model. These are discussed in Section 4.
The expression (3) is defined as the U-model. This model has the following advantages:

1. The control-oriented U-model is more general than other parameterizing approaches, such as the polynomial NARMAX model [8], the Hammerstein model etc.

2. The sampled data representation of many nonlinear continuous time systems can be of the form as follows:

\[ y(t) = \sum_{j=0}^{M} \alpha_j(t)u^j(t - 1) \]

3. The U-model exhibits a polynomial structure in the current control \( u(t - 1) \).

4. Due to its polynomial structure, the nonlinear algebraic equations, which need to be solved to obtain the output value of the controller, are also polynomials in \( u(t - 1) \), unlike other models which lead to complex non-linear algebraic equations.

2.2. Approximation function: Gaussian RBF neural network

Neural networks are parameterized nonlinear functions. Their parameters are, for instance, the weights and biases of the network. Adjustment of these parameters results in different shaped nonlinearities. Typically these adjustments are achieved by a gradient descent approach on an error function that measures the difference between the output of the neural network and output of the actual system. In other words, a neural network is adjusted to serve as an approximator for an unknown function that is only known by how it specifies output values for the given input values. Additionally there is no restriction on the unknown function to be linear. In this way, neural networks provide a logical extension to create nonlinear adaptive control schemes.

To understand the approximation task, consider a set of training samples \( \{x(i), y(i)\} \), with \( x(i) \) and \( y(i) \) as input and output vectors respectively. The purpose of function approximation is to identify a mapping from \( x \) to \( y \), that is, \( y = F(x) \) such that the expected sum of square approximation error \( E[|y - F(x)|^2] \) is minimized. To this end, Neural Networks have successfully been used as good approximators that require no knowledge of the function structure [3]. RBF is a well known neural network that makes use of the radial basis function. A radial basis function \( \varphi(||x - m_i||) \) has the same value for all neural inputs \( x \) that lie on a hypersphere with center \( m_i \), i.e., it exhibits radial symmetry. The RBF neural network is used in the following way: given a set of points \( \{y(k); 1 \leq k \leq K\} \) and the values of an unknown function evaluated on these \( K \) points \( \{y(k) = F(x(k)); 1 \leq k \leq K\} \), the RBF approximates...
\[ F(x) \text{ in the form} \]
\[ \hat{F}(x) = \sum_{i=1}^{C} w_i \varphi \left( \frac{\|x - m_i\|}{\sigma_i} \right) \]

This is a weighted linear combination (weights \( w_i \)) of a family of radial basis functions \( \varphi \left( \frac{\|x - m_i\|}{\sigma_i} \right) \) such that the sum of square approximation error at these sets of training samples

\[ \sum_{k=1}^{K} \left[ y(k) - \hat{F}(x(k)) \right]^2 \]

is minimized. Here \( m_i \) and \( \sigma_i \) represent the center and the width parameters, respectively. Several radial basis functions exist for use in the RBF neural networks. Among these, the Gaussian RBF is the most commonly used and is given by,

\[ \varphi(x) = \exp \left( \frac{\|x - \mu\|^2}{\sigma^2} \right) \]

In this case, the width parameter is the same as the standard deviation of the Gaussian function. One of the most attractive properties of the Gaussian RBF is its universal approximation capability [3].

2.3. Internal model control

IMC is one of the most popular control strategies used in industrial process control. Its main features are its simple structure, fine disturbance rejection capabilities and robustness [10]–[15]. IMC can be used for both linear and non-linear systems [16] and is especially suitable for the design and implementation of the open-loop stable systems. Many industrial processes happen to be intrinsically open-loop stable. A detailed analysis of the properties of IMC has been given in [17] Important characteristics of the IMC are summarized with the following properties.

1. Property P1 (Dual Stability): If the plant and the controller are input-output stable and the model is a perfect representation of the plant; then the closed-loop system is input-output stable.

2. Property P2 (Perfect Control): If the inverse of the operator describing the plant model exists, and this inverse is used as the controller, and the closed-loop system is input-output stable with this controller; then the control will be perfect.

3. Property P3 (Zero Offset): If the inverse of the steady state model operator exists, and the steady state controller operator is equal to this, and the closed-loop system is input-output stable with this controller. Then offset free control is attained for asymptotically constant inputs.

3. U-Model based adaptive IMC for nonlinear dynamic plants

In this section we propose a new adaptive IMC based on U-model for tracking of nonlinear dynamic plants with unknown structure. In the following we refer to the block diagram given in section 1 (figure 1).

3.1. The plant

We assume a stable nonlinear dynamic plant whose functional parameters or the functional structure need not be known.

