Abstract

The solid oxide fuel cell power plant is known to be a potential alternative in the electric utility. However, the output voltage of the solid oxide fuel cell changes with the load variations. Model predictive control is part of a family of optimization-based control methods, which are based on online optimization of future control moves. This paper proposes a model-based controller for the regulation of a solid oxide fuel cell. The performances using both the linear and fuzzy Hammerstein models are evaluated with constraints.

1. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$C_p$</td>
<td>heat capacity of the cell unit (J/kg K)</td>
</tr>
<tr>
<td>$E_0$</td>
<td>standard reversible cell potential (V)</td>
</tr>
<tr>
<td>$F$</td>
<td>Faraday’s constant ($C/mol$)</td>
</tr>
<tr>
<td>$R_{an}$, $R_{cd}$</td>
<td>anode (cathode) total rate of production of species ($mol/s$)</td>
</tr>
<tr>
<td>$r$</td>
<td>ohmic resistance ($\Omega m^2$)</td>
</tr>
<tr>
<td>$T_0$</td>
<td>temperature constant (K)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>stack solid average temperature (K)</td>
</tr>
<tr>
<td>$V$</td>
<td>compartment volume ($m^3$)</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>cell voltage (V)</td>
</tr>
<tr>
<td>$V_o$</td>
<td>volume of the cell unit ($m^3$)</td>
</tr>
<tr>
<td>$V_o$</td>
<td>open-circuit reversible potential (V)</td>
</tr>
<tr>
<td>$x_i$</td>
<td>mole fractions of species</td>
</tr>
<tr>
<td>$x_{ai}, x_{ci}$</td>
<td>anode (cathode) inlet mole fractions</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>electron transfer coefficient of the reaction at the electrode</td>
</tr>
<tr>
<td>$\alpha_r, \beta_r$</td>
<td>ohmic resistance constants</td>
</tr>
<tr>
<td>$\eta_{act}$</td>
<td>activation losses (V)</td>
</tr>
<tr>
<td>$\eta_{con}$</td>
<td>concentration losses (V)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>total gas components in anode or cathode</td>
</tr>
<tr>
<td>$\tau_{H2}$</td>
<td>time constant associated with the hydrogen flow and is a function of temperature (s).</td>
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2. Introduction

The main features of the solid oxide fuel cell (SOFC) are all solid-state construction and high-temperature operation. The combination of these features leads to a number of unique characteristics and advantages for this type of fuel cell, including flexibility in cell and stack designs, manufacturing processes, and power plant sizes.

Padullés et al. [1] develop a SOFC model which includes species dynamics, but it does not consider temperature dynamics. Hall and Colclaser [2] modeled a 3-kW SOFC but they did not take into account dynamics of the chemical species. Achenbach develops a mathematical model of a planar SOFC, which concentrates on effects of temperature changes on output voltage response [3]. Temperature dynamics is modeled in a three-dimensional (3-D) vector space. The same author investigated the transient behavior of a stand-alone SOFC caused by a load change in [4]. It shows that the relaxation time of the output voltage is highly related to the effect of temperature dynamics. Sedghisigarchi and Feliachi present a model based on electrochemical and thermal
equations [5]. The application of the AutoRegression with eXogenous signal (ARX) identification algorithm to compute linear system models is presented in [6]. The Nonlinear AutoRegressive Exogenous (NARX) approach is used in [7] to analyze the dynamics of this fuel cell.

For nonlinear dynamic systems, the conventional techniques of modeling and identifications are difficult to implement and sometimes impracticable. However, others techniques based on fuzzy logic are more and more used for modeling this kind of process [8]. Among the different fuzzy methods, the Takagi-Sugeno model has attracted most attention [9]. In fact, this model consists of if-then rules with fuzzy antecedents and mathematical functions in the consequent part. The task of system identification is to determine both the nonlinear parameters of the antecedents and the linear parameters of the rules consequent. A fuzzy logic control of three-phase inverters for fuel cell systems is presented in [10].

Takagi-Sugeno fuzzy models are suitable to model a large class of nonlinear systems [11], [12]. Fuzzy modeling and identification from measured data are effective tools for the approximation of uncertain nonlinear systems. Most attention has been devoted to Single-Input, Single-Output (SISO) or Multi-Input, Single-Output (MISO) systems. Recently, also methods have been proposed to deal with Multi-Input, Multi-Output (MIMO) systems [13]-[15].

The Hammerstein models are special kinds of nonlinear systems where the nonlinear block is static and is followed by a linear system. These models have applications in many engineering problems and therefore, identification of Hammerstein models has been an active research area for many years. There exist a large number of research papers in the literature on the topics of Hammerstein model identifications [16]-[19].

In this paper, a fuzzy Hammerstein (FH) model to represent SOFC is introduced where the static nonlinearity is represented by a fuzzy model. Therefore, the model can be identified with the help of input-output data.

Model predictive control (MPC) has been an active field of research during the last three decades, driven both by numerous successful applications of the technology [20]-[22] and by the research interests of the academia. The main reason of this success is the ability of MPC to control multivariable systems under constraints in an optimal way. In model predictive control, the control action is computed by solving an optimization problem on line in each sampling period. This is the main difference from conventional control, where a precomputed control law is employed. Many applications of MPC based on fuzzy prediction models have been reported [23]-[26].

The paper is organized as follows. In Section 3, general principles of SOFC are explained. In Section 4, the FH model is introduced. The MPC is formulated in Section 5. Simulation examples illustrating the performance of SOFC are presented in Section 6, and finally, conclusions are provided in Section 7.

3. Solid Oxide Fuel Cell Dynamic Model

The proposed model is based on the following assumptions.
1) The SOFC model is fed with hydrogen and air, therefore the fuel processor dynamics is not considered.
2) A uniform gas distribution among cells is supposed.
3) All of cells have the same temperature and current density [1].
4) The channels that transport gases along the electrodes have a fixed volume, but their lengths are small, so that it is only necessary to define one single pressure value in their interior.

Fig. 1 shows the suggested model of SOFC.

![Figure 1. Complete model of SOFC.](image)

3.1. Electrochemical model

The variation in concentration of each species can be defined as follows [28], [29]:

$$\frac{V}{RT} \frac{d}{dt} p_i = N_i^{in} - N_i^{O} - N_i^{r}$$

(1)

In agreement with the basic electrochemical relationships, the molar flow that reacts can be calculated as:

$$N_i^{r} = \frac{N_i^{O}}{2F} = 2K_r I$$

(2)

The cell utilization is specified using

$$u = \frac{N_i^{O}_{n_2}}{N_i^{n_2}}$$

(3)

For orifice that is choked [27], molar flow of any gas through the valve is proportional to its partial pressure inside the channel according to the following expressions [1]:
Considering the hydrogen partial pressure,

\[ \frac{V}{RT} \frac{d}{dt} P_{H_2} = N_{H_2}^0 - N_{H_2}^- - 2K_I \]

(5)

Applying the Laplace transformation to the above equations, yields the following formulations:

\[ p_{H_2} = \frac{1}{K_H} + \frac{S}{\tau_{H_2}} \left( N_{H_2}^0 - 2K_I \right) \]

(6)

\[ \tau_{H_2} = \frac{V}{K_{H_2}RT} \]

(7)

3.2. Thermal model

The fuel cell power output is closely related to the temperature of the cell unit. The heat storage in the thin fuel unit gas or oxidant gas layer is neglected. The thin fuel unit or oxidant gas layers are lumped to the cell unit [2]-[4].

The energy balance equation for each cell unit is as follows:

\[ M^o C^o \frac{dT_1}{dt} = N_{H_2}^0 \left( \sum_{i=1}^{N_u} \frac{\dot{q}_R}{T_R} \right) - \sum_{i=1}^{N_u} \frac{\dot{q}_{R_i}}{T_R} R_{a_i} + \sum_{i=1}^{N_u} \frac{\dot{q}_{R_i}}{T_R} R_{a_i} - P_{ac} \]

(8)

Under the ideal gas supposition the partial molar enthalpies are calculated using

\[ R = R^o + \int_{T_L}^{T_R} c_p(u) du \]

(9)

and coefficients of the specific heats \( c_{p,i} \),

\[ c_{p,i} = a_i + b_i T + c_i T^2 + d_i T^3 \]

(10)

are encountered in standard reference tables. The reference enthalpy stands for energy at standard reference temperature and considers the heat of formation for each gas species to account for energy change on chemical reaction.

3.3. Nernst’s Equation

The stack is connected in series. Applying Nernst’s equation and Ohm’s law, the stack output voltage is [28]-[30]:

\[ V_{ac} = V_o - \eta_{act} - \eta_{cone} \]

(11)

\[ V_o = N_{ac} \left( E^o + \frac{RT}{2F} \ln \frac{x_{H_2} X_{O_2}^{0.5}}{x_{H_2}^0} \right) \]

(12)

Concentration loss equation is given by [2], [28]-[30]:

\[ \eta_{con} = \frac{RT}{nF} \ln \left( 1 - \frac{i}{i_0} \right) \]

(13)

Activation loss equation is as follows [29]:

\[ \eta_{act} = \frac{RT}{\alpha nF} \ln \left( \frac{i}{i_0} \right) \]

(14)

\( \alpha \) is the transfer coefficient, which is considered to be the fraction of the change in polarization that leads to a change in the reaction rate constant and its value is usually 0.5 for the fuel cell application.