3.2. The identifying model

We propose to identify the plant online using the radial basis nonlinear moving average model with the following structure:

\[ y_M(t) = a_1 u(t-1) + \hat{b}_1 \Phi(u(t-1)) \]
\[ + \hat{b}_2 \Phi(u(t-2)) + \ldots + \hat{b}_n \Phi(u(t-n)) \]  

(4)

Where the parameter \( a_1 \) is selected in advance and the parameters \( \hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n \) are estimated using the normalized least mean square algorithm. \( \Phi \) can be any function used in neural networks. Here we use the Gaussian radial basis function because of its characteristics outlined in section 2.2

3.3. The control law

To simplify the synthesis of the control law we introduce the equivalent U-model for the radial basis nonlinear model 4 as:

\[ y_M(t) = a_0(t) + a_1(t)u(t-1) \]  

(5)

where

\[ a_0(t) = \hat{b}_1 \Phi(u(t-1)) + \hat{b}_2 \Phi(u(t-2)) \]
\[ + \ldots + \hat{b}_n \Phi(u(t-n)) \]
\[ a_1(t) = a_1 \]

The controller output \( u(t-1) \) can now be found easily using the Newton-Raphson algorithm recursively with \( U(t) \) as a root solver as follows (see figure 1):

\[ u_{i+1}(t-1) = u_i(t-1) - \frac{\sum_{j=0}^{K} a_j(t)u_j(t-1) - U(t)}{\sum_{j=0}^{K} a_j(t)u_j(t-1) - \frac{dU(t)}{du(t-1)}} \]  

(6)

where the subscript \( i \) is the iteration index [5]. Using the U-model of (5) which is linear with respect to the
control term \( u(t - a) \) in (6), the controller has the simplified form as follows:

\[
u(t - 1) = \frac{U(t) - \alpha_0(t)}{\alpha_1(t)}
\]

(7)

It must be noted that the proposed scheme leads to a very simple and general control law. This approach is therefore expected to prove extremely useful in the area of nonlinear control.

### 3.4. System operation

As shown in figure 1, the output of the controller \( u(t) \) is fed to both the unknown plant and the radial basis nonlinear moving average filter. The mismatch error \( \varepsilon \) input to the filter is the difference between the output of the plant \( y_P(t) \) and the output of the radial basis nonlinear moving average filter \( y_M(t) \). The filter parameters are updated using normalized least mean square algorithm such that the error \( \varepsilon \) is minimized.

A copy of the filter parameters which are the parameters of the U-model is fed to the controller online and the controller calculates the inverse of the unknown plant using the Newton-Raphson method based on the U-model of the plant.

If the plant to be controlled is unstable then it is first stabilized using any known robust control techniques and then the controller scheme proposed here can be applied considering the entire stabilized system as an unknown plant to achieve tracking of the input reference signal. It must be noted that the proposed system uses the noise rejection capability of the IMC, the approximation power of the Gaussian RBF and the control-oriented nature of the U-model to achieve adaptive identification and tracking of nonlinear plants.

### 3.5. Stability of the proposed structure

Since the proposed structure is essentially IMC-based we use the dual stability property (section 2.3) of the IMC to discuss the stability of the proposed structure. The dual stability condition requires that

1. **Plant and Controller be input output stable**
2. **Model must be a perfect representation of the plant**

Since in our case

1. **The plant is assumed to be either stable or to have been stabilized first**
2. **The Gaussian RBF possesses the universal approximation property and is therefore capable of approximating the plant [3]**

Therefore, the proposed closed loop system is input-output stable.

### 3.6. Simulations

To demonstrate the application of the proposed scheme, we carried out simulations on the following nonlinear Hammerstein model

\[
y(t) = 0.5y(t - 1) + x(t - 1) + 0.1x(t - 2)
\]

\[
x(t) = 1 + u(t) - u^2(t) + 0.2u^3(t)
\]

The system was modelled according to (4), and then its equivalent U-model (5) was used to synthesize the control law (7). The first parameter \( a_1 \) was selected as 5, while the number of linear combination weights was four \((\hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4)\) All weights were initialized to 0 and the step size was chosen to be 0.1. The system performed well. Adaptive tracking was achieved and the results are given in figures 2, 3, 4 and 5.

![Figure 2. Tracking](image1.png)

![Figure 3. Control Input](image2.png)

### 4. Conclusions

A novel technique, involving U-model based IMC, was proposed for the adaptive control of nonlinear dynamic plants. The robustness of the IMC and the ability of Neural Networks to identify arbitrary nonlinear functions, were combined with the control-oriented nature of the U-model to achieve adaptive tracking of stable nonlinear plants. The effectiveness of the proposed scheme was demonstrated with the help of simulations for the adaptive control of the
Figure 4. Difference Between Plant and Model Outputs

Figure 5. Parameter Estimates

Hammerstein model. It is noted that the current contribution is expected to prove extremely useful in the area of nonlinear control.

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References