Tafel plots provide a visual understanding of the activation polarization of a fuel cell. They are used to measure the exchange current density, given by the extrapolated intercept at \( \eta_{act} = 0 \) which is a measure of the maximum current that can be extracted at negligible polarization, and the transfer coefficient (from the slope).

The usual form of the Tafel equation that can be easily expressed by a Tafel Plot is

\[ \eta_{act} = a + b \ln i \]

(15)

where \( a = \left( -RT/\alpha nF \right) \ln i_0 \) and \( b = RT/\alpha nF \).

The term \( b \) is called the Tafel slope, and is obtained from the slope of a plot of \( \eta_{act} \) as a function of \( \ln i \).

The ohmic losses are dependent on the arithmetic average of cathode inlet and exit temperature. The ohmic resistance is expressed by [30]:

\[ r = \alpha \exp \left[ \beta \left( \frac{1}{T} - \frac{1}{T_0} \right) \right] \]

(16)

4. Fuzzy Hammerstein model

The fuzzy Hammerstein model consists of a series connection of a memoryless nonlinearity, \( f \), and linear dynamics, \( G \), where \( y = [y_1, \ldots, y_n]^T \) is the output vector, \( u = [u_1, \ldots, u_n]^T \) the input vector, and \( v = [v_1, \ldots, v_n]^T \) represents the transformed input variables [31].

If the static nonlinearity is separately parameterized, \( f (\cdot) \) can be formulated as a set of functions \( v_h = f_h (u) \) for \( h = 1, \ldots, n_u \). In this paper, the functions \( f_h (u) \) are represented by zero-order Takagi-Sugeno fuzzy models formulated as a set of
From a given input vector, \( \mathbf{u} \), the output of the fuzzy model, \( v_h \), is inferred by computing the weighted average of the rule consequents:

\[
\hat{v}_h = \frac{\sum_{j=1}^{N_h} \beta_j(\mathbf{u}) p_j}{\sum_{j=1}^{N_h} \beta_j(\mathbf{u})}
\]  

(18)

The static nonlinearity is followed by a multivariable linear dynamic ARX model. Hence, the Nonlinear AutoRegressive Moving Average with eXogenous input model (NAARX) representation of the MIMO Hammerstein model is given by

\[
\hat{y}(k) = \sum_{i=1}^{n_y} A_i y(k-1) + \sum_{j=1}^{n_u} B_j f(u(k-i-n_d))
\]  

(19)

where \( y(k), \ldots, y(k-n_y+1) \) and \( u(k-n_d), \ldots, u(k-n_d-n_u+1) \) are the lagged outputs and inputs of the linear dynamic system, where \( n_y \) and \( n_u \) denote the maximum lags for the past outputs and inputs, and \( n_d \) is the discrete time delay. \( A_1, \ldots, A_{n_y} \) and \( B_1, \ldots, B_{n_u} \) are \( n_x \times n_y \) and \( n_x \times n_u \) matrices, respectively.

Hence, a compact form of the fuzzy Hammerstein model that represents a SISO process is formulated as,

\[
\hat{y}(k) = \sum_{i=1}^{n_y} a_i y(k-1) + \sum_{j=1}^{n_u} b_j \sum_{p=1}^{N} \beta_p(u(k-i-n_d)) p_j
\]

\[
= \sum_{i=1}^{n_y} a_i y(k-1) + \sum_{j=1}^{n_u} \sum_{p=1}^{N} b_j \beta_p(u(k-i-n_d))
\]

(20)

\( a, b \) are the linear parameters, while the parameters \( p \) belonging to the fuzzy model, are called the nonlinear parameters. The structure of the resulting model is shown in Fig. 2, where \( q \) denotes the shift operator, i.e.,

\[
u(k)q^{-1} = u(k-1).
\]

5. Model Predictive Control

5.1. Theoretical background

Model Predictive Control (MPC) refers to a class of control algorithms in which a dynamic process model is used to predict and optimize system performance.

![Figure 3. Strategy of model predictive controller.](image)

MPC is rather a methodology than a single technique. The methodology of controllers belonging to the MPC family is characterized by the following strategy illustrated in Fig. 3.

As shown in Fig. 3, in MPC, the future outputs (fuel cell voltage) for a determined prediction horizon \( H_p \) are predicted at each instant \( k \) using a prediction model. These predicted outputs \( \hat{y}(k+1), j = 1, \ldots, H_p \) depend on the state of the model at the current time \( k \) (given, for instance, by the past inputs and outputs) and on the future control signals \( u(k+j) \).

The control signal (fuel flow) change only inside the control horizon, \( H_c \), remaining constant afterwards,

\[
u(k+j) = u(k+H_c-1), \quad j = H_c, \ldots, H_p
\]

(21)

The set of control signals is calculated by optimizing a cost function in order to keep the process as close as possible to the reference trajectory (fuel cell voltage reference). \( \omega(k+j), j = 1, \ldots, H_p \). This criterion usually requires a quadratic function of the errors between the predicted output signal and the reference trajectory. The control effort is included in the objective function in most cases. An explicit solution can be obtained if the criterion is quadratic, the model is linear and there are no constraints. Otherwise an iterative optimization method has to be used.

In practice all systems are subject to restrictions. The actuators have a limited field of action, as in the case of
valves. Constructive reasons, safety or environmental ones can cause limits in the system variables such as fuel flow or maximum temperatures and pressures. All of them lead to the introduction of constraints in the MPC problem.

Usually, input constraints like

$$u_{\text{min}} \leq u(k+j) \leq u_{\text{max}}, \quad j = 1, ..., H_c$$  \hspace{1cm} (22)

$$\Delta u_{\text{min}} \leq \Delta u(k+j) \leq \Delta u_{\text{max}}, \quad j = 1, ..., H_c - 1$$  \hspace{1cm} (23)

are hard constraints in the sense that they must be satisfied. Conversely, output constraints can be viewed as soft constraints because their violation may be necessary to obtain a feasible optimization problem:

$$y_{\text{min}} \leq y(k+j) \leq y_{\text{max}}, \quad j = j_1, ..., H_p$$  \hspace{1cm} (24)

where \(j_1\) represents the lower limit for output constraint enforcement.

5.2. Linear model based predictive control

The basic idea is to use the linear model to predict the future system behavior. This model is used throughout the entire prediction horizon. Even if this model is very accurate at the linearization point, its accuracy decreases over the prediction horizon. As a consequence, there may be a significant prediction error at \(k + H_p\).

5.3. Fuzzy Hammerstein model based predictive control

Due to the relatively simple block-oriented structure, the application of Hammerstein models in MPC is more straightforward than the application of the general NARX or NAARX models. In this section, the FH model is implemented in MPC by inverting the fuzzy model that represents static nonlinearity [32]. As the remaining part of the prediction model is the linear dynamic part of the FH model, the MPC optimization can be solved by quadratic programming.

The combination of the inverse fuzzy model and the nonlinear system results in a transformed dynamical system. This system is linear if the system is of the Hammerstein type and the static nonlinearity is identical to the fuzzy model.

As the inversion of the single-input single-output and multiple-input single-output fuzzy model is a straightforward analytical procedure, the computational demand of the controller is quite comparable to the linear generalized predictive control (GPC). This is a significant advantage compared to other nonlinear models which require the use of nonlinear programming or linearization techniques.

In order to cope with the model-plant mismatch and also with disturbances (load changes), the internal model control (IMC) scheme [33] is used. The resulting scheme is depicted in Fig. 4.

5.4. Optimization

In general, the GPC algorithm computes the control sequence \(\{\Delta u(k+j)\}_j, j = 1, ..., H_c\), such that the following quadratic cost function is minimized:

$$J(H_{pl}, H_{pe}, H_c, \lambda) = \sum_{j=H_{pl}}^{H_{pe}} (\omega(k+j) - \tilde{y}(k+j))^2 + \lambda \sum_{j=1}^{H_c} \Delta u^2(k+j-1)$$  \hspace{1cm} (25)

Here, \(\tilde{y}(k+j)\) denotes the predicted system output, \(\omega(k+j)\) the modified setpoint that is assumed to be known in advance, \(H_{pl}\) is the minimum costing horizon, \(H_{pe}\) is the maximum costing or prediction horizon, \(H_c\) is the control horizon, and \(\lambda\) is the move suppression coefficient.

6. Results

6.1. Fuzzy Hammerstein model

The inputs to the FH model are the average temperature, the fuel flow, the air flow, and the current. The output of the FH model is the voltage. The average temperature is taken as the arithmetic average of that of the cathode inlet and cathode exhaust, under constant air flow.

The linear subsystem in the Hammerstein model is represented using the rational orthonormal bases with fixed poles studied in [34], [35],

$$B_l(q) = \sqrt{\frac{1 - k_l^2}{q - \xi_l}} \prod_{l = 0}^{l-1} \left(\frac{1 - \xi_l q}{q - \xi_l}\right) \quad l \geq 1$$  \hspace{1cm} (26)

$$B_0(q) = \sqrt{\frac{1 - k_0^2}{q - \xi_0}}$$  \hspace{1cm} (27)
where \((\zeta_0, \zeta_1, \ldots, \zeta_{n-1})\) are the poles of the bases.

In order to determine the model order of the linear subsystem, as well as initial guesses for the location of the poles of the bases, the same input–output data are used to identify a linear model of the process using a subspace method. System Identification Toolbox for use with MATLAB [36] is used for the identification of the linear model. As a result of the identification process a fourth order model is estimated as the linear part of the Hammerstein model.

In this paper the rules of the fuzzy system are designed based on the available a priori knowledge and the parameters of the membership, and the consequent are adapted in a learning process based on the available input-output data.

For a good model performance, the antecedent fuzzy sets on the input variables are designed. Fig. 5 shows the membership functions corresponding to temperature, current and fuel flow. The nominal operating conditions of the column considered in this example are given in Table 1 [37], [38].

| Table 1. Operating point data. |
|---------------------|-----------------|
| **Power**          | 100 kW          |
| **Stack voltage**  | 286.3 V         |
| **Stack current**  | 300 A           |
| **Number of cells**| 384             |
| **Number of stacks**| 1               |
| **Open circuit voltage for each cell** | 0.935 V          |
| **Input fuel flow** | 1.2 \(10^{-5}\) kmol s\(^{-1}\) |
| **Input air flow** | 2.4 \(10^{-3}\) kmol s\(^{-1}\) |
| **Cell area**      | 1000 cm\(^2\)   |
| **Cell temperature** | 1000 °C        |
| **Transfer coefficient** \(\alpha\) | 0.5             |
| **Ohmic resistance constant** \(\beta\) | -2870           |
| **Ohmic resistance constant** \(\alpha_r\) | 0.2             |
| **Temperature constant** \(T_0\) | 923 K           |
| **Limiting current** | 0.8 A/m\(^2\)   |
| \(K_{H2}\)        | 8.43 \(10^{-4}\) kmol/(atm s) |

6.2 Model based predictive control

The amount of fuel flow can be controlled according to the current. The current is proportional to the terminal load.

It is evident that an increase in the load growths current, which in turn decreases output voltage of the fuel cell. The increase in current makes fuel flow rate bigger. This increase becomes power flow bigger from the SOFC to the load.

Fig. 6 shows the load step changes. Fig. 7 represents the output current due to these changes in the load.

Two different controller configurations are used to illustrate the performance of the SOFC.

• **Linear MPC using a model obtained by ARX identification of the complete system model at nominal operating conditions** [6].

• **Fuzzy Hammerstein MPC.**

The MATLAB implementation of quadratic programming is used [39]. The MPC parameters are selected according to the tuning rules given in [40]. The two methods use identical control parameters.

The prediction horizon is set at a sufficiently large value (50 sampling periods, i.e. 500 s) in such a way that the system steady-state response is partly present in the predicted system response. The control horizon is set to one sampling period. The use of shorter prediction horizons would produce manipulated variable variations mainly on the basis of the high frequency process dynamics, while with longer prediction horizons the controller would tend to behave like a steady-state controller. Furthermore, a longer control horizon would have dramatically increased the computational burden and made the controller still more difficult to implement in a real-time application because it would have been accompanied by an increased number of optimization variables.
The focus of the following simulations is to contrast the performance of the MPC framework using the Hammerstein model and the linear model with the complete SOFC model. The performances using both the linear and Hammerstein models are evaluated with constraints. Fig. 8 shows the output voltage response of the SOFC due to the change in fuel flow input. Comparing the responses, one will notice that the Hammerstein model yields a significantly better closed-loop response.

Limiting fuel flow to a specific value produces a fixed output power. Output power will have a time delay in following the load power. This is due to the SOFC time constants.

7. Conclusions

This paper has described a model based controller (MPC) for the regulation of a SOFC.

The first step in designing an MPC system is the derivation of a model that the controller will use for the optimization. This model should be as accurate as possible, while being simple enough to allow for repeated calculations during the optimization.

An MPC is used to control the output voltage of the SOFC by manipulating the fuel flow to keep the voltage at a desired value. The performance of the closed-loop system improves when the Hammerstein model is used instead of the linear model.

References


